CLASSICAL MECHANICS

$\left\{ \begin{matrix} \text{States} \\ \text{of system} \end{matrix} \right\} \longleftrightarrow M$ smooth manifold

$\{ \text{Observables} \} \longleftrightarrow C^\infty(M, \mathbb{R})$

NOTE: $\mathcal{A} := C^\infty(M, \mathbb{R})$ is associative, commutative algebra.

FURTHER NOTE: $C^\infty(M, \mathbb{R})$ may also have a Poisson bracket

1. (AS) $\{f, g\} = -\{g, f\}$
2. Derivation $\{f, \{gh\}\} = \{f, g\}h + g\{f, h\}$
3. Jacobi $\{f, \{g, h\}\} + \text{cyclic} = 0$

QUANTUM MECHANICS

$\left\{ \begin{matrix} \text{States} \\ \text{of system} \end{matrix} \right\} \longleftrightarrow \mathcal{H}$ Hibert space

$\left\{ \text{Observables} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{Self-adjoint} \\ \text{operators on } \mathcal{H} \end{matrix} \right\}$

COMMUTATION RULES

eg $\quad [Q, P] = i\hbar$

for position, momentum operators

DIRAC QUANTIZATION

- Start with classical system M, $C^\infty(M,\mathbb{R})$ with Poisson bracket

- Quantizing means
  "to any classical observable f
  associate a quantum observable $\hat{f}$"
  with new, non-commutative product $*$

REQUIREMENTS:

- $\hat{f} * \hat{g} = \widehat{f \cdot g} + o(\hbar)$

- $\hat{f} * \hat{g} - \hat{g} * \hat{f} = i\hbar \, \widehat{\{f, g\}} + o(\hbar^2)$

TERMINOLOGY "Original product is deformed in the direction of the Poisson bracket"

FORMAL DEFORMATIONS

- Let A associative algebra
- A formal deformation of A

  is an associative multiplication $*$

  on $A[[\lambda]]$ st:

$$\boxed{x * y = \sum_{r=0}^{\infty} \mu_r(x,y)\lambda^r}$$

- REQUIREMENTS:
  - μ_0 = original product
  - μ_r are bilinear : $A \times A \to A$