Introduction

**HISTORY OF MACROECONOMICS**

- In 1930, there is no macroeconomics – only classical economics (microeconomics).
- When the great depression came, it was not expected nor could be explained – started macroeconomics.
- M. Keynes is the father of macroeconomics – used mathematical models.

**MACROECONOMIC VARIABLES**

- Output – how to measure it?
- Unemployment rate.
- Inflation rate – affects purchasing power.

**OBJECTIVES OF MACROECONOMICS**

- Low unemployment.
- Low inflation.
- Stable but fast growing economy.

**ANATOMY OF ECONOMIES**

- There are four major players: households, government, foreigners, firms.
- They meet in three markets: factor, goods, financial.

**OUTPUT**

Have only one good that represents all goods – GDP (gross domestic product).

**Definition of GDP**
Market value of all final goods and services produced within a country in a given period of time.

- **Market value**: what the price should be.
- **of all**: legal, commercially sold.
- **final**: capital and consumption goods, but no intermediate goods.
- **Goods and services**: both tangible and intangible products.
- **produced**: reselling doesn’t count.
- **within a country**: geographical.
- **in a given period of time**: time constraint (ex: quarterly, yearly).

**Approaches**

- **Output approach.**
  - Final good approach: \( \text{GDP} = \sum P_i Q_i \).
  - Value added approach: \( \text{GDP} = \sum VA_i \). The value added is the value of output minus the value of used intermediate goods.

- **Demand approach.**
  - \( \text{GDP} = \text{Private Consumption (C)} + \text{Gross Investment (I)} + \text{Government Spending (G)} + \text{Net Export (NX)} \).
  - Private Consumption (60-70%): durable goods, nondurable goods, services.
  - Gross Investment (15-17%): fixed investment (machinery, building (residential, non-residential)), inventories.
  - Government Spending (15-20%): federal, provincial, local.
  - Net Export NX = X – Q (±5%): >0 (trade balance surplus), <0 (trade balance deficit), >0 (balanced trade).

- **Income approach.**
  - Idea: When output is sold, somebody in the economic earns it.
  - \( \text{GDP} = \text{Labor Compensation} + \text{Capital Return} + \text{Rent} \).
  - Doesn’t quite add up to GDP because of indirect taxes and depreciation.

**INFLATION AND PRICES**

**GDP**

- Nominal GDP, \( \sum P_i Q_i \) – has both price and quantity in it.
- Real GDP, \( \sum P_i Q_i \) – based on a base year, and yields the change of aggregate quantity.

\[ \text{GDP deflator} = \frac{\text{nominal GDP}}{\text{real GDP}} \times 100, \text{ or } P = \frac{\$Y}{Y} \text{ so } Y = PY. \text{ This is the price of the aggregate good.} \]

- From here, we can calculate the inflation of \( P - \text{GDP deflator inflation} \).

**Chained Method**

\[ \text{Real GDP Growth Rate} = \left( \frac{\sqrt{\sum P_i Q_i} \sum P_{i-1} Q_i}{\sqrt{\sum P_{i-1} Q_{i-1}} \sum P_i Q_{i-1}} - 1 \right) \times 100. \]

**Cost of Living**

- Cost of Living = \( \sum P_i Q_{i,b} \) – \( Q_b \) (the bundle) is based in a base year.
• Note: The bundle contains only household goods and is different from GDP.

• Consumer Price Index: \( \text{CPI} = \frac{\text{Cost of Living}}{\text{Base Year}} \times 100 \).

**Unemployment Rate**

  • In symbols, \( P = L + NL \Leftrightarrow 1 = \frac{L}{P} + \frac{NL}{P} \).
  • \( \frac{L}{P} \) is the participation rate.

• Labor Force = Employed + Unemployed.
  • In symbols, \( L = E + U \Leftrightarrow 1 = \frac{E}{L} + \frac{U}{L} \).
  • \( \frac{E}{L} \) is the unemployment rate.

**Economies in the Short Run**

**Assumptions**

• The price level \( (P) \) is fixed.
• Capital stock \( (K) \) and labor force \( (L) \) are fixed.
• The output capacity \( Y = F(K, L) \) is fixed.
• \( Y \) can go above or below output capacity \( Y \) by over-utilization or under-utilization of capital stock and labor force.

**Goods Market**

The demand for goods \( Z = C + G + I \) (we assume closed economy, so \( NX = 0 \)).

• Behavioral function for \( C \) is \( C = C(Y_d) = c_0 + c_1 Y_d \). Since \( Y_d = Y - T \), \( C = c_0 + c_1(Y - T) \).
  • \( c_0 \): Autonomous consumption.
  • \( c_1 \): Marginal propensity to consume (MPC).
  • We assume the tax function \( T = \bar{T} \) is a lump sum tax.

• Behavioral function for \( G \) is \( G = \bar{G} \), an exogenous variable.
• Behavioral function for \( I \) is \( I = \bar{I} \), an exogenous variable.

So finally, \( Z = c_0 + c_1(Y - \bar{T}) + \bar{G} + \bar{I} \).

The supply equation is the sum of all value added \( Y \).

So, by setting \( Z = Y \), we can solve for equilibrium GDP.

**Stability**
Inventories is the mechanism to stability in the short run.

- At $Y_1$, demand exceeds supply – inventory levels decrease. When firms see this, they increase output, which moves GDP back up to $Y^*$.
- At $Y_1$, supply exceeds demand – inventory levels increase. When firms see this, they decrease output, which moves GDP back down to $Y^*$.

**Intervention**

Notice that the equilibrium GDP ($Y^*$) may not be at the potential output ($\bar{Y}$).
- If $Y^* < \bar{Y}$, it is a recession.
- If $Y^* > \bar{Y}$, it is a boom/expansion.

How to bring economy to $\bar{Y}$? By shifting $Z$ up through interventions.
- Shift $G$ – fiscal policies.
- Shift $C$ – ask people to consume more by campaigns.

**Fiscal Policies**
- Expansionary fiscal policies: increase $G$, decrease $T$.
- Contractionary fiscal policies: decrease $G$, increase $T$.

**The G and T Multipliers**

Notice $Y = c_0 + c_1 (Y - T) + G + I \Rightarrow Y = c_0 + c_1 Y - c_1 T + G + I \Rightarrow Y (1 - c_1) = c_0 - c_1 T + G + I \Rightarrow$ 

$Y^* = \frac{c_0 - c_1 T + G + I}{1 - c_1}$. 

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\[ \frac{\Delta Y}{\Delta G} = \frac{1}{1-c_1} \implies \Delta Y = \frac{1}{1-c_1} \Delta G, \text{ and } \frac{1}{1-c_1} \text{ is called the G-multiplier.} \]

\[ \frac{\Delta T}{\Delta G} = - \frac{c_1}{1-c_1} \implies \Delta Y = -\frac{c_1}{1-c_1} \Delta T, \text{ and } -\frac{c_1}{1-c_1} \text{ is called the T-multiplier.} \]

**FINANCIAL MARKET**

The financial market is the “place where extra resources are lent and borrowed” or the “demand and supply of assets”.

**Assets**

- Extra resources earned by households is converted into assets to preserve the purchasing power for future.
- Assume two assets:
  - Money (cash, demand deposits): Good for transaction (liquid), but doesn’t pay interest.
  - Bonds: Not good for transaction (not liquid), but pay interest.
- We need a combination/balance/portfolio of both assets – liquid vs. return.

**Demand for Money**

The behavioral function for demand of money is \( M^d = L\left( i, \frac{\bar{M}}{\bar{Y}} \right) \) (textbook uses \( M^d = \bar{Y} \cdot L\left( i \right) \). \( \bar{Y} \) is the transaction demand, and \( L\left( i \right) \) is the speculative demand.

\[ \bar{Y}_2 > \bar{Y}_1 \]

\[ M^d = \bar{Y}_2 L(i) \]

\[ M^* = \bar{Y}_1 L(i) \]

**Supply of Money**

\( M^* = \bar{M} \) fixed supply decided by central bank.
Changes in Equilibrium
If income increases $Y_1 < Y_2$, then:

Increase in income $\Rightarrow$ increase in demand for money $\Rightarrow$ demand for bonds decreases $\Rightarrow$ selling of bonds increases $\Rightarrow$ price of bonds decreases $\Rightarrow$ interest rates increases.

Monetary Policies
Expansionary monetary policies: $\overline{M}$ increases.
Contractionary monetary policies: $\overline{M}$ decreases.

Note: Ideally, the central bank and the government should be independent.

The central bank can increase/decrease the money supply by open market operations.
- If the central bank buys bonds, then $\overline{M} \uparrow \Rightarrow P_B \uparrow \Rightarrow i \downarrow$.
- If the central bank sells bonds, then $\overline{M} \downarrow \Rightarrow P_B \downarrow \Rightarrow i \uparrow$.

Money Multiplier
\[ M = \frac{1}{(r+e)(1-c)+c} \] 
\( r \) is required reserve ratio, \( e \) is excess reserve retained, \( c \) is cash kept.

**GOODS MARKET AND FINANCIAL MARKETS TOGETHER**

**Goods Market**

Change the behavioral function for investment: 
\[ I = I(Y, i) \]

IS-curve (investment-saving curve): Gives all the possible equilibriums in the goods market.

Shocks: Changes not due to policy-makers. For example, consumer confidence \( (c_0 \text{ in } C = c_0 + c_1(Y - T)) \), investor optimism (function change in \( I = I(Y, i) \)).

Note: Any expansionary shocks or policies will shift the IS-curve to the right.
Financial Market

We had \( M^d = SY \cdot L(i) \) \( M' = \frac{\bar{M}}{P} \) \( \Rightarrow M^d = M^s \) equilibrium. But goods market is measured in real GDP!

Since \( Y = \frac{SY}{P} \), so change the model to \( \frac{M^d}{P} = Y \cdot L(i) \) \( \frac{M^s}{P} = \frac{\bar{M}}{P} \) \( \Rightarrow M^d = M^s \) equilibrium.

LM-curve: Gives all possible equilibriums in financial market.

![Financial Market Diagram](image)

Note: Any expansionary shocks or policies will shift the IS-curve down.

Goods Market and Financial Market Linked

IS-curve: \( Y = c_0 + c_1(Y - T) + I(Y, i) + G \).

LM-curve: \( \frac{\bar{M}}{P} = YL(i) \).

![Goods Market and Financial Market Linked Diagram](image)

**The Dynamics of Changes in Equilibrium Points**

**Note**

The financial market is always in equilibrium – always on the working LM-curve.

**Example**
OPEN ECONOMIES

- Open goods market – domestic and foreign goods.
- Open financial market – domestic currency and bonds, foreign currency and bonds.
- Open factor market – domestic labor and capital, foreign labor and capital.

Assumption
Free flow of goods (no tariffs, no quotas).

Nominal Exchange Rate
- Nominal exchange rate (NER) is the price of foreign currency in terms of domestic currency.
- $E$ increases (market determined): depreciation; $E$ decreases (market determined): appreciation.
- $E$ increases (central bank determined or fixed): devaluation; $E$ decreases (central bank determined or fixed): revaluation.

Real Exchange Rate
- Real exchange rate (RER) is the price of foreign goods in terms of domestic goods. $\varepsilon = \frac{E_P^s}{P}$. 
- $\varepsilon$ increases: real depreciation; $\varepsilon$ decreases: real appreciation.

Arbitrage
Suppose you want to invest $1 in bonds.
- Domestic bonds: $(1+i_t)$ after time $t$.
- Foreign bonds: $(1+i_t^*)\frac{E_t^s}{E_t}$ after time $t$. 
By arbitrage, the expected return of all assets are equal. So \( (1 + i_t) = (1 + i^*_t) \frac{E_{t+1}^e}{E_t} \) \( \Rightarrow \) 
\[
(1 + i_t) = (1 + i^*_t) \left( \frac{E_{t+1}^e - 1}{E_t} \right) \Rightarrow (1 + i_t) = (1 + i^*_t) \left( 1 + \frac{E_{t+1}^e - E_t}{E_t} \right) \Rightarrow i_t = i^*_t + \frac{E_{t+1}^e - E_t}{E_t} .
\]
This is the uncovered interest parity (UIP) condition, and \( \frac{E_{t+1}^e - E_t}{E_t} \) is the expected depreciation rate.

**Open Goods Market**

\[
Z = C + I + G - \varepsilon Q + X = C + I + G + NX .
\]
- \( Z \) is the demand for domestic goods.
- \( C + I + G - \varepsilon Q + X = C + I + G + NX \) is the domestic demand for goods (including foreign goods).

**Behavioral Functions**

\[
Z = c_0 + c_1(Y - T) + I(Y, i) + G - \varepsilon Q(Y, \varepsilon) + X(Y^*, \varepsilon) .
\]

**Equilibrium**

At equilibrium, \[
\begin{align*}
ZZ &= C + I + G - \varepsilon Q + X = DD + NX \\
Z &= Y \\
\end{align*}
\]
gives \( Y = C + I + G - \varepsilon Q + X . \)

**Fiscal Policies and Shocks**
- Suppose expansionary fiscal policy: $G\uparrow$ or $T\downarrow$.
- Result: Deterioration of trade deficit.

- Suppose foreigners become richer: $Y^*\uparrow$.
- Result: Improvement of trade deficit.

**Marshall Lerner Condition**
Consider an increase in $\varepsilon$ (real depreciation). The effect on $NX$ is unclear. However, if the import and export are elastic enough, then $X\left(Y^*, \varepsilon\right) - Q\left(Y, \varepsilon\right) = NX\left(Y^*, Y, \varepsilon\right)$.

**J-Curve**
Suppose a country devalues its currency to reduce its trade deficit. Then in the very short run (ex: a few days), $NX = X\left(Y^*, \varepsilon\right) - Q\left(Y, \varepsilon\right)$ will decrease sharply since $\varepsilon$ is lower, but import and exports cannot respond yet. So the economy is actually worse in the very short run, but will improve after that.
OPEN FINANCIAL MARKET

Uncovered Interest Parity Condition

Assuming the expected exchange rate is fixed (exogenous),

\[ i_t = i^* + \frac{E_{t+1}^e - E_t}{E_t} \Rightarrow i = i^* + \frac{E^e - E}{E} \].

Solving for \( E \), we get \( E = \frac{E^e}{1 + i^* - i} \) (UIP condition).

Shocks

1) If \( i^* \uparrow \Rightarrow i > i^* + \frac{E^e - E}{E} \) \( \Rightarrow \) switch to domestic bonds \( \Rightarrow \) capital inflow \( \Rightarrow \) demand for domestic currency increases \( \Rightarrow \) appreciation (\( E \downarrow \)). This is a movement along the UIP curve.

2) If \( i^* \downarrow \Rightarrow i < i^* + \frac{E^e - E}{E} \) \( \Rightarrow \) switch to foreign bonds \( \Rightarrow \) capital outflow \( \Rightarrow \) supply for domestic currency increases \( \Rightarrow \) depreciation (\( E \uparrow \)). This is a movement along the UIP curve.

3) If \( i = i^* \Rightarrow i > i^* + \frac{E^e - E}{E} \) \( \Rightarrow \) switch to domestic bonds \( \Rightarrow \) capital inflow \( \Rightarrow \) demand for domestic currency increases \( \Rightarrow \) appreciation (\( E \downarrow \)). This is a downward shift of the UIP curve.

4) If \( E^e \uparrow \Rightarrow i < i^* + \frac{E^e - E}{E} \) \( \Rightarrow \) switch to foreign bonds \( \Rightarrow \) capital outflow \( \Rightarrow \) supply for domestic currency increases \( \Rightarrow \) depreciation (\( E \uparrow \)). This is a rightward shift of the UIP curve.

GENERAL EQUILIBRIUM OF OPEN MARKETS

\[
\begin{align*}
IS : Y &= c_0 + c_1(Y - T) + I(Y, i) + G + NX \left( Y^*, Y - \frac{E^e}{1 + i^* - i} + (ML) \right) \\
LM : \frac{M}{P} &= YL(i)
\end{align*}
\]


**Exchange Rate Regimes**

**Flexible Exchange Rate Regime**
Exchange rate determined by the market (supply and demand).

**Pegged Exchange Rate Regime**
Fix the exchange rate with a particular country (usually the US).

**Crawling Pegged Exchange Rate Regime**
Fixed, but revised once in a while. \( \varepsilon = \frac{EP^*}{P} \Rightarrow \frac{\Delta \varepsilon}{\varepsilon} = \frac{\Delta E}{E} + \frac{\Delta P^*}{P^*} - \frac{\Delta P}{P} \), but since \( \varepsilon \) is fixed, so \[
\frac{\Delta \varepsilon}{\varepsilon} = 0 \Rightarrow \frac{\Delta E}{E} = \frac{\Delta P}{P} - \frac{\Delta P^*}{P^*}.
\]

**Credible Pegged Exchange Rate Regime: Mundell-Fleming Model**
- Pegged \( E = \bar{E} \), credible \( \bar{E}^* = \bar{E} \).
- UIP condition \( i = i^* + \frac{\bar{E}^* - E}{E} \Rightarrow i = i^* \).
- In this case, monetary policies are useless, i.e. \( LM \) curve fixed – central bank forced to defend the exchange rate. However, fiscal policies shifts both \( IS \) and \( LM \) curves.

**Economies in Medium Run**

**Idea**
We let \( P \) be endogenous instead of fixed.

**Labour Market, Unemployment Rate**
In medium run, \( \{ y = \bar{Y}, \quad U = U_n \} \). So in a boom \( \{ y > \bar{Y}, \quad U < U_n \} \), and in a recession \( \{ y < \bar{Y}, \quad U > U_n \} \).

**Reasons for Unemployment**

**Frictional Unemployment (Job Search)**

\[ U = \frac{s}{s+f} \]

\( f \): job finding rate

\( s \): job separation rate

**Wait Unemployment (Real Wage Rigidity)**

- Minimum wage laws – market can’t adjust.
- Labour market and collective bargaining – monopoly of workers.
- Efficiency wages – “if pay more, more productivity”.
  - Nutritional and health issues.
  - Turnover issues – always have a pool of unemployed to “scare” the workers.
  - Adverse selection – hidden information.
  - Moral hazard – hidden action, lack of monitoring.

**Equilibrium**

**Wage Setting Relation**

\[ W = P^e F\left( U, z \right) \]

\( z \) increases if a factor increases \( U \) at any \( W \), or if a factor increases \( W \) at any \( U \).
Price Setting Relation
If market competitive: $P = W$.
If market non-competitive: $P = W(1 + \mu)$, where $\mu$ is the market (market power).

The Equilibrium
In the medium run, $\begin{cases} Y = \bar{Y} \\ U = U_n \end{cases}$. So we get $P = P^e$.

\[ \begin{align*}
WS : \frac{W}{P} &= F(U, z) \\
PS : \frac{W}{P} &= \frac{1}{1 + \mu}
\end{align*} \]

AGGREGATE SUPPLY AND DEMAND CURVE

Aggregate Supply Curve
\[ AS : P = P^e (1 + \mu) \cdot F \left( 1 - \frac{Y}{\bar{Y}}, z \right). \]

Increasing: $Y \uparrow \Rightarrow 1 - \frac{Y}{\bar{Y}} = u$ (unemployment) $\downarrow \Rightarrow W$ production cost $\uparrow \Rightarrow P \uparrow$.

If $P^e \uparrow$, $AS$ shifts up (ask for higher wages), so $P \uparrow$.

Aggregate Demand Curve
Use \[ IS : Y = C + G + I(i, Y) \]
\[ LM : \frac{\bar{M}}{P} = Y \cdot L(i) \]

\[ AD : Y = Y \left( G, T, \frac{\bar{M}}{P} \right) \].

If \( G \uparrow \) or \( T \downarrow \Rightarrow IS \) shifts right \( \Rightarrow \) all points shifts right \( \Rightarrow AD \) shifts up.
If \( \bar{M} \uparrow \Rightarrow LM \) shifts down \( \Rightarrow \) all points shifts right \( \Rightarrow AD \) shifts up.
So all expansionary policies shifts \( AD \) right.

**Equilibrium**

1) \( Y > \bar{Y} \), \( u < u_n \) – boom, danger of inflation.
2) \( Y < \bar{Y} \), \( u < u_n \) – stagflation, hell.
3) \( Y < \bar{Y} \), \( u > u_n \) – recession, high unemployment.

**SHOCKS AND POLICIES**

**Expansionary Monetary Policy**
- Short run: \( i \downarrow, Y \uparrow, P \uparrow \).
- From shout run to medium run: People adjust their \( P^e \uparrow \), which will cause \( P \uparrow \).
- Medium run equilibrium: When \( P^e = P \), everything is the same, but only \( P \) higher! This phenomenon is called “neutrality of money” (money doesn’t matter).

**Supply Side Shocks and Policies**
Suppose there is a technological progress. This will shift \( \bar{Y} \) to the right.

Short Run: \( i \downarrow, Y \uparrow, P \downarrow, \frac{W}{P} \uparrow, u_n \downarrow \).

Medium Run: \( P \downarrow, Y \uparrow \) further.
So, a positive supply side shock will permanently increase \( \bar{Y} \) and decrease \( u_n \).

**Example: Weaker Unions**
This is characterized by $z \downarrow$.
In the short run, $z \downarrow \Rightarrow W \downarrow \Rightarrow$ production cost $\downarrow \Rightarrow P \downarrow \Rightarrow \frac{M}{P} \uparrow \Rightarrow i \downarrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \Rightarrow u \downarrow$ (partially offsets $W \downarrow$). Note that $\frac{W}{P}$ is constant.
In the medium run, $P^c \downarrow \Rightarrow W \downarrow \Rightarrow$ production cost $\downarrow \Rightarrow P \downarrow \Rightarrow \frac{M}{P} \uparrow \Rightarrow i \downarrow \Rightarrow I \uparrow \Rightarrow Y \uparrow$. Note that $\frac{W}{P}$ is constant.

**PHILLIPS CURVE, OKUN’S LAW, AND AGGREGATE DEMAND**

**Phillips Curve**
The phillips curve is given by $\pi_t - \pi_{t-1} = -\alpha(u_t - u_n)$ or $\pi_t - \pi_{t-1} = -\alpha(u_t - u_n)$ if we assume $\pi_t = \pi_{t-1}$, where $\pi_t$ is the rate of inflation. It is actually derived from the aggregate supply curve $AS : P = P^c(1 + \mu)F(u, z)$. Note that if $u_t < u_n$, we have accelerated inflation.

**Okun’s Law**
If the output growth rate is $g_Y$, then $u_t - u_{t-1} = -\beta(g_Y - \bar{g}_Y)$, where $\bar{g}_Y$ is the normal growth rate.

**Aggregate Demand**
$AD : g_Y = g_M - \pi_t \Leftrightarrow \pi_t = g_M - g_Y$. This is derived from the aggregate demand curve $AD : Y(G, T, \frac{M}{P})$.

**Together**
Putting the three relations together, we get
\[
\begin{align*}
\pi_t - \pi_{t-1} &= -\alpha(u_t - u_n) \\
u_t - u_{t-1} &= -\beta(g_Y - \bar{g}_Y) \\
\pi_t &= g_M - g_Y
\end{align*}
\]

**MEDIUM RUN EQUILIBRIUM WITH INFLATION IN CLOSED ECONOMIES**
We have
\[
\begin{align*}
\text{AS: } P &= P^e (1 + \mu) F(u, z) \\
\text{AD: } Y &= Y \left( \frac{\bar{P}}{P}, G, T \right)
\end{align*}
\]
and
\[
\begin{align*}
\text{FC: } \pi_t - \pi_t^e &= -\alpha(u_t - u_n) \\
\text{OK: } u_t - u_{t-1} &= -\beta \left( g_{Y_t} - \bar{g}_Y \right) \\
\text{AD: } \pi_t &= g_{M_t} - g_{Y_t}
\end{align*}
\]
Assume that before \( t = 0 \), \( g_M = 0 \) and \( \bar{M} \) is fixed; at \( t = 0 \), unanticipated change \( g_M > 0 \).

In the first year, \( O \) to \( A \) unanticipated, then eventually \( O \) to \( B \).

Next year, \( B \) to \( C \) anticipated;
\( \bar{M} \uparrow \Rightarrow \) shift in \( AD \) \( \Rightarrow \) go straight to \( C \).

\[ P^e \uparrow \Rightarrow \text{shift in } AS \]

**Fighting Inflation**

Suppose in medium run \( u_t = u_{t-1} = u_n \)
\[
\begin{align*}
\text{FC: } \pi_t &= \pi_t^e \\
\text{OK: } g_{Y_t} &= \bar{g}_Y \Rightarrow \pi_t = g_{M_t} - \bar{g}_Y
\end{align*}
\]

Consider
\[
\begin{align*}
\text{FC: } \pi_t^e - \pi_t &= -\alpha(u_t - u_n) \\
\text{OK: } u_t - u_{t-1} &= -\beta \left( g_{Y_t} - \bar{g}_Y \right)
\end{align*}
\]
Want to reduce inflation to \( \bar{\pi} \). How do we know \( \pi_t^e \)? There are three scenarios:

1) No credibility: \( \pi_t^e = \pi_{t-1} \) (adaptive expectation theory).
2) Half credibility: \( \pi_t^e = \frac{1}{2} \pi_{t-1} + \frac{1}{2} \bar{\pi} \).
3) Full credibility: \( \pi_t^e = \bar{\pi} \).
To decrease inflation, have to decrease $M$, but this is costly! We measure the cost of different policies/scenarios by:
1) Extra unemployment rate.
2) Output fall.
3) Duration of policies.

**MEDIUM RUN EQUILIBRIUM WITH INFLATION IN OPEN ECONOMIES WITH FLEXIBLE EXCHANGE RATE REGIME**

In medium run, $P \uparrow$, $P^e \uparrow$; $e$, $NX$, $i$, $I$, $C$, $G$, $Y$ the same.

**MEDIUM RUN EQUILIBRIUM IN OPEN ECONOMIES WITH PEGGED EXCHANGE RATE REGIME**

Credible Regime
Suppose “consumer pessimism” $c_0 \downarrow$. From the short-run equilibrium at point $A$:

1) Doing nothing policy $B$. In medium-run $P^e \downarrow \Rightarrow P \downarrow \Rightarrow \frac{M^e}{P} \uparrow \Rightarrow LM \downarrow \Rightarrow \varepsilon = \frac{E^p}{P} \uparrow \Rightarrow NX \uparrow \Rightarrow IS \rightarrow$, so $Y = C + I + G + NX$.

2) Devaluation policy $C$. In medium-run

$$\bar{E} \uparrow$$

$$\Rightarrow \varepsilon = \frac{E^p}{P} \uparrow \Rightarrow NX \uparrow \Rightarrow IS \Rightarrow \{ \text{upward pressure on } i \} \Rightarrow \{ \text{capital inflow} \} \Rightarrow \{ \text{central bank buys foreign exchange} \} \Rightarrow LM \downarrow,$$

so $Y = C + I + G + NX$.

3) Expansionary fiscal policy $D$. In medium-run $G \uparrow$, so $Y = C + I + G + NX$.

4) Tax cut policy $D$. In medium-run $T \downarrow$, so $Y = C + I + G + NX$.

**Non-Credible: Exchange Rate Crisis**
Suppose “consumer pessimism" $c_0 \downarrow$. From the short-run equilibrium at point $A$:

1) Doing nothing policy $B$. $i < i^* + \frac{E^c - \bar{E}}{\bar{E}}$, so capital outflow. Central bank must sell foreign exchange to keep $\bar{E}$, but must sell a lot, so it will collapse at some point.
2) Devaluation policy $C$. Capital inflow.

**Long Run**

**ASSUMPTIONS**

1) $Y = F(K, L)$, $K$ and $L$ are now variables.
2) Money and price don’t matter, only $\bar{Y}$ matters.

**STYLIZED FACTS**

We develop models to explain these observed facts. For industrialized countries:

1) Output per capita growth rates have decreased over time.
2) Countries with lower output per capita grow faster.
3) Output per capita have had 2% increase in output per capita per year on average in past century.
   Note: 1 and 2 gives Conditional Convergence Theory — all countries goes to the same place.
THE PRODUCTION FUNCTION

\[ Y = F(K, N) \] is the aggregate production function.

Example: Cobb-Douglas Production Function
\[ Y = 2K^{0.3}N^{0.7} \] is the Cobb-Douglas production function.

Example: Technological Progress
Original: \[ Y = 2K^{0.3}N^{0.7} \]. \( K = 100, N = 200 \Rightarrow Y = 324.9 \).
New: \[ Y = 3K^{0.3}N^{0.7} \]. \( K = 100, N = 200 \Rightarrow Y = 487.4 \).

Properties of the Production Function
1) Constant return to scale (CRTS): \( \lambda Y = F(\lambda K, \lambda N) \).
2) Diminishing Returns to Factors (DRTF): Increase in only one factor will cause output to increase less every time.

![Graph of production function](image)

Production Function per Capita
\[ Y = F(K, N) \] and \( \lambda = \frac{1}{N} \) gives \( \frac{Y}{N} = F\left( \frac{K}{N}, \frac{1}{N} \right) \).

Notation: Denote \( y = \frac{Y}{N} \) and \( k = \frac{K}{N} \), then \( y = f(k) \).

Example
\[ Y = 2K^{0.3}N^{0.7} \Rightarrow \frac{Y}{N} = \frac{2K^{0.3}N^{0.7}}{N} = 2\left( \frac{K}{N} \right)^{0.3} \left( \frac{N}{N} \right)^{0.7} = 2\left( \frac{K}{N} \right)^{0.3} \Rightarrow y = 2k^{0.3} = f(k) \).

ENDOGENOUS GROWTH MODEL: SIMPLE SOLOW MODEL

Assumptions
1) \( Y_t = F(K_t, N_t) \) is CRTS and DRTF, so can rewrite \( y_t = f(k_t) \).
2) Demand side GDP \( Y_t = C_t + I_t + G_t \) and income side GDP \( Y_t = C_t + T_t + S_t \) gives
\[ I_t + G_t = T_t + S_t \Rightarrow I_t = S_t + (T_t - G_t) \]. Assume no government, so \( I_t = S_t \).

3) \( C_t = c_0 + c_1(Y_t - T_t) \). Assume \( T = 0 \) and \( c_0 = 0 \), we get \( C_t = c_1Y_t \Rightarrow S_t = Y_t - C_t = Y_t - c_1Y_t = (1-c_1)Y_t \), so \( S_t = sY_t \).

4) Capital accumulation dynamics: \( \frac{K_{t+1}}{K_t} = \frac{I_t - \delta K_t}{K_t} \) (inflow - outflow).

5) \( \frac{\Delta N_t}{N_t} = \frac{N_{t+1} - N_t}{N_t} = g_N \). Note: We first assume \( g_N = 0 \), so \( N_{t+1} = N_t = N \).

Derivations
\[
\frac{K_{t+1} - K_t}{N_t} = \frac{I_t - \delta K_t}{N_t} \Leftrightarrow K_{t+1} - K_t = sY_t - \delta K_t \Leftrightarrow \frac{K_{t+1}}{N_t} - \frac{K_t}{N_t} = \frac{sY_t}{N_t} - \delta \frac{K_t}{N_t} \Leftrightarrow k_{t+1} - k_t = sf(k_t) - \delta k_t .
\]

- At \( k_t = k_1 \), \( sf(k_1) > \delta k_1 \Rightarrow k_{t+1} > k_t \), so capital will increase.
- At \( k_t = k_2 \), \( sf(k_2) < \delta k_2 \Rightarrow k_{t+1} < k_t \), so capital will decrease.
- At \( k_t = k^* \), \( sf(k^*) = \delta k^* \) (inflow = outflow), therefore \( k_{t+1} - k_t = 0 \). The point \( \left(k^*, y^*\right) \) is called the steady state equilibrium.

Note
At the steady state equilibrium, the rate of growth of \( y, k, Y, K \) are all 0. This is the problem with Solow Model (see Stylized Facts 3).

Golden Rule Saving Rate
To maximize consumption, we need a “Golden Rule Saving Rate” \( s_g \) such that
\[
\begin{align*}
  f'(k) &= \delta \\
  s_g f'(k) &= \delta k \\
  s_g f(k) &= k
\end{align*}
\]

ENDOGENOUS GROWTH MODEL: SOLOW MODEL WITH POPULATION GROWTH
Assumption

Now we let \( g_N = \frac{\Delta N_t}{N_t} = \frac{N_{t+1} - N_t}{N_t} \neq 0 \). Then \( k_{t+1} - k_t = sf(k_t) - (\delta + g_N)k_t \).

Steady State Equilibrium

At steady state equilibrium:
- Growth rates of \( k \) and \( y = 0 \).
- Growth rates of \( K \) and \( Y = g_N \).

**Endogenous Growth Model: Solow Model with Population Growth and Labour Augmenting Technological Progress**

**Assumptions**

1. We change our production function from \( Y = F(K, N) \) to \( Y = F(K, AN) \) where \( AN \) is the “effective labour force”.
2. \( g_N = \frac{\Delta N_t}{N_t} = \frac{N_{t+1} - N_t}{N_t} \neq 0 \), \( g_A = \frac{\Delta A_t}{A_t} = \frac{A_{t+1} - A_t}{A_t} \neq 0 \).

**Derivation**

\( Y = F(K, AN) \Rightarrow \frac{Y}{AN} = \frac{F(K, AN)}{AN} \). Define \( \dot{y} = \frac{Y}{AN} \) and \( \dot{k} = \frac{K}{AN} \). Then \( \dot{k} = \dot{F}(\dot{k}) \). Thus, the capital accumulation dynamics is \( \dot{k}_{t+1} - \dot{k}_t = sf(\dot{k}_t) - (\delta + g_N + g_A)\dot{k}_t \).

**Steady State Equilibrium**
At steady state equilibrium:

- Growth rates of per effective worker capital ($\hat{k}$) and per effective worker output ($\hat{y}$) are 0.
- Growth rates of per worker capital ($k$) and per worker output ($y$) are $g_A \neq 0$.
- Growth rates of total capital ($K$) and total output ($Y$) are $g_N + g_A \neq 0$.

**Endogenous Growth Model: AK Model**

**Assumptions**

1) The production function is $Y = AK$ (CRTS, but not DRTF).

2) $g_N = \frac{\Delta N_t}{N_t} = \frac{N_{t+1} - N_t}{N_t} \neq 0$.

**Derivations**

$Y = AK \Rightarrow \frac{Y}{N} = A \frac{K}{N} \Rightarrow y = Ak$, and thus $k_{t+1} = sAk_t - (g_N + \delta)k_t$.

**Result**

There is no steady state equilibrium. It grows forever. $\frac{k_{t+1} - k_t}{k_t} = \Delta k_t = sA - g_N + \delta$ and $\frac{y_{t+1} - y_t}{y_t} = \Delta y_t = sA - g_N + \delta$. 
CONDITIONAL CONVERGENCE

Each country converges to its own steady state equilibrium.

TWO PERIOD ECONOMY

Note
Close economy: \( \begin{align*}
Y &= C + I + G \\
Y &= C + S + T \\
\Rightarrow I &= S + (T - G).
\end{align*} \)

Open economy: \( \begin{align*}
Y &= C + I + G + NX \\
Y &= C + S + T \\
\Rightarrow I &= S + (T - G) - NX.
\end{align*} \)

Note
In an open economy, \( NX = Y - C - I - G \). In long run, we assume \( G = 0 \), so \( NX = Y - C - I \).

Current and Capital Account Dynamics
Let \( \beta^f \) denote foreign assets. Then \( \beta^f_{t+1} = \beta^f_t + i \left( \frac{1}{1+i} \right) + (Y_t - C_t - I_t) \) (return) \( NX \), \( \beta^f_{t+1} - \beta^f_{t-1} = i \left( \frac{1}{1+i} \right) + (Y_t - C_t - I_t) \) (capital account) \( \text{current account } = CA \).

Two Period Economy
Case 1 – Only Lending/Borrowing Possibilities, No Investment Possibilities
Use \( C_1 = C_2 \) to approximate the optimal point.

Note:
- Slope of budget line is \( -(1+i) \).
- \( \beta^f_0 = \beta^f_1 = 0 \).

Two Period Economy
Case 2 – Only Investment Possibilities, No Lending/Borrowing Possibilities
Use \( C_1 = C_2 \) to approximate the optimal point.
Two Period Economy  
Case 3 – Both Investment and Lending/Borrowing Possibilities  
The optimal point is at \( MPK = i \).

**Expectations**

**Definition: Real Interest Rate**  
The real interest rate, \( r \), is the rate of return to investment/goods.

**Definition: Nominal Interest Rate**  
The nominal interest rate, \( i \), is the real interest rate + inflation (+ risk).

**Relationship Between Real and Nominal Interest Rates**

\[
\begin{align*}
\text{Time } t & \quad \rightarrow \quad \text{Time } t + 1 \\
1 \text{ unit of good} & \quad \rightarrow \quad 1 + r_i \text{ units of good} \\
P_t & \quad \downarrow \\
SP_t & \quad \rightarrow \quad S(1 + i_t)P_t \quad \text{or} \quad \frac{S(1 + i_t)}{P_{t+1}} 
\end{align*}
\]

So, \( 1 + r_i = \frac{(1 + i_t)P_t}{P_{t+1}} \). We also have that \( \pi_t^c = \frac{P_{t+1}^c - P_t}{P_t} = \frac{P_{t+1}^c}{P_t} - 1 \). Therefore, \( 1 + r_i = (1 + i_t) \frac{1}{\pi_t^c + 1} \), and thus \( r_i = i_t - \pi_t^c \) or \( i_t = r_i + \pi_t^c \).

**Compounding and Discounting**

**Compounding**  
\[
FV_N = PV\left(1 + i_1\right)\left(1 + i_2\right) \cdots \left(1 + i_{t+N-1}\right),
\]

where \( FN_N \) is the future value at time \( N \), and \( PV \) is the present value.

**Discounting**  
\[
PV = \frac{FV_N}{\left(1 + i_1\right)\left(1 + i_2\right) \cdots \left(1 + i_{t+N-1}\right)}.
\]

**Project Evaluation**

1) Present Discounted Value: \( \$V_t = Z_t + \frac{Z_{t+1}^c}{1 + i_t} + \frac{Z_{t+2}^c}{(1 + i_t)(1 + i_{t+1})} + \cdots + \frac{Z_{t+N}^c}{(1 + i_t)(1 + i_{t+1}) \cdots (1 + i_{t+N-1})} \), where \( Z_t \) is the future value/cash flow.

2) Expected Present Discounted Value: \( \$V_t^e = Z_t^c + \frac{Z_{t+1}^c}{1 + i_t^e} + \frac{Z_{t+2}^c}{(1 + i_t^e)(1 + i_{t+1}^e)} + \cdots + \frac{Z_{t+N}^c}{(1 + i_t^e) \cdots (1 + i_{t+N-1}^e)} \).
Special Cases

1) Annuity. It pays $Z$ for $N$ years starting next year with interest rate $i$. Thus its present value is
$$V = \frac{Z}{i} + \frac{Z}{i^2} + \cdots + \frac{Z}{i^N} = Z \left[ \frac{1-\left(1+i\right)^{-N}}{i} \right].$$

2) Annuity Due. It pays $Z$ for $N$ years starting this year with interest rate $i$. Thus its present value is
$$V = Z + \frac{Z}{i} + \frac{Z}{i^2} + \cdots + \frac{Z}{i^N} = Z \left[ \frac{1-\left(1+i\right)^{-N}}{i} + 1 \right].$$

3) Perpetuity. It pays $Z$ forever starting next year with interest rate $i$. Thus its present value is
$$V = \frac{Z}{i} + \frac{Z}{i^2} + \cdots = \lim_{N \to \infty} Z \left[ \frac{1-\left(1+i\right)^{-N}}{i} \right] = \frac{Z}{i}.$$

4) Delayed Annuity. It pays $Z$ for $N$ years starting at year $K$ with interest rate $i$. Thus its present value is
$$V = Z \left[ \frac{1-\left(1+i\right)^{-N}}{i} \right] \frac{1}{\left(1+i\right)^K} = Z \left[ \frac{1-\left(1+i\right)^{-N}}{i\left(1+i\right)^K} \right].$$

5) Delayed Perpetuity. It pays $Z$ forever starting at year $K$ with interest rate $i$. Thus its present value is
$$V = \frac{Z}{i} \left[ \frac{1}{\left(1+i\right)^K} \right] = \frac{Z}{i\left(1+i\right)^K}.$$

Policy/Shocks on Real Interest Rate

Recall that
$$IS : Y = C + G + I(Y, r)$$
$$LM : \frac{M}{P} = Y \cdot L(i)$$
Suppose $\pi^e \uparrow \Rightarrow r \downarrow \Rightarrow I \uparrow \Rightarrow IS \text{ shifts right} \Rightarrow AD \text{ shifts right}.$$

Bonds and Stocks

<table>
<thead>
<tr>
<th>Bonds</th>
<th>Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Issued by government or by private sector (debt financing).</td>
<td>Issued by private sector (equity financing).</td>
</tr>
<tr>
<td>Predetermined payment (face value, coupons).</td>
<td>Paid out of company profits (dividends), but a company can choose to retain earnings.</td>
</tr>
</tbody>
</table>

Bonds

From discounting, we know
$$P_{1,t} = \frac{Z}{1+i_{1,t}}, \quad P_{2,t} = \frac{Z}{(1+i_{1,t})(1+i_{1,t+1})} = \frac{P_{1,t+1}}{1+i_{1,t}},$$
so, for stripped coupon bonds (no coupons),
$$P_{N,t} = \frac{Z}{(1+i_{1,t})(1+i_{1,t+1})\cdots(1+i_{1,t+N-1})} = \frac{P_{N-1,t+1}}{1+i_{1,t}} = \frac{P_{N-2,t+2}}{1+i_{1,t+1}} = \cdots.$$
Yield To Maturity
The yield is the annual interest rate \( i_{N,t} \) that is constant for \( N \) years.

Thus for bonds,

\[
\frac{SZ}{(1 + i_{1,t}) \cdots (1 + i_{N,t} + N - 1)} = \left( \frac{SZ}{1 + i_{N,t}} \right)^N = \left( 1 + i_{1,t} \right) \cdots \left( 1 + i_{N,t} + N - 1 \right) \Rightarrow i_{N,t} = \frac{1}{N} \left( i_{1,t} + \cdots + i_{N,t} + N - 1 \right).
\]

Also note that \( i_{N,t} = \left( \frac{SZ}{P_{N,t}} \right)^N - 1 \).

We can draw a yield curve, plotting yield against time. From this yield curve, one can conclude the market’s expectation of future interest rates.

Stocks
Ex-dividend price \( Q = \frac{SD_{i+1}^P + SD_{i+2}^P}{1 + i_{i,t}} + \cdots + \frac{SD_{i+N}^P}{(1 + i_{i,t})(1 + i_{i+1}) \cdots (1 + i_{i+N-1})} \).

The fundamentals that affect stock prices are: current interest rate, expected interest rates, expected output.

Expectations and Consumption

Consumption Theories
1) Keynesian Consumption Theory: \( C_t = c_0 + c_1 Y_t \) (no taxes).
2) Milton Friedman: Permanent Theory of Consumption.

Permanent Theory of Consumption and Life Cycle Theory of Consumption arise because consumption does not depend only on current income; consumers are forward-looking (foresighted).

PERMANENT INCOME THEORY

Assumptions
1) MPC out of permanent \( Y^P \) income is larger, closer to 1.
2) MPC out of transitory \( Y^T \) income is smaller, closer to 0.

Example
Suppose \( C_t = 0.9 Y_t^P \) and \( Y_t^P = \frac{1}{5}(Y_{t-2} + Y_{t-1} + Y_t + Y_{t+1} + Y_{t+2}) \). Then \( C_t = 0.18(Y_{t-2} + Y_{t-1} + Y_t + Y_{t+1} + Y_{t+2}) \) or

\[
C_t = 0.18(Y_{t-2} + Y_{t-1} + Y_{t+1} + Y_{t+2}) \left( \frac{1}{c_0} \right) + \left( \frac{1}{c_1} \right) 0.18 Y_t. \]

Here, \( MPC_P = 0.9 \) and \( MPC_T = 0.18 \).

LIFE CYCLE THEORY
Assumptions
Consumption is a function of total wealth. \( C_t = C(W_t) \).
Wealth consists of:
1) Human capital (expected present discounted value of lifetime after tax labour income).
2) Nonhuman capital (financial assets, physical assets).

Note
In reality, \( C_t = C(W_t, Y_t - T_t) \).

Market Failure
Cannot use future assets as collateral!

Note
Sometimes, with higher income, one enjoys life more. \( C_t = C(Y_t, r^e, r^c, T_t, T^e) \).