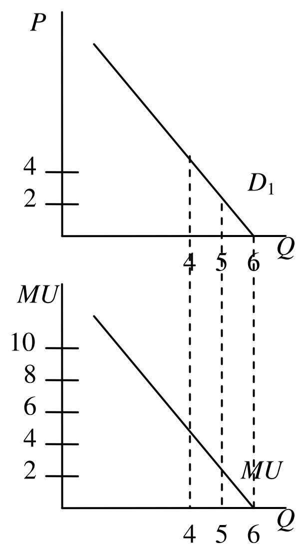
Utility Theory

One Good Utility Theory

- "Cardinal" measure of utility – precise, accurate, numerical measurement.
- Assume one good *X*:
  - Total Utility = $\text{TU}(X)$.
  - Marginal Utility = $\text{MU}(X) = \dfrac{\Delta\,\text{TU}(X)}{\Delta X}$.
- Diminishing marginal utility – "Can get too much of a good thing"..

Example: Deriving Demand



- MU drives D – diminishing marginal utility leads to a negatively sloped D.
- Constant marginal utility of money – constant value everywhere.

Consumer Surplus

- The difference of the value of the commodity and how much the consumer spend on it.
- Consumer surplus comes about because every unit is the same price, but consumers value each unit differently.

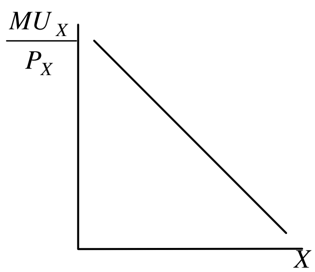
Example

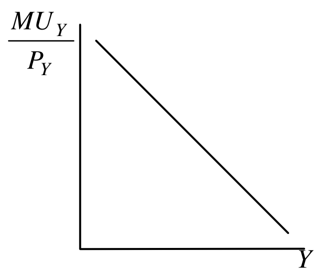
Given $p = 12 - 2q$, at $p = 2$, $CS = \frac{1}{2}(5)(12-2) = 25$.

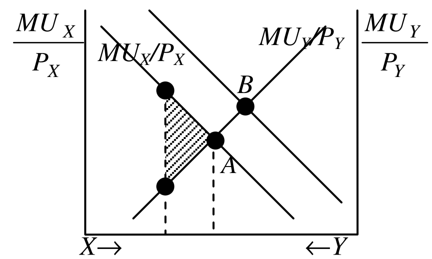
- Note the $5 difference is due to the extra area from the triangles.

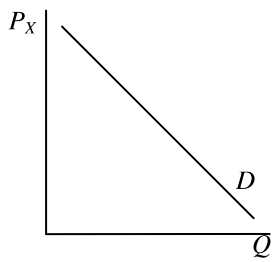
Two Good utility Function

- $\text{TU}(X, Y) = \text{TU}(X) + \text{TU}(Y)$ – *X* and *Y* are not related goods.
- Equilibrium: $\dfrac{MU_X}{P_X} = \dfrac{MU_Y}{P_Y}$









- $\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$.
- If $P_X\downarrow$, then $Q_X\uparrow$.

Indifference Theory

INDIFFERENCE CURVES

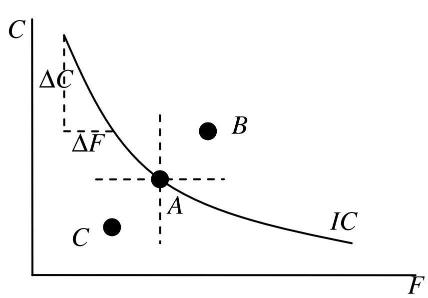
Definition

All points with the same level of satisfaction.

Assumptions

- Consumers are able to rank – given A and B, they are able to pick the preferred one.
- Transitivity/rational/consistent – if A > B > C, then A > C.
- More is preferred to less – $MU > 0$.

Derivation

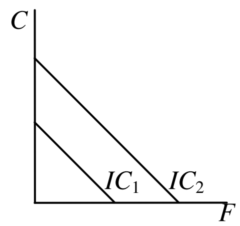


- *B* is preferred to *A* (more of both goods) and *A* is preferred to *C* (more of both goods), so *B* and *C* are not on *IC* – *IC* negatively sloped.
- $\frac{\Delta C}{\Delta F} < 0 \Rightarrow C\uparrow, F\downarrow$.
- Typically non-linear:
  ΔTU due to $\Delta C + \Delta TU$ due to $\Delta F = 0$

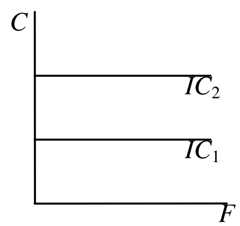
$$\Delta C\left(\frac{\Delta TU}{\Delta C}\right) + \Delta F\left(\frac{\Delta TU}{\Delta F}\right) = 0$$
$$\Delta C \cdot MU_C + \Delta F \cdot MU_F = 0$$
$$\frac{\Delta C}{\Delta F} = -\frac{MU_F}{MU_C}$$

- *MRS*: Willingness to trade on *IC* (holding *TU* constant) – $MRS_{F/C} = \left|\frac{MU_F}{MU_C}\right|$.
- *IC* convex because of marginal utility – diminishing marginal rate of substation.
- *IC*'s can't cross – there is one and only one *IC* through any point.

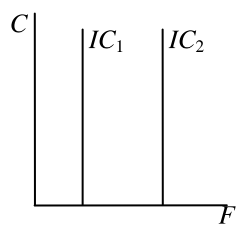
Special Cases



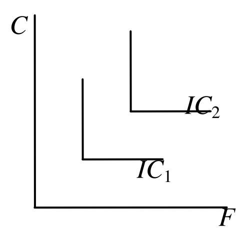
F and *C* are perfect substitutes.
- *MRS* constant.
- *IC* is linear – slope of *IC* = *MRS*.



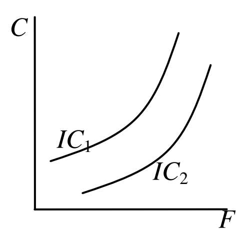
F is a "neutral" good (adds no satisfaction).
- *IC* is horizontal.



C is a "neutral" good (adds no satisfaction).
- *IC* is vertical.

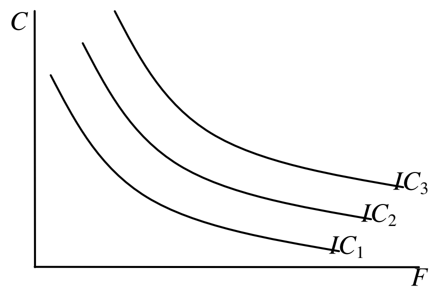


F and *C* must be consumed in fixed proportions.
- Increases in only one good adds no satisfaction.



C is a "bad" ($MU_C < 0$).
- Additional units of *C* will decrease satisfaction.
- $IC_2 > IC_1$.

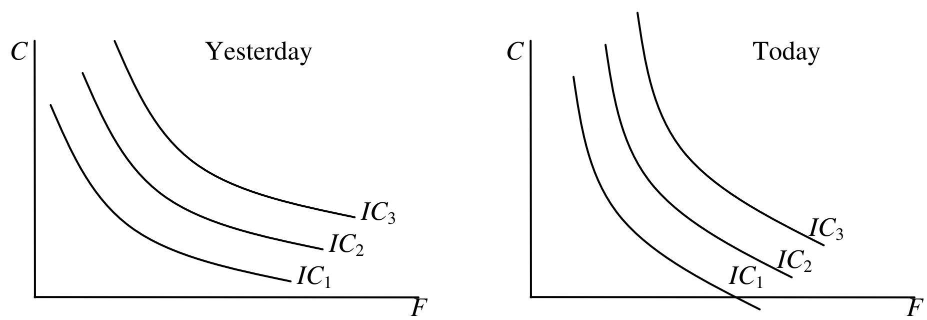
INDIFFERENCE MAP



- In a typical indifference map, higher *IC* represents higher level of satisfaction.

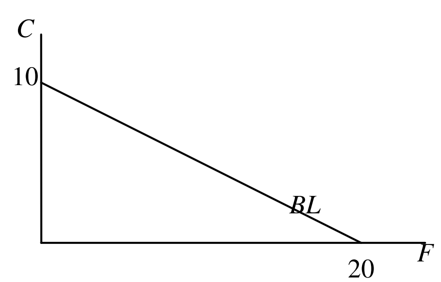
Change in Taste

Suppose more *F* is preferred (MU_F increases).



- Since $MRS = \left|\frac{MU_F}{MU_C}\right|$, increase in MU_F means steeper *IC*'s.
- $MRS_{\text{Today}} > MRS_{\text{Yesterday}}$.

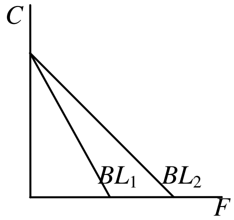
Budget Constraint



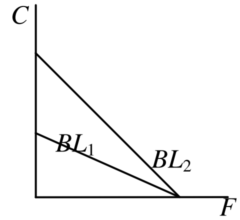
Suppose $I = \$100$, $P_F = \$5$, $P_C = \$10$.

- $I = P_F \cdot F + P_C \cdot C$, so $\$100 = \$5F + \$10C$.
- *C* intercept: $\frac{I}{P_C} = \frac{\$100}{\$10} = 10$.
- *F* intercept: $\frac{I}{P_F} = \frac{\$100}{\$5} = 20$.
- Slope of *BL*: $-\frac{\frac{I}{P_C}}{\frac{I}{P_F}} = -\frac{P_F}{P_C} = -\frac{1}{2}$.

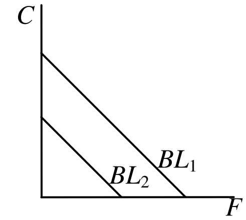
Changes to *BL*



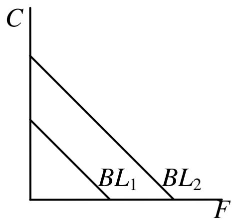
P_F increases.



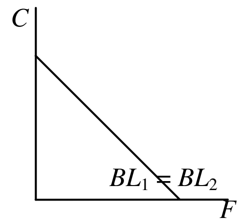
P_C decreases.



P_F and P_C increase proportionally.

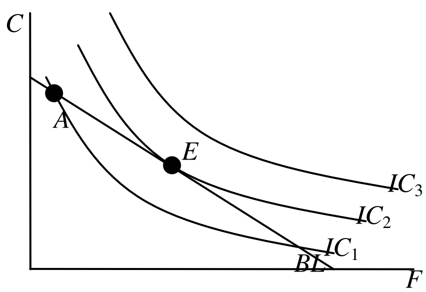


Income increases.



P_F, P_C, and I increase/decrease proportionally.

Equilibrium

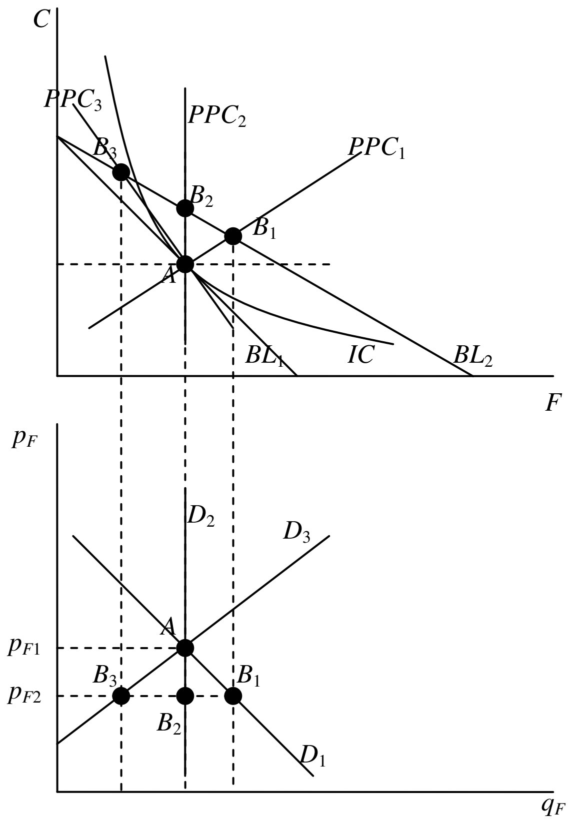


- Equilibrium is at the point of tangency of the *IC* to the *BL* – $\text{slope}_{IC} = \text{slope}_{BL} \Rightarrow \frac{MU_F}{MU_C} = \frac{P_F}{P_C}$.
- The equilibrium *IC* is the highest attainable *IC* – maximum satisfaction.

- At *A*, the market is saying *F* is cheaper than the consumer values it. The consumer should consumer more *F* and move towards *E*, where the consumer feels what the market prices say.

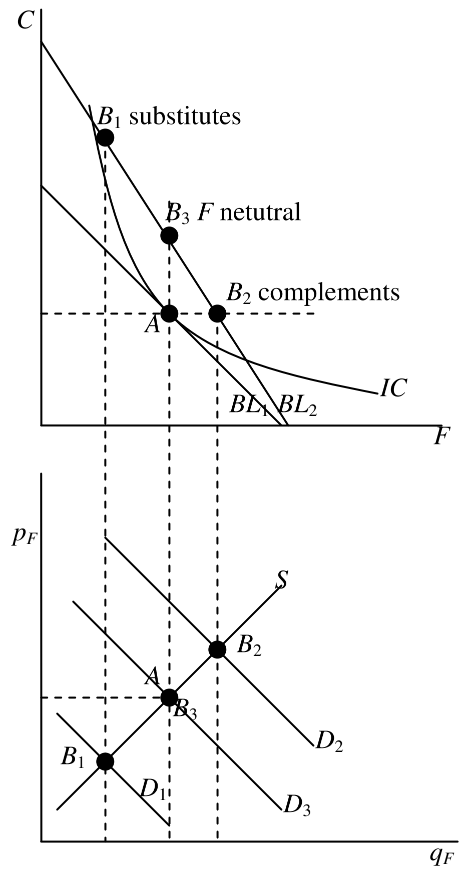
Price and Income Effects

PRICE EFFECTS



- If P_F falls, there are 3 possibilities: more *F*, same *F*, less *F*.
- Price Consumption Curve: connects *A* and *B*.

Substitute, Complement, and Neutral Goods

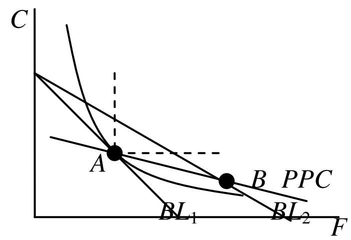


- Suppose P_C fell. Assuming negatively sloped demand curves, then:
    - If F and C are substitutes, $C\uparrow$ and $F\downarrow$.
    - If F and C are complements, $C\uparrow$ and $F\uparrow$.
    - If F is a neutral good, $C\uparrow$ and F constant.

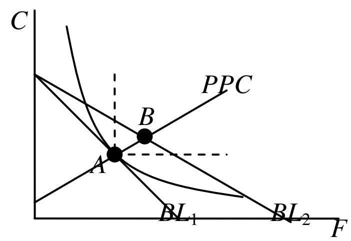
Elasticity

- $\varepsilon_D = -\dfrac{\%\Delta q_D}{\%\Delta p}$.
- Arc elasticity – elasticity between two points.
    - $\varepsilon_D = -\dfrac{\Delta q_D / \text{Avg } q_D}{\Delta p / \text{Avg } p}$ (mid-point formula).
- Point elasticity – elasticity at a point.
    - $\varepsilon_D = -\dfrac{\Delta q_D / q_D}{\Delta p / p} = -\dfrac{\Delta q_D}{\Delta p} \cdot \dfrac{p}{q_D}$.
- If $p\downarrow$, then:
    - $TR\uparrow$ means $\varepsilon_D > 1$ – relatively elastic.
    - $TR\downarrow$ means $\varepsilon_D < 1$ – relatively inelastic.
    - TR constant means $\varepsilon_D = 1$ – unit elastic.
- Readily available substitutes – ε_D large.

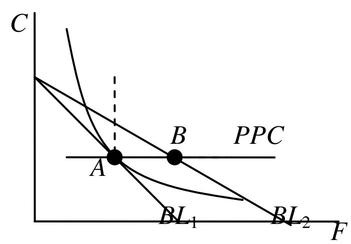
- Time period – the longer the time period, the larger ε_D.
- Percentage of income – the larger percentage, the larger the ε_D.



- $\varepsilon_D > 1$ for *F*.
- Spend more on *F*, so spend less on *C* – more *F*, less *C*.



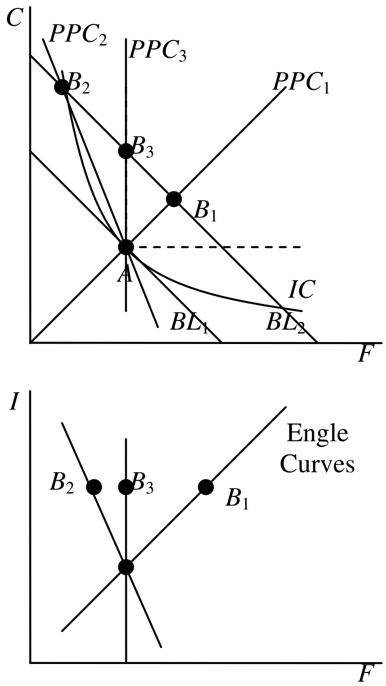
- $\varepsilon_D < 1$ for *F*.
- Spend less on *F*, so spend more on *C* – more *F* and *C*.



- $\varepsilon_D = 1$ for *F*.
- Spend same on *F* and *C* – more *F*, *C* constant.

INCOME EFFECTS

- Assume P_F, P_C constant; only *I* changes.



Indifference Graphs

- B_1: normal good.
- B_2: inferior good.
- B_3: neither/neutral good.

Engle Curves

- Positively sloped: normal good.
- Negatively sloped: inferior good.
- Vertical: neither/neutral good.

Elasticity

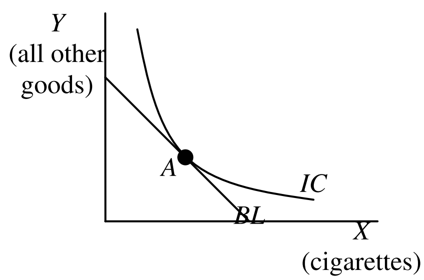
- Income elasticity of demand: $\varepsilon_I = \dfrac{\%\Delta q_D}{\%\Delta I}$.
    - > 0 – normal.
    - < 0 – inferior.
    - > 1 – income elastic/luxury.
    - < 1 – income inelastic/non-luxury.

Applications of Indifference Theory

Change in Consumer Tastes

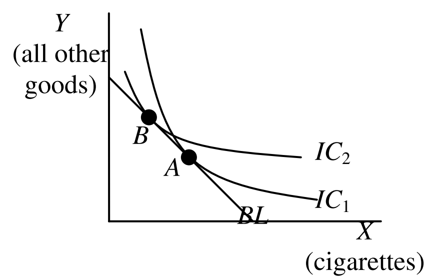
Health Canada now requires that cigarette boxes contain a warning about the health hazards of tobacco.

If consumers totally ignore it:



- No change in anything.

If consumers alter their preference:

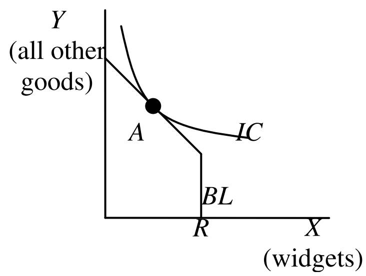


- *IC*s flatter – change in indifference map.
- $MU_X\downarrow$, so $MRS_{X/Y}\downarrow$.

Rationing of Goods

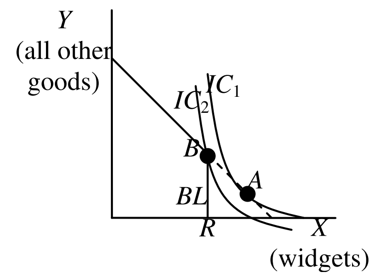
Suppose that widgets are in "short supply" and the government has decided in its wisdom not to let the price of widgets rise in order to clear the market. Instead the government restricts the amount a consumer can acquire by issuing ration coupons.

Consumers who don't like widgets:



- No change in anything.

Consumers who like widgets:



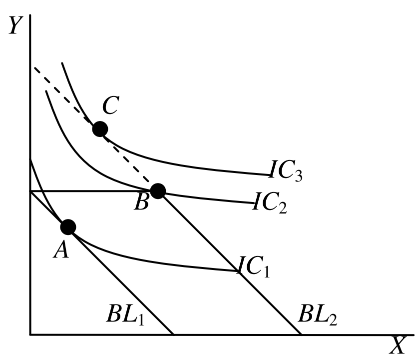
- They are restricted – can't get the amount of widgets they want.
- *IC* falls – satisfaction falls.

- Enforcement needed – rations may be sold to those who want them.

Goods In-Kind

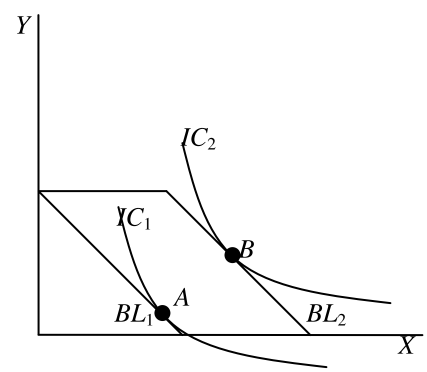
Government often establish programs to assist people in need by providing them with particular goods or services "for free". This is called an "in-kind" subsidy. Public housing is one example. The alternative of course is to provide recipients with cash transfers equivalent to the "in-kind" subsidy.
A similar situation exists with respect to benefits packages provided by employers to employees. The employer could provide the package or could provide an increased wage instead.

Consumers who don't like *X* (good in-kind):



- Goes to the corner solution.
- Would buy more *Y* if *X* was money.

Consumers who like *X* (good in-kind):



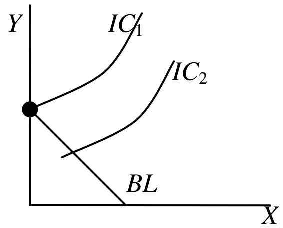
- Higher *IC* – more satisfaction.

- Fungibility – if the units in-kind can be sold, then the consumers may reach maximum satisfaction instead of the corner solution.

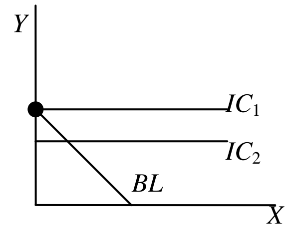
Charitable Donations

Why do people make charitable donations? After all, they do not receive any commodities in return.

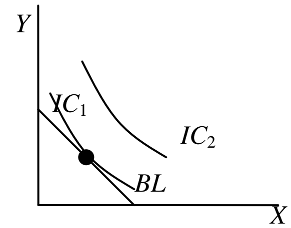
- People donate because it gives them satisfaction – it is a "good".



- Charitable donation is a "bad".



- Charitable donation is neutral.



- Charitable donation is a "good".

Income and Substitution Effects

Conceptual Basis

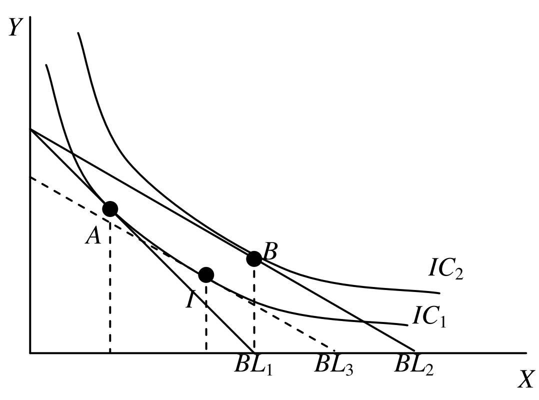
- Suppose the price of coffee falls. A consumer would buy more.
    - Cheaper – substitute other goods with coffee.
    - Can buy more coffee – higher income.

- If $P_X\downarrow$, then $\Delta X = \Delta X_{SE} + \Delta X_{IE}$.
    - ΔX_{SE} focuses on ΔP_X, holding I constant.
    - ΔX_{IE} focuses on ΔI, holding P_X constant.

Hicks Method

- The Hicks Method holds satisfaction constant.

Normal Goods



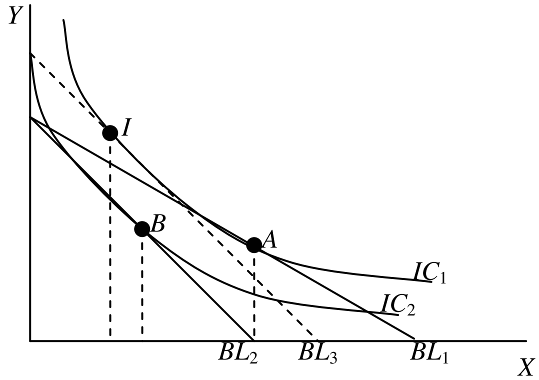
Suppose the price of X falls. X is a normal good.

- A to I: Substitution Effect.
- I to B: Income Effect.

- This produces a negatively sloped demand curve.

- The Substitution Effect is always negative with price.
- The Income Effect depends on the Engle curve.

Inferior Goods

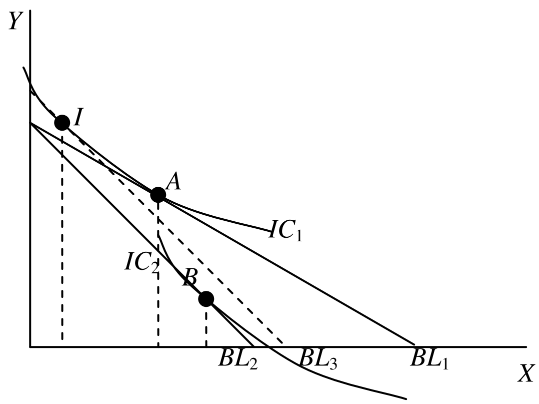


Suppose the price of *X* rises. *X* is an inferior good with *SE* > *IE*.

- *A* to *I*: Substitution Effect.
- *I* to *B*: Income Effect.

- This still produces a negatively sloped demand curve.

Giffen Goods



Suppose the price of *X* rises. *X* is an inferior good with *IE* > *SE*.

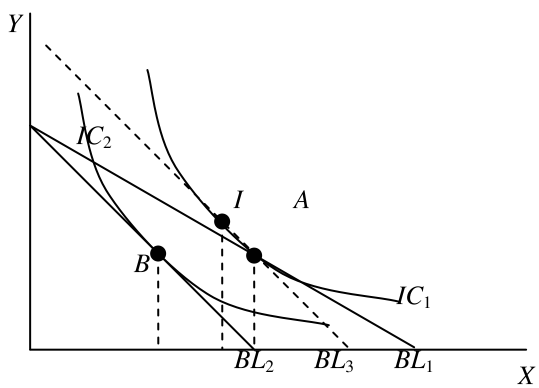
- *A* to *I*: Substitution Effect.
- *I* to *B*: Income Effect.

- This produces a positive sloped demand curve.

SLUTSKY METHOD

- The Slutsky Method holds real income constant.

Normal Goods



Suppose the price of *X* falls. *X* is a normal good.

- *A* to *I*: Substitution Effect.
- *I* to *B*: Income Effect.

- This produces a negatively sloped demand curve.

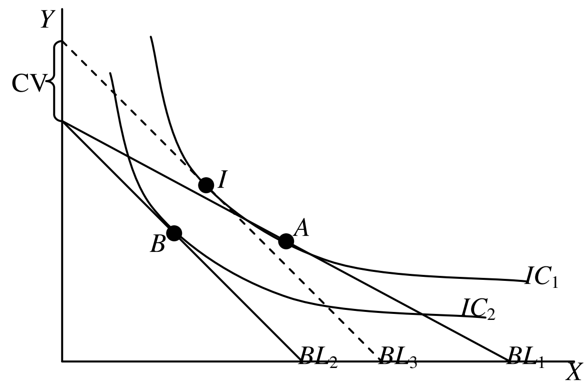
- The Substitution Effect is always negative with price.
- The Income Effect depends on the Engle curve.

Compensated Demand and Consumer Surplus

IMPACT OF A PRICE CHANGE

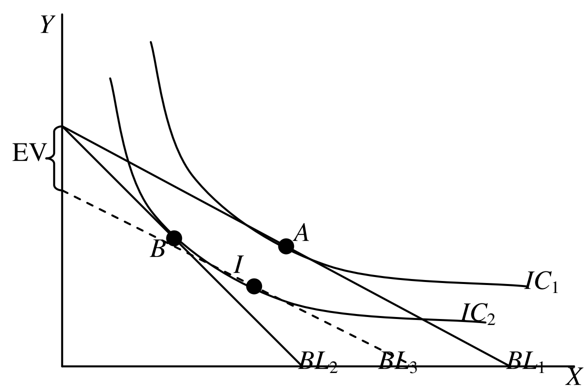
- Suppose the price of X went up. Assume X is normal.

Compensating Variation



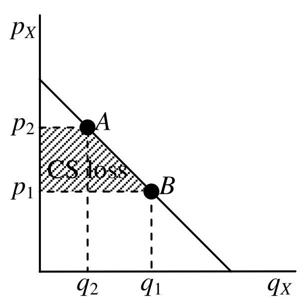
- Idea: Compensate the consumer for the price change.
- How much income is needed to bring the consumer back to the same level of satisfaction before the price change? BL_2 to BL_3, the compensating variation.
- Equivalent to the Hicks substitution effect.

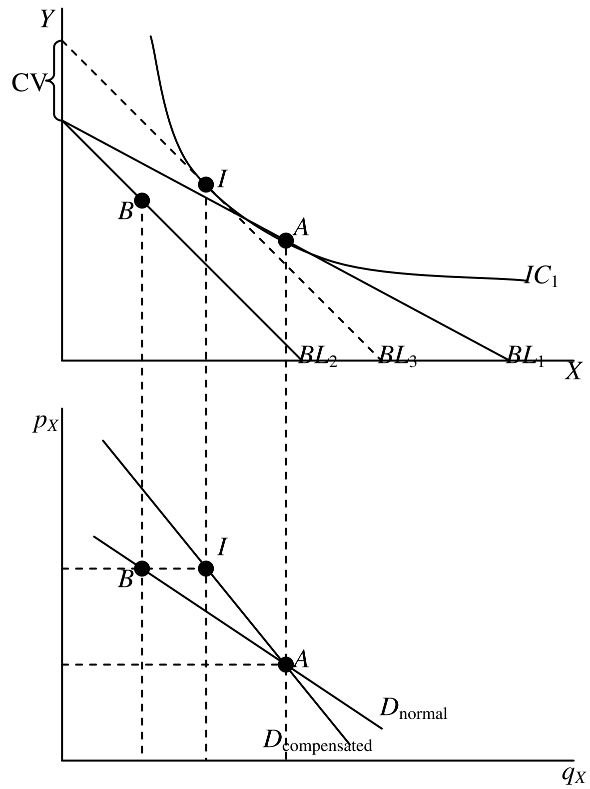
Equivalent Variation



- Idea: The amount of income equivalent to the price change.
- How much income need to take away for the consumer to be at the second indifference curve under original condition.? BL_1 to BL_2, the equivalent variation.

Consumer Surplus and the Compensated Demand Schedule



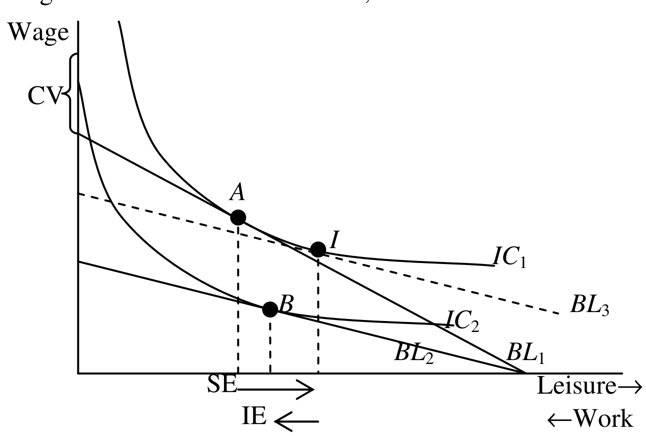


- Why compensated demand schedule? Have to look at SE only – relative price change.
- *D* based on SE is always negative.
- Use CV to get *D*.

Application to the Labour Market

Impact of a Wage Change

- Wage rate falls. Leisure is normal, SE > IE.

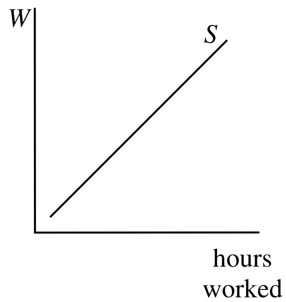


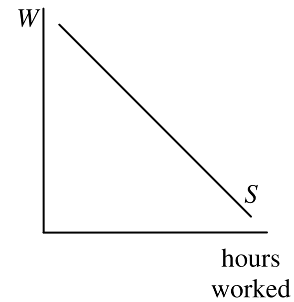
- Slope = -wage rate.
- SE: Work less – opportunity cost of leisure is down.
- IE: Work more to make up for less income.
- If SE > IE, work less.
- If IE > SE, work more.
- Those who value leisure more will have steeper ICs.

Supply Curve of Labour

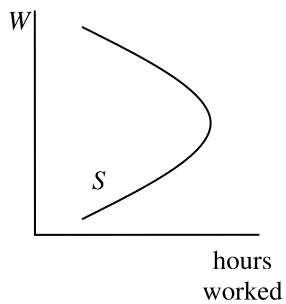
Work more as *W* increases.

Work less as *W* increases.



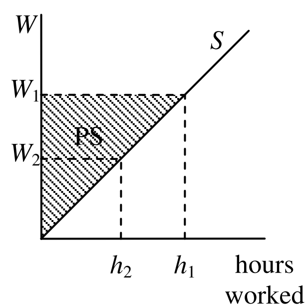


The Real World



- Those who have lower income will work more when wages increase.
- Those who have higher income will work less when wages increase.

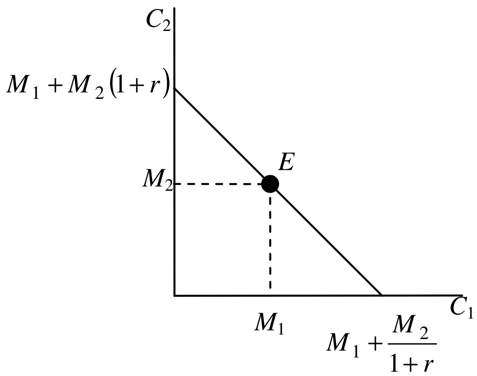
Producer Surplus



Two Period Consumption Model

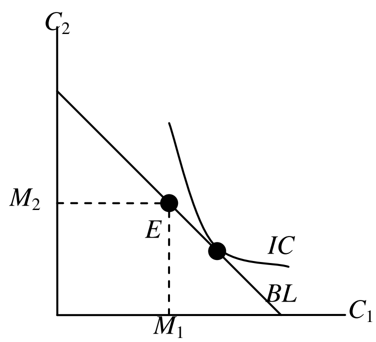
- Assumptions: No inflation, no price change, borrow rate = saving rate.
- Interest rate return will allowing a consumer to buy more – "save today, spend tomorrow".

THE BUDGET LINE

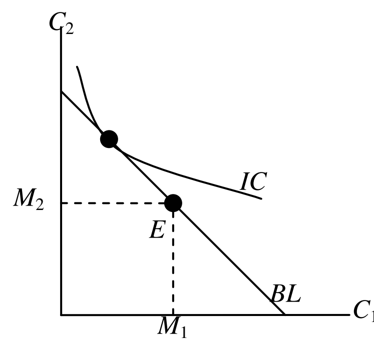


- Given M_1 and M_2:
  - $\max C_2 = M_1 + M_2(1+r)$.
  - $\max C_1 = M_1 + \frac{M_2}{1+r}$.
- E is the endowment point – neither borrow nor save.

INDIFFERENCE CURVES



- Borrower – steep indifference curves.



- Saver – flatter indifference curves.

Changes In Interest Rates

Suppose interest rates rise.

- A borrower will borrow less – SE and IE work the same way.
- A saver will either save more or less – depends in size of SE and IE.
  - If SE > IE, save more.
  - If IE > SE, save less.

RATE OF TIME PREFERENCE

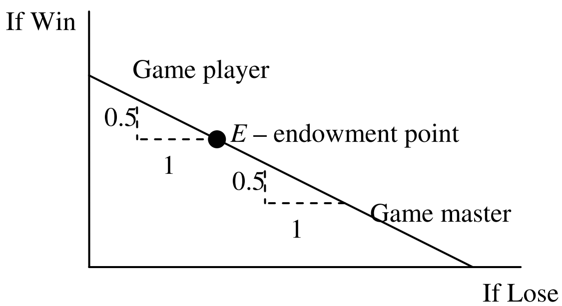
- Indifference curves represent the willingness to sacrifice tomorrow to enjoy today.
- Slope of IC = $-(1+p)$, where p is the rate of time preference.
- Slope of BL = $-(1+r)$, where r is the interest rate.
- At equilibrium, $-(1+p) = -(1+r) \Rightarrow 1+p = 1+r \Rightarrow p = r$. Thus, a consumer changes his/her rate of time preference to match the interest rate to reach maximum satisfaction.

Choice Under Uncertainty: Expected Value Approach

PROBABILITY, ODDS, AND EXPECTED VALUE

- Suppose the game is coin-toss. There are only two outcomes: head or tail.
  - The probability of head is $\rho_H = 0.5$ and tail is $\rho_T = 0.5$.
  - The odds of head to tail is $\frac{\rho_H}{\rho_T} = \frac{0.5}{0.5} = \frac{1}{1} = 1:1$. In general, the odds of X to Y is $\frac{\rho_X}{\rho_Y}$.
- The expected value of a game is $EV = \rho_1 O_1 + \rho_2 O_2 + \rho_3 O_3 + \cdots$, where ρ_i is the probability and O_i is the value of the outcome.
  - If $EV > 0$ – better than fair.
  - If $EV < 0$ – unfair.
  - If $EV = 0$ –fair.

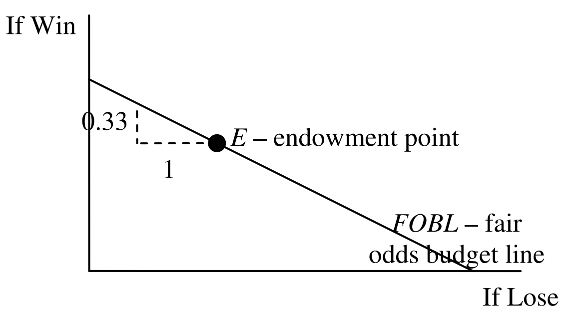
INDIFFERENCE APPROACH TO UNCERTAINTY



Suppose you:

- Lose \$1 if a spade is drawn.
- Win \$0.5 if any other card is drawn.

- $EV = \frac{3}{4}(\$0.5) + \frac{1}{4}(-\$1) = 0.125 > 0$.
  This is a better than fair game.



Suppose you:

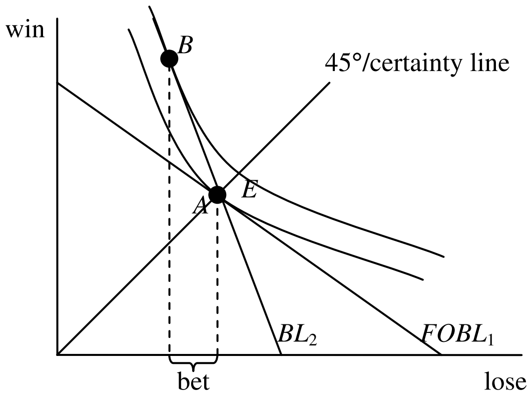
- Lose \$1 if a spade is drawn.
- Win \$0.33 if any other card is drawn.

- $EV = \frac{3}{4}(\$0.33) + \frac{1}{4}(-\$1) = 0$. This is a fair game.

PREFERENCES

Risk Averse

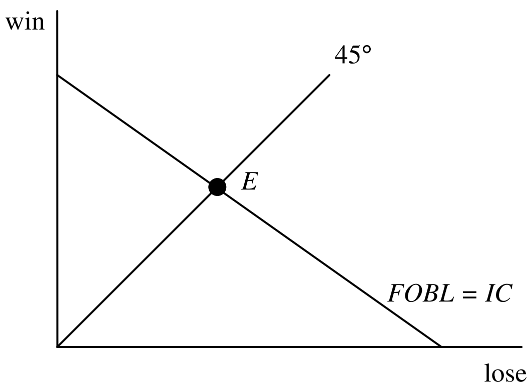
A risk averse person will not take a fair bet.



- If odds are better than fair, a risk averse person will bet.

Risk Neutral

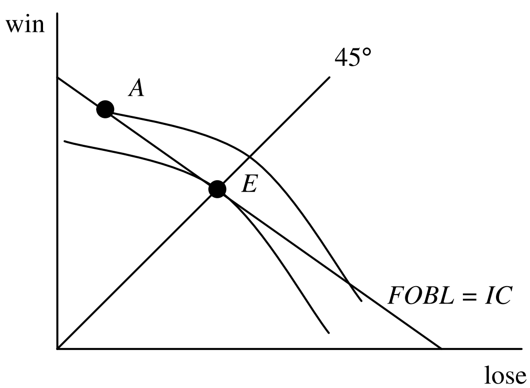
A risk neutral person is indifferent between a fair bet or its certainty equivalent (EV).



- The IC coincides with the FOBL.

Risk Seeker

A risk seeker will take a fair bet.



- *A* is preferred to *E*.
- The equilibrium will be a corner solution.

APPLICATIONS

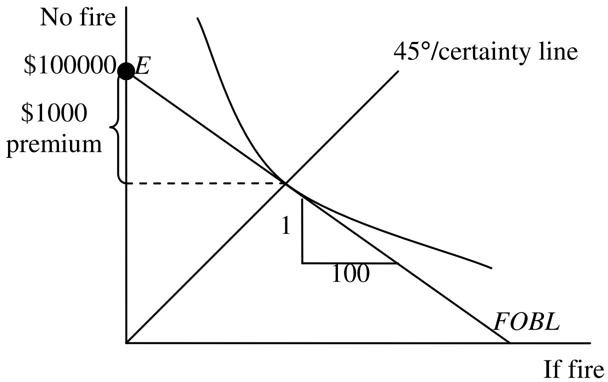
- Risk-free-rate-of-interest: Interest paid by highly secure deposits, for example government bonds.
- Risk premiums: The higher return needed to induce consumers to buy non risk-free deposits.

Diversification

- "Spread the risk" – lowers the risk.
- Trade-off: Lower return than high-risk-high-return winner – sacrifice the opportunity to "win big".

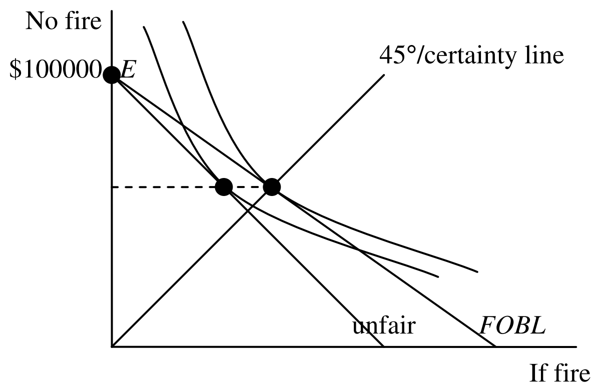
Insurance

- Certainty – coverage, premium.



- House = $100000
- Probability of fire = 0.01
- Probability of no fire = 0.99
- Fair premium = EV = 1000

- A risk averse person will buy full insurance if odds are fair.



- In the real world, odds are unfair because insurance companies have expenses and need to make money.
- A risk averse person will buy less insurance – not fully insured.

Choice Under Uncertainty: Expected Utility Approach

Example

If the price to play a game of coin toss is $49, and the payoffs are $100 if tails and $0 if heads, will a person play given their utility function?

Utility Function	Expected Utility if no play	Expected Utility if play	Conclusion
$U = U(W) = \sqrt{W}$	$EU = \sqrt{49} = 7$	$EU = \frac{1}{2}\sqrt{100} + \frac{1}{2}\sqrt{0} = 5$	No play
$U = U(W) = W$	$EU = 49$	$EU = \frac{1}{2}(100) + \frac{1}{2}(0) = 50$	Play

- In general, $EU = \sum_{i=1}^{n} \rho_i \cdot U(X_i)$, measured in jollies.

PREFERENCES

Given risk situation $(\rho_1, \rho_2; W_1, W_2)$, $EV = \rho_1 W_1 + \rho_2 W_2 = W_E$.

- Risk averse: $U(W_E) > \rho_1 \cdot U(W_1) + \rho_2 \cdot U(W_2)$.
- Risk neutral: $U(W_E) = \rho_1 \cdot U(W_1) + \rho_2 \cdot U(W_2)$.
- Risk averse: $U(W_E) < \rho_1 \cdot U(W_1) + \rho_2 \cdot U(W_2)$.

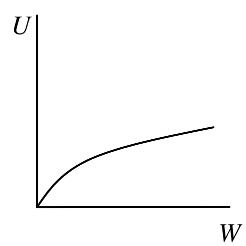
Example

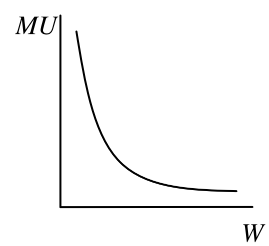
Given risk situation $\left(\frac{1}{4}, \frac{3}{4}; 100, 0\right)$ and the cost is $25.

Utility Function	$U(W_E)$	Expected Utility	Conclusion
$U(W) = \sqrt{W}$	$U(W_E) = \sqrt{25} = 5$	$EU = \frac{1}{4}\sqrt{100} + \frac{3}{4}\sqrt{0} = 2.5$	No play.
$U(W) = W$	$U(W_E) = 25$	$EU = \frac{1}{4}(100) + \frac{3}{4}(0) = 25$	Indifferent.
$U(W) = W^2$	$U(W_E) = (25)^2 = 625$	$EU = \frac{1}{4}(100)^2 + \frac{3}{4}(0)^2 = 2500$	Play.

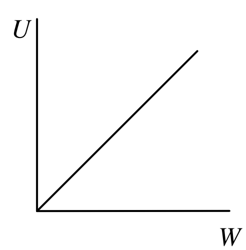
Utility Functions

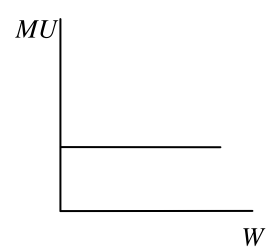
Risk averse:
$U(W) = \sqrt{W}$



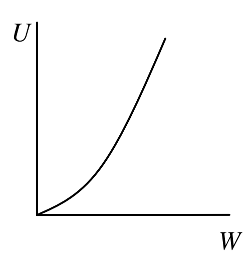


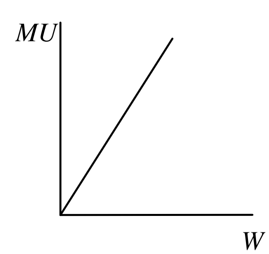
Risk neutral:
$U(W) = W$





Risk seeker:
$U(W) = W^2$

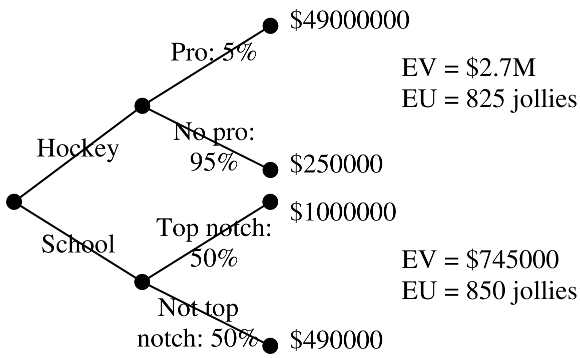




RESERVATION PRICE

- Reservation price is the maximum price consumers are willing to pay.
- A consumer wouldn't pay more than the expected utility – don't want negative EU.

RISKY PATHWAYS



- On an EV basis, hockey wins.
- On an EU basis, school wins.

- Risk averse – don't value high rewards as much.