CLASSICAL MECHANICS

\{ \text{States of system} \} \longleftrightarrow M \quad \text{smooth manifold}

\{ \text{Observables} \} \longleftrightarrow C^\infty(M,\mathbb{R})

**Note:** \( A := C^\infty(M,\mathbb{R}) \) is associative, commutative algebra.

**Further Note:** \( C^\infty(M,\mathbb{R}) \) may also have a Poisson bracket

1. (AS) \[ \{ f, g \} = - \{ g, f \} \]
2. Derivation \[ \{ f, \{ g, h \} \} = \{ f, g \} h + g \{ f, h \} \]
3. Jacobi \[ \{ f, \{ g, h \} \} + \text{cyclic} = 0 \]
QUANTUM MECHANICS

\{ \text{States of system} \} \rightarrow \mathcal{H} \quad \text{Hilbert space}

\{ \text{Observables} \} \leftarrow \{ \text{Self-adjoint operators on } \mathcal{H} \} \quad \text{Commutation Rules}

\text{eg} \quad [Q, P] = i\hbar

\text{for position, momentum operators}
DIRAC QUANTIZATION

1. Start with classical system $M, \mathcal{C}^{\infty}(\mathcal{M})$ with Poisson bracket.

2. Quantizing means "to any classical observable $f$ associate a quantum observable $\hat{f}$" with new, non-commutative product $\ast$.

REQUIREMENTS:

1. $\hat{f} \ast \hat{g} = \hat{f}g + o(\hbar)$

2. $\hat{f} \ast \hat{g} - \hat{g} \ast \hat{f} = i\hbar \{f, g\} + o(\hbar^2)$

TERMIOLOGY "Original product is deformed in the direction of the Poisson bracket"
Formal Deformations

Let $A$ be an associative algebra.

A formal deformation of $A$ is an associative multiplication $*$ on $A[[x]]$ such that:

$$x*y = \sum_{r=0}^{\infty} \mu_r(x,y) x^r$$

Requirements:

- $\mu_0 = \text{original product}$
- $\mu_r$ are bilinear: $A \times A \to A$