Productivity Curves

Definitions
- Very short run: $q = \bar{q}(\bar{K}, \bar{L})$ – perfectly inelastic supply curve.
- Short run: $q = \bar{q}(K, L)$.
- Long run: $q = q(K, L)$.
- Very long run: $q = q(K, L)$ – the production function $q$ itself changes.

**SHORT RUN PRODUCTIVITY CURVES**
- Total productivity: $TP_L = q = \bar{q}(\bar{K}, L)$.
- Average productivity: $AP_L = \frac{TP_L}{L} = \frac{q}{L}$.
- Marginal productivity: $MP_L = \frac{\Delta TP_L}{\Delta L} = \frac{\Delta q}{\Delta L}$ – decreases (diminishing marginal return).
LONG RUN PRODUCTIVITY CURVES

In the long run, \( q = q(K, L) \) – there are various levels of \( K \) and \( L \) to produce a specific level of output.

**Isoquants**
- Hold \( q \) constant and see what combinations of \( K \) and \( L \) are required.
\[ q = q(K, L). \]

\[ q_2 > q_1 - q_2 \text{ represents a higher output.} \]

\[ \text{Negatively sloped} - \text{if } K \text{ decreases, } L \text{ must increase to keep } q \text{ constant.} \]

\[ \text{slope } = -MRTS_{L/K} = -\frac{MP_L}{MP_K} = \frac{\Delta K}{\Delta L}. \]

\[ \text{Convex because of the law of diminishing returns} - MRTS \text{ decreases as } L \text{ increases and } K \text{ decreases.} \]

### Non-Typical Isoquants

**Perfect Substitutes**

\[ L \]

\[ K \]

\[ q_1 \]

\[ q_2 \]

- MRTS constant.

**Fixed Proportions**

\[ L \]

\[ K \]

\[ q_1 \]

\[ q_2 \]

- MRTS = 0.

### Returns to Scale

Let \( q_1 = q(K, L) \) and \( q_1 = q(2K, 2L) \) - amount of inputs are doubled.

- Constant return to scale: \( q_2 = 2q_1 \) (output doubled).
- Increasing return to scale: \( q_2 > 2q_1 \) (output more than doubled).
- Decreasing return to scale: \( q_2 < 2q_1 \) (output less than doubled).

### Math Examples of Long Run Production Functions

- Constant return to scale: \( q = q(K, L) = \sqrt[3]{K \cdot L} \).
- Increasing return to scale: \( q = q(K, L) = K \cdot L. \)
- Decreasing return to scale: \( q = q(K, L) = \frac{1}{3} \sqrt[3]{K \cdot L} \).

### Problem

“It’s impossible for a production function which exhibits increasing returns to scale to also adhere to the ‘law of diminishing returns’”. Do you agree?

- No, it is possible. The “law of diminishing returns” is in the short-run, but “returns to scale is in the long-run.”
Cost Curves

- Two inputs, one output, fixed input prices.
- \( TC = P_K \cdot K + P_L \cdot L \).

**ISOCOST LINE**

- \( \frac{TC}{P_K} + \frac{TC}{P_L} = 0 \).
- Slope: \( \frac{TC}{P_K} = -\frac{P_L}{P_K} \).

**OPTIMUM PRODUCTION POINT**

**Output Maximization, Fixed Costs**

- A firm would like \( q_3 \), but it can’t get there.
- \( q_1 \) is possible, but can do better.
- \( q_2 \) is the maximum output for a given cost budget.
  - Slope of \( q = \text{slope of } c \), so \( \frac{MP_L}{MP_K} = -\frac{P_L}{P_K} \).
- At equilibrium, \( MRTS_{L/K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K} \).
Cost Minimization, Fixed Output

- Can’t reach $q$ with $c_1$ – not enough money.
- $q$ is possible with $c_1$, but costs too much – can do better.
- $c_2$ is the minimum cost for a given output target.
  - Slope of $q$ = slope of $c$, so $\frac{MP_L}{MP_K} = -\frac{P_L}{P_K}$.
- At equilibrium, $MRTS_{L/K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K}$.

Problem

Consider a firm that has a production budget of $X$ and faces fixed input prices for $K$ and $L$. It knows that the optimum output is 100 units. Suppose the wage rate rises. What happens?

- Increase $TC$.
- Use less $L$ than before – $\frac{K}{L}$ ration increases.

LONG RUN COST CURVES

Expansion Path

- The expansion path is transformed into the total cost schedule.
Cost Curves

- Constant return to scale.
- Decreasing returns to scale.
- Increasing returns to scale.

Total Cost, Average Cost, and Marginal Cost Curves

- Long-run total cost has increasing, constant, then decreasing returns to scale.

**Short Run Cost Curves**

- In the short run, \( q = q(K, L) \).
- \( SRTC = \bar{P}_K \cdot \bar{K} + \bar{P}_L \cdot L = TFC + TVC \).
In the short run, firms are not always producing at the lowest cost – forced to use fix $K$.
- Need a higher isocost line to produce $q_1$, $q_3$.
- $q_2$ is the only point with the lowest cost in both long run and short run.

**Long Run and Short Run Average Cost**

- $LRAC$ is the lower envelope of all $SRAC$s.
- Why are $SRAC$ and $SRMC$ u-shaped? Law of diminishing marginal returns leads to increasing costs.
  - $AP_L = \frac{q}{L}$, $AVC = \frac{\bar{P}_L \cdot L}{q} = \bar{P}_L \left( \frac{1}{AP_L} \right)$. So as the $AP_L$ decreases, $AVC$ increases.
  - $MP_L = \frac{\Delta q}{\Delta L}$, $MC = \frac{\Delta TVC}{\Delta q} = \frac{\Delta \bar{P}_L \cdot L}{\Delta q} = \bar{P}_L \left( \frac{\Delta L}{\Delta q} \right) = \bar{P}_L \left( \frac{1}{MP_L} \right)$. So as the $MP_L$ decreases, $MC$ increases.
Short Run Average Cost and Marginal Cost Curves

Profit Maximizing Behaviour of Firms

**ECONOMIC PROFITS AND COSTS**

- $\pi = TR - TC = pq - TC = pq - (p_1x_1 + p_2x_2 + \cdots)$.
  - $\pi$: economic profit.
  - $TC$: economic cost.

- $\pi = 0 \iff TR = TC$: normal profits – every input is being paid at market value.
- $\pi > 0 \iff TR > TC$: above average rate of return.
- $\pi < 0 \iff TR < TC$: revenues not covering all economic costs.
**Revenues**

![Graph of Revenues]

**Profit Maximizing Rules**

1) How much to produce?
   - \( \pi = TR - TC \), so \( \Delta \pi = \Delta TR - \Delta TC = MR - MC \).
   - Produce at \( MR = MC \) for maximum profit.

2) When do you shut down?
   - In the long run, if \( \pi < 0 \), leave!
   - In the short run, produce at a loss if \( TR > TVC \Rightarrow \frac{TR}{q} > \frac{TVC}{q} \Rightarrow AR = p > AVC \).

**Perfect Competition**

**The Assumptions of Perfect Competition**

1) Price takers.
2) Free entry.
3) No strategic behaviour.
4) Buyers are price takers.

**Attributes of Competitive Markets**

1) Many sellers.
2) Many buyers.
3) Homogeneous products/services.
4) Full information – consumers fully informed.
**COMPETITION IN SHORT RUN**

**Profit Maximization Rules**

1) Point of maximum profit if producing is \( MR = MC \Rightarrow p = MC \).

![Graph showing profit maximization](image1)

2) Produce if \( p > AVC \) or \( TR > TVC \).

![Graph showing supply curves](image2)

**Supply Curves**

![Graph showing supply curves for firm and industry](image3)

**Equilibrium**

At this equilibrium, firms make positive economic profits.
**Competition in Long Run**

- If \( \pi > 0 \), new firms will enter – above normal rate of return.
- New firms will enter until \( \pi = 0 \) – SRS shifts down.

**Equilibrium**

- A and B are both long run equilibrium points if it is a constant cost industry.

**Heterogeneous Suppliers**

- There is some input that distinguishes the firms – some have lower cost curves, some higher.

**Example**

- In the beginning, only low cost curves firms (A).
- If demand rises, medium cost curves firms (B) get induced in.
- If demand rises again, high cost curves firms (C) get induced in also.
Long Run Supply

- The industry $LRS$ is upward sloping (ignoring the discontinuities).

Economic Rent: Example

- The landlord will charge higher rents if $D$ is very high – can extract all the economic profits.
- The landlord will get all the benefits of fertile land.

THE “OPTIMALITY” OF COMPETITION

- No collusive behaviour.
- Firms always producing at lowest possible costs.
  - $p = MC$: consumers’ evaluation = market’s evaluation.

Applications of Supply and Demand Under Competition

TAX INCIDENCE

Add a per unit tax of $T$.

- Firms will make economic losses – exit until $\pi = 0$. 
Tax Burdens

Elasticity

- $e_D$: The more elastic, the more producers bear.
- $e_S$: The more elastic, the more consumers bear.

Alternate Way of Showing Tax Burdens

Tariff on Imports

- Domestic equilibrium: $p_1, q_1$. 
If imports are allowed, then the new equilibrium is \( p_W, q_2 \), with domestic production at \( q_3 \) and importing \( q_2 - q_3 \).

Once tariffs are imposed, the equilibrium is \( p_W + T, q_4 \), with domestic production at \( q_5 \) and importing \( q_4 - q_5 \).

**PRICE CEILING**

**Example: Rent Control**

- Assume demand increases.
  - If there is no intervention, then:
    - **VSR**: Substantial price increase (\( R_2 \)).
    - **SR**: As new units open up, price will decrease (\( R_3 \)).
    - **LR**: New buildings are built, price decrease further (\( R_4 \)).
    - Over-time, price will be higher than original level.
  - If there is a rent control, there will be a shortage.
    - **Winners**: People who already have a unit.
    - **Losers**: People who are shut out.

**Monopoly**

**THE UNDERPINNINGS OF MONOPOLY THEORY**

**Assumptions**

1) Not a price taker – price setter.
2) No free entry or barriers to entry.
3) Buyers are price takers.
4) No (close) substitutes.
5) Full information.
6) Firms faces the market demand schedule.
The Demand Curve and Marginal Revenue

- \( MR < p = AR \)
- \( MR = p - \left( \frac{\Delta p}{\Delta q} q = p \left( 1 - \frac{\Delta p}{\Delta q} \frac{q}{p} \right) = p \left( 1 - \frac{1}{\varepsilon_D} \right) \). \]
  - In perfect competition, \( MR = p \left( 1 - \frac{1}{\infty} \right) = p \).
  - If \( \varepsilon_D \neq 0 \), then \( MR < p = AR \).
  - \( MR = 0 \) when \( \varepsilon_D = 1 \).

Profit Maximization

Short Run:
- Produce where \( MR = MC \).
- Produce a positive quantity if \( p > AVC \).

Long Run:
- Operate only if \( p > LRATC \).

Supply Schedule of a Monopoly

- No supply schedule – will only produce at a single point.

First Degree Price Discrimination

- The monopolist sells each unit at the highest price it will command.
- The monopolist extracts the consumer surplus.

Without discrimination:

With first degree discrimination:

Second Degree Price Discrimination

There are two versions:
1) An approximation of first degree price discrimination – each block has a different price.
2) Fee and a price per unit.

- Set $p = AC$ – just enough to cover costs.
- Set the fee equal to consumer surplus.
- This is identical to first degree price discrimination – maximum profit.

**THIRD DEGREE PRICE DISCRIMINATION**

- Identify groups of people who are in separate and distinct markets.
- Different price sensitivities in consumers – if a firm can distinguish between the differences, then it can take advantage.
  - Higher price, lower price sensitivity.
  - Lower price, higher price sensitivity.
- For example, “Lady’s Day at Ballpark” is a segmentation of market – must prevent resale (arbitrage).

**Math Example**

\[ p_1 = 12 - 0.5q_1, \quad MR_1 = 12 - q_1, \quad p_2 = 12 - q_2, \quad MR_2 = 16 - 2q_2. \quad MC = 2. \]

- Market 1: $q_1 = 10$, $p_1 = 7$, $\pi_1 = 50$, $\epsilon_D = \frac{p_1}{p_1 - MR_1} = \frac{7}{7 - 2} = 1.4$.
- Market 2: $q_2 = 7$, $p_1 = 9$, $\pi_1 = 49$, $\epsilon_D = \frac{p_1}{p_1 - MR_1} = \frac{9}{9 - 2} = 1.28$.

**NATURAL MONOPOLY**

- Definition: The entire relevant $LAC$ is decreasing (before $MC = MR$).
- Perfect competition is unsustainable because of falling $MC$ – $\pi$ increases when $q$ increases.
- In the long run, there is economies of scale – each firm would want to expand to take advantage.
- So it makes more sense to let one firm supply the market instead of forcing perfect competition.
Government Intervention: Price Ceiling

- Government should set price ceiling at \( LRAC = p \) (then \( \pi = 0 \)) – the natural monopoly won’t produce when \( LRAC > p \).

Government Intervention: Tax

- If government chooses to regulate a natural monopoly, it taxing it less than 100\% of profit will let \( MC = MR \) occur at the same \( p \) and \( q \) – produces at the same position.
- Consumers, however, would favour lower prices – maybe price ceiling is a better choice.

Cartels and Monopolistic Competition

PRODUCT CARTELS

- Competitive firms want to get positive profits – collusion (cartels).

- \( \pi^+ \) is addition profits – incentive to cheat ( \( p = MC \) ).
- Cartels will want to prevent cheating and prevent new entries.
  - Quotas guarantee profits provided costs don’t change – quotas (“right to produce”) becomes valuable.
  - New producers who buy quota now has extra fixed cost – profits will be less than the original cartel members.
- Advantage of buying additional quotas from other firms is economies of scale – \( SAC \) decreases.
- To squeeze more profits, a firm will build smaller plants to get to \( LRAC \).

MONOPOLISTIC COMPETITION

Assumptions

- Homogeneous but differentiated products – ex: different brands of cereal.
- Consumers have a degree of brand loyalty – downward sloping demand schedule (more inelastic than perfect competition).
- Free entry in the long run.
Since \( \pi > 0 \), new firms will enter in the long-run, creating new differentiated brand.

- Industry demand doesn’t change, so every existing brand’s/firm’s market share decreases – demand schedule for each firm shifts inward.
- Firms will stop entering when \( \pi = 0 \) and \( p = ac \).

**Issues**

- Firms are not producing at minimum cost – have excess capacity in the long-run.
- If a firm innovates, its market share increases. Since other firms were already at \( \pi = 0 \), they will suffer losses – a firm must either innovate or exit.

**Duopoly**

**Assumptions**

- Strategic planning – “what will the other firms do, therefore what should I do”.
- Buyers have full information, but no strategic buying.

**Math Example**

A monopoly’s demand and cost schedules:

- \( D : P = 200 - Q \), \( MR : P = 200 - 2Q \).
- \( MC = AC = 0 \).

So maximum profit occurs at \( Q = 100 \), \( P = 100 \), \( \pi = 100000 \).
Cournot

The second firm assumes the first firm’s output is constant. Then it faces a residue demand schedule.

The first firm will react to the second firm, assuming it produces constant output.

Summary:

<table>
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<th></th>
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<th>Firm 2 enters</th>
<th>Firm 1 reacts</th>
<th>Firm 2 reacts</th>
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<td>100</td>
<td>75</td>
<td>75</td>
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<tr>
<td>(q_2)</td>
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<td>50</td>
<td>50</td>
<td>62.5</td>
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<td>(Q)</td>
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<td>150</td>
<td>125</td>
<td>137.5</td>
</tr>
<tr>
<td>(P)</td>
<td>100</td>
<td>50</td>
<td>75</td>
<td>62.5</td>
</tr>
</tbody>
</table>

Reaction Schedule

- \(\frac{\Delta \pi_1}{\Delta q_1} = 24 - 2q_1 - q_2\), so \(R_1: q_1 = 12 - \frac{q_2}{2}\) (for maximum profit).
- \(\frac{\Delta \pi_2}{\Delta q_3} = 24 - 2q_2 - q_1\), so \(R_2: q_2 = 12 - \frac{q_1}{2}\) (for maximum profit).

Cournot in General

- Each firm: \(q_i = \frac{24}{n+1}\), \(\pi_i = \left(\frac{24}{n+1}\right)^2\).
- The industry: \(Q = nq_i = \frac{n}{n+1} \times 24\), \(P = 34 - \frac{n}{n+1} \times 24\), \(\pi = n\pi_i = n\left(\frac{24}{n+1}\right)^2\).
- Note: As \(n \to \infty\), \(\pi_i \to 0\), and \(P \to 10\), \(Q \to 24\).

Bertrand

- Assumes constant price.

“Smart” and Naïve

- Smart: Does not assume the other firm will produce a constant quantity/price (knows the reaction schedule of the other firm).
- Naïve: Assumes the other firm will produce a constant quantity/price.
- The smart firm can force the other firm to produce at a quantity which will maximize its own profits.
Theory of Factor Markets

General Rule

- How much of an input should a firm employ?
- A firm should continue to employ more of an input as long as the addition revenue from the additional output produced by the additional unit of input exceeds the additional cost from the addition unit of input.
- Stop when Marginal Revenue Product (MRP) equals Marginal Factor Cost (MFC).
- Rule: $\text{MRP} = \text{MFC} \iff \text{MR} \times \text{MP} = \text{MFC}$.

Competitive Input and Output Markets

- In competition, $\text{MRP} = \text{MFC}$ translates to $P \times MP_L = P_L$ or Value of Marginal Produce ($VMP_L$) = Wage Rate ($W$).

Demand For Labour In The Short-Run

- Negatively sloped – for a firm and in the aggregate.
- Diminishing marginal product is the main driver – as $L$ increases $MP_L$ decrease, so $W$ decreases.
- To obtain equilibrium, the aggregate supply schedule is needed.

- A firm would not produce in III – additional unit of input will reduce $TP$ since $MP$ is negative.
- A firm would not produce in I – $MP$ above $AP$ means $SMC = P$ is below $AVC$. 
Aggregate Demand Schedule

- Intuition: sum of VMP of each individual firm.
- Issue/problem: If more input is employed, $Q_o$ increases, which means $P_o$ decreases – the firm will now employ less input!

**COMPETITIVE INPUT MARKET AND MONOPOLY OUTPUT MARKET**

Profit Maximization Rule

$$P_L = MR \times MP_L$$

(Marginal Revenue Product).

Monopoly vs. Perfect Competition

- Monopoly will produce less output than the perfectly competitive market – use less input.

**MONOPSONY INPUT MARKET AND COMPETITIVE OUTPUT MARKET**

- Monopsony – the sole employer of labour input.
- Example: Mining company in a pure mining town.

- General Rule: $MFC = MRP$.
- In this case, $MFC = VMP$ – so that’s why $W^* < VMP_L$. 

**Government Control**

- Establish a minimum wage – $W_{\text{Min}}$ becomes $MFC$.
- In monopsony, the minimum wage will cause the firm to hire more workers at a higher wage rate.

**Bilateral Monopoly**

- Monopsony buyer of input, and monopoly seller of input (ex: union).
- $MC = MR$ to maximize union’s profit.
- $MFC = VMP$ to maximize the firm’s profit.
- “Indeterminate” solution – collective bargaining is needed.