Asset Markets

What Are Assets
Something acquired today that will have purchasing power later.
1. Bonds, stocks, treasury-bills.
2. Physical assets – houses, machinery, etc.
3. Human capital.

Assumptions
1. No transaction costs.
2. Short sales of securities permitted.
3. Agents limited by wealth constraint – can't acquire liabilities today that they will be unable to honour in the future.
4. No arbitrage condition.

Rate of Return

Ex-Post Nominal Rate of Return
- Suppose t-bill price last year is $955, and t-bill price this year (today) is $1000. Let \( r \) = rate of return. Then
  \[
  1 + r = \frac{\text{payoff}}{\text{previous price}} = \frac{1000}{955} \Rightarrow r = 0.04712 = 4.712\% .
  \]
- Here, \( r \) is the interest rate a year ago.
- Suppose stock price last year is $52, and stock price this year is $50. Then
  \[
  1 + R = \frac{\text{current price}}{\text{previous price}} = \frac{50}{52} \Rightarrow R = -0.0385 = -3.85\% .
  \]
- \( r \) and \( R \) are the ex-post nominal rates of return.

Ex-Ante Nominal Rate of Return
Suppose current price \( S_0 = $50 \), and price one year from now \( S_1 = ? \). Then
  \[
  1 + \hat{R} = \frac{S_1}{S_0} \Rightarrow \hat{R} = ?
  \]
is the ex-ante nominal rate of return (a random variable). It matters for decision making.

Ex-Ante Real Rate of Return
Let \( P_0 = \) current consumption price and \( P_1 = \) future consumption price. Then
  \[
  1 + \hat{R} = \frac{S_1}{S_0} \frac{P_0}{P_1} = \frac{S_1}{P_1} \frac{P_0}{S_0} = (1 + \hat{R}) \frac{P_0}{P_1} .
  \]

Asset Pricing Under Perfect Certainty
Suppose we have:
- Two consumption periods, current period 0 and future period 1.
- Consumption price 0 = consumption price 1.
- \( r = \hat{R} = 4\% \).
Suppose \( S_0 = $90 \), then \( \hat{R} = 15.56\% \). This is impossible because one can borrow $90 and buy stocks in period 0, and get $104 and repay $90 \times 1.04 = $93.60 \text{ in period 1. This is impossible because this is arbitrage.}
Therefore, the equilibrium price for the stock \( S_0 = $100 = \frac{104}{1.04} = \text{present discounted value} \).

Discounting
Let \( P_0^1 \) denote the price today of a one-period discount bond that will pay $1 in period 1. Then
Let \( P_{0}^{(1)}(1 + r_{0}) = \$1 \Leftrightarrow P_{0}^{(1)} = \frac{\$1}{1 + r_{0}} \).

Let \( P_{0}^{(2)} \) denote the price today of a one-period discount bond that will pay \$1 in period 2. Then
\[
P_{1}^{(2)} = \frac{\$1}{1 + r_{1}} \Leftrightarrow P_{0}^{(2)} = \frac{\$1}{(1 + r_{0})(1 + r_{1})}.
\]

Let \( r_{0}^{(2)} \) denote the 2-year rate of interest in period 0.
\[
P_{0}^{(2)} = \frac{\$1}{(1 + r_{0}^{(2)})^2} \Rightarrow r_{0}^{(2)} = \sqrt{\frac{\$1}{P_{0}^{(2)}}}.
\]

**Yield To Maturity (a.k.a. Internal Rate of Return)**

The yield to maturity of asset \( y \) that pays \$\( D_{T} \) in period \( T \) is \( i_{0}(y) \) such that
\[
P_{0}(y) = \sum_{t=1}^{\infty} \frac{D_{t}}{(1 + i_{0}(y))^{t}}.
\]

**Equity/Share Price**

The ex-dividend price of a share in period 0 is \( S_{0}^{ex} = \frac{D_{1}}{1 + r_{0}} + \frac{D_{2}}{(1 + r_{0})(1 + r_{1})} + \cdots + \frac{D_{T}}{(1 + r_{0})(1 + r_{1})\cdots(1 + r_{T-1})} \).

The pre-dividend price of a share in period 0 is \( S_{0}^{pre} = S_{0}^{ex} + D_{0} \).

If dividend \( D \) is constant and interest rate \( r \) is constant and the share pays forever, then
\[
S_{0}^{ex} = \sum_{t=1}^{\infty} \frac{D}{(1 + r)^{t}} = \frac{D}{r} \quad \text{and} \quad S_{0}^{pre} = \frac{D}{r} + D.
\]

If dividends grow at the constant rate \( g \) per time period and \( g < r \), then
\[
S_{0}^{ex} = \sum_{T=1}^{\infty} D_{0} \left( \frac{1 + g}{1 + r} \right)^{T} \quad \text{and} \quad S_{0}^{pre} = D_{0} \left( \frac{1 + g}{1 + r} \right)^{1} - \frac{D_{1}}{r - g}.
\]

**Utility Maximization in a Two-Time Period**

Lifetime utility: \( U_{0} = U(c_{0}, c_{1}) \).

Endowment: \((y_{0}, y_{1})\).

Initial wealth: \( W_{0} = y_{0} + \frac{y_{1}}{1 + r} \).

Budget constraint: \( c_{t} = y_{t} + (1 + r)(y_{t-1} - c_{0}) \quad \text{or} \quad c_{0} + \frac{c_{1}}{1 + r} = W_{0} \).

Maximum utility when \( \frac{\text{MRS}}{\text{slope of highest indifference curve}} = \frac{\text{MRT}}{-(1 + r)} \).

**Fisher Separation Theorem**

Corporate decisions independent of individual preference.

**Asset Pricing Under Uncertainty**

**Expected Utility Theory**

Utility is given by \( U(C) \). An individual will choose between \( A \) and \( B \) according to highest utility, ie prefer \( A \) iff \( E[U(A)] \geq E[U(B)] \).
Definitions
1. Certainty equivalent: The value $CE$ that solves $U(CE) = E[U(W)]$.
2. Risk premium: The maximum amount $\Pi$ that an individual would pay to exchange the risky wealth $W$ for $\bar{W}$; so $U(\bar{W} - \Pi) = E[U(W)] = U(CE) \Rightarrow \Pi = \bar{W} - CE$.

Types of Utility Functions
Let $W = W_0 + X$, where $X$ is a random variable with $E[X] = 0$ and $\text{var}(X) = \sigma^2 > 0$. Then the Markowitz approximation for $\pi$ is $\pi \approx \frac{1}{2} \sigma^2 \left[ -U''(W) \right]$. 

Measure of Risk Aversion
Pratt Measure of Absolute Risk Aversion (ARA)
Pratt measure of ARA: $\frac{-U'''(W_0)}{U'(W_0)}$.
Note that this implies ARA decreases as $W_0$ increases.

Arrow Measure of Relative Risk Aversion (RRA)
Arrow measure of RRA: $\frac{-W_0 U''(W_0)}{U'(W_0)}$.
This is much more reasonable.

Class of CRRA Utility Function
$U(W) = \frac{W^{1-y}}{1-y}$, for $y \geq 0$ and $U(W) = \ln W$, for $y = 1$.
Then, $\text{RRA} = -W \frac{U''}{U'} = y$.

Portfolio Choice
- Assume two time periods: 0, 1. Household has initial wealth $W$ at the start of period 0.
- Lifetime utility is $E[U(c_0, c_1)] = U(c_0) + \beta E[U(c_1)]$, where $\beta$ is the time preference parameter ($\beta \approx 0.96$ empirically).
- Household chooses:
  - $c_0$, so $W_0 = W - c_0$ is invested.
  - $\alpha W_0$ invested in risky asset ($r_1$ with $E(r_1) = r_1$ and $\text{var}(r_1) = \sigma^2_1$), and $(1 - \alpha)W_0$ invested in risk-free asset ($r_f$).
• Household has portfolio with return \( r_p = \alpha r_1 + (1 - \alpha) r_f \). So \( c_1 = W_0 (1 + r_p) \).

• Household will maximize \( E [U(c_0, c_1)] = U(W - W_0) - \beta E[U(W_0(1 + r_p))] \), or equivalently,
  \[
  \max_{\alpha} E[U(W_0(1 + \alpha r_1 + (1 - \alpha) r_f))] \quad . \text{The derivative is}
  \]

\[
E[U'(W_0(1 + \alpha r_1 + (1 - \alpha) r_f))(r_1 - r_f)] = 0 = E[U'(W_0(1 + \alpha r_1 + (1 - \alpha) r_f))|E[(r_1 - r_f)] + \text{cov}(\quad)
\]

- Case A: \( \tilde{r}_1 - r_f > 0 \Rightarrow \alpha > 0 \).
- Case B: \( \tilde{r}_1 - r_f < 0 \Rightarrow \alpha < 0 \).
- Case C: \( \tilde{r}_1 - r_f = 0 \Rightarrow \alpha = 0 \).

Result
If there are two households, the more risk averse one will have lower \( \tilde{\alpha} \), where \( \tilde{r}_1 > r_f \).

Approximation
Assume CRRA \( U(c) = \frac{c^{1-y}}{1-y} \), \( y \geq 0 \), \( U(c) = \ln c \), \( y = 1 \). Then \( \max_{\alpha} \beta E[U(c_1)] = \max_{\alpha} E[U(1 + r_p)] \). Note that as \( y \) increases, \( \tilde{\alpha} \) decreases.

An approximate for \( E[U(1 + r_p)] \) is \( E[U(1 + r_p)] = U(1 + r_f) + \frac{\sigma^2}{2}(1 + \bar{r}_p) = V(\bar{r}_p, \sigma_p) \), where \( \frac{\partial V}{\partial \bar{r}_p} > 0 \) and \( \frac{\partial V}{\partial \sigma_p} < 0 \).

This is a good approximation because:
1. If \( r_1 \) is normal, then \( E[U(1 + r_p)] = V(\bar{r}_p, \sigma_p) \).
2. Most assets are very close to normal.

Preferences
Indifference curves of \( V(\bar{r}_p, \sigma_p) \):

- The are upward sloping because \( \sigma \) is a “bad”.
- The slope is the MRS.
- The more risk averse an individual is, the steeper the curves.

Market Opportunities
If an individual invests \( \tilde{\alpha} \) in risky asset, \( 1 - \tilde{\alpha} \) in risk-free asset, then \( \bar{r}_p = \alpha \bar{r}_1 + (1 - \alpha) r_f = r_f + \alpha (\bar{r}_1 - r_f) \) and \( \sigma_p^2 = \tilde{\alpha}^2 \sigma_1^2 \Leftrightarrow \sigma_p = \alpha \sigma_1 \).
• An individual will only choose the effect set of portfolio (upper branch) – it has higher rate of return for each $\sigma$.

• The equation of the effective set is $\tilde{r}_p = r_f + \left( \frac{\tilde{r}_1 - r_f}{\sigma_1} \right) \sigma_p$.

• The slope is the MRT.

**Equilibrium**
The equilibrium is when $\text{MRS} = \text{MRT}$, i.e. when the indifference curve is tangent to the market opportunity curve.

**Market Opportunities: Two Risky Assets**
Let $\alpha$ invested in riskier asset 1, and $1-\alpha$ invested in less risky asset 2. Then $r_p = \alpha r_1 + (1-\alpha) r_2$ with $\tilde{r}_p = \alpha \tilde{r}_1 + (1-\alpha) \tilde{r}_2$ and $\sigma_p^2 = \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2 \alpha (1-\alpha) \rho_{1,2} \sigma_1 \sigma_2$ where $\rho_{1,2} = \frac{\text{cov}[r_1, r_2]}{\sigma_1 \sigma_2}$.

If $\rho_{1,2} = 1$, then $\sigma_p = \alpha \sigma_1 + (1-\alpha) \sigma_1$.

If $\rho_{1,2} = -1$, then $\sigma_p = \alpha \sigma_1 - (1-\alpha) \sigma_1$. 

If \(-1 < \rho_{1,2} < 1\) (usual case), then 
\[ \sigma_p = \sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2 \alpha (1-\alpha) \rho_{1,2} \sigma_1 \sigma_2}. \]

The efficient set is all portfolios with \( r_p > \bar{r}_{\text{min var}} \).

Note: 
\[ \alpha_{\text{min var}} = \frac{\sigma_2^2 - \rho_{1,2} \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \rho_{1,2} \sigma_1 \sigma_2}. \]

**Market Opportunities: Two Risky Assets and a Risk-Free Asset**

Suppose \( \bar{\bar{r}}_1 > \bar{\bar{r}}_2 > r_f \) and \( \bar{\bar{\sigma}}_1 > \bar{\bar{\sigma}}_2 \).

Tangency portfolio: \( \alpha_T \) in asset 1, \( 1-\alpha_T \) in asset 2, and thus 
\( r_T = \alpha_T \bar{r}_1 + (1-\alpha_T) \bar{r}_2 \).

Agent's Portfolio: \( r_p = w r_r + (1-w) r_f \), and \( \sigma_p = w \bar{\sigma}_r \).

**Two-Fund Theorem**

Any agent will have the same risky portfolio, the tangency portfolio.

**Market Opportunities: All Risky Assets and a Risk-Free Asset**

CML (capital market line) is the opportunity set available to every investor. This is the efficient set.
Note: The two-fund theorem still applies here.

Consider all risky assets 1, 2, ..., N. Let asset i have value $V_i$, and $V_M = \sum_{i=1}^{N} V_i$.

The market portfolio is $\frac{V_1}{V_M}$ in asset 1, $\frac{V_2}{V_M}$ in asset 2, etc. Then $\mu_M = \sum_{i=1}^{N} \frac{V_i}{V_M} \mu_i$.

Note: The tangency portfolio = market portfolio.
Note: Lending = borrowing, so average agent is at M.

An agent's portfolio is $\hat{\mu}_p = w \hat{\mu}_M + (1-w) \mu_f$ with $\hat{\sigma}_p = w \sigma_M$.

Equation of CML is $\hat{\mu}_p = \mu_f + \frac{\hat{\mu}_M - \mu_f}{\sigma_M} \hat{\sigma}_p$.

**Security Market Line**

Now consider asset j. If an agent invests $\alpha$ in market portfolio and $1-\alpha$ in asset j, then $\hat{\mu}_p = \alpha \hat{\mu}_M + (1-\alpha) \hat{\mu}_j$ and

$\sigma_p = \sqrt{\alpha^2 \sigma_M^2 + (1-\alpha)^2 \sigma_j^2 + 2 \alpha(1-\alpha) \sigma_{M,j}}$. When $\alpha = 1$, the slope of CML is $\frac{\mu_M - \mu_f}{\sigma_M}$. Thus

$\hat{\mu}_j = \mu_j + \frac{\text{cov}(\hat{\mu}_M, \mu_j)}{\sigma_M^2}(\hat{\mu}_M - \mu_f) = \mu_j + \beta_j (\hat{\mu}_M - \mu_f)$; the bigger $\beta_j$, the riskier it is!

**Diversifying Portfolio**

Notice $\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_j^2$, i.e. total risk = systematic risk + unsystematic risk. An agent is only compensated for systematic risk.

Empirical Security Market Line: Since $E(r_j) = \mu_j$ not observable, write $\mu_j = \hat{\mu}_j + \beta_j (\hat{\mu}_M - \mu_f) + \epsilon_j$ with $E(\epsilon_j) = 0$ and $\text{cov}(\epsilon_j, \mu_f) = 0$.

**CML**

R

$w < 1$

$sigma$

$0 < w < 1$

$\hat{\mu}_j$
If \( r_p = w_1 r_1 + \cdots + w_k r_k, \) \( \sum w_i = 1 \), then \( r_p = r_f + \beta_p (r_M - r_f) + \sum w_i \varepsilon_i, \beta_p = \sum w_i \beta_i \) with \( r_p = r_f + \beta_p (r_M - r_f) \) and 
\[ \sigma_p = \beta_p^2 \sigma_M^2 + \text{var} \left( \sum w_i \varepsilon_i \right). \]
For large \( k \), \( \sigma_p \approx \beta_p^2 \sigma_M^2 \); the unsystematic risk disappears!

Notice \( r_p = \alpha r_m + (1 - \alpha) r_f = r_f + \alpha (r_M - r_f) \), so \( \beta_p = \alpha. \)

**Application**

Suppose the payoff is \( X \) in period 1, with \( X \) and \( \sigma_X^2 \). The price of \( X \) in period 0 is 
\[ P_X = \frac{X}{1 + r_f + d}. \]

What is \( d \)? Since \( 1 + r_f + d = \frac{X}{P_X} = 1 + r_f \) and \( r_j = r_f + \beta_j (r_M - r_f) \), so \( d = \beta_j (r_M - r_f) \).

**No Risk Free Asset**

Let \( Z \) be the zero-beta portfolio, that is \( \beta_z = 0 \iff \text{cov} (r_Z, r_M) = 0 \).

The slope at \( M \) is \( \frac{r_M - r_z}{\sigma_M^2} \). So \( r_j = \beta_j (r_M - r_Z) \) (empirical: \( r_j = \beta_j (r_M - r_Z) + \varepsilon_j \)).

Note that in this model, different agents will hold different risky assets, with the average agent holding \( M \).

**State Preference Theory**

Suppose agents A and B has endowments:

<table>
<thead>
<tr>
<th>Time 0</th>
<th>( y_0^A )</th>
<th>( y_0^B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time 1</td>
<td>( y_1^A )</td>
<td>( y_1^B )</td>
</tr>
<tr>
<td></td>
<td>( U^A(c_0, c_1) )</td>
<td>( U^B(c_0, c_1) )</td>
</tr>
</tbody>
</table>

For each agent,
- Time 0: endowment \( y_0 \), utility \( U(c_0) + E(U(c_1)) \).
- Time 1: endowment \( y_1^A \) with probability \( \pi_1 \) (state 1), endowment \( y_1^B \) with probability \( \pi_2 \) (state 2), etc.

Suppose there are “pure securities”, with payoffs in time 1:

<table>
<thead>
<tr>
<th>Security</th>
<th>Price at Time 0</th>
<th>Payoff in State 1</th>
<th>Payoff in State 2</th>
<th>Payoff in State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure 1</td>
<td>( q_1 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pure 2</td>
<td>( q_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Pure 3</td>
<td>( q_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Pure A</td>
<td>( q_A = 3q_1 + 2q_2 + 4q_3 )</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
Asset Pricing: Pure Securities
Pure Securities: Any security in the market is a linear combination of pure securities with price \( p_i \) and $1 payout in the \( i \)-th state. So if \( S \) and any security and \( s_i \) is its payoff in the \( i \)-th state, then \( P_s = p_1 s_1 + \cdots + p_n s_n \).

Hedge Portfolio: A risky portfolio that replicated a risk-free portfolio. If it contains \( a_i \) amount of security \( i \) (priced \( P_i \)), then \( a_1 P_1 + a_2 P_2 + \cdots + a_n P_n = \frac{1}{1+r_f} \).

Risk Neutral Probabilities: If an agent is risk neutral, then for any security \( S \), \( P_S = \mathbb{E}(S) = \pi^{RN}_1 s_1 + \cdots + \pi^{RN}_n s_n \); \( \pi^{RN}_i \) is the risk neutral probability of state \( i \). Notice \( \pi^{RN}_i = \left(1+r_f\right)p_i \).

Continuous Time Process
Definition
The current period is date 0. $1 in a risk-free debt at date 0 is worth $ \( e^{r_f T} \) at date \( T \). Here \( r_f \) is the continuously compounded/instantaneous rate of interest.
Note: Time is in years, so \( T=1 \) is one year.

Value of Equity
The value of a share of equity at date \( T \) is \( S_T = S_0 e^{\int_0^T r_s(t)dt} \).
Note: \( \int_0^T r_s(t)dt \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right) \), so \( S_T \) is log-normal (i.e \( \ln \left(\frac{S_T}{S_0}\right) = \int_0^T r_s(t)dt \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right) \)).

Options
Call Option
Written on a stock with price \( S_S \), there is:
- exercise buying price \( X \),
- maturity date \( T \).
European Call Options: At time \( T \), one can “exercise” – sell at \( S_T \) and buy at \( X \). So the payoff will be \( \max(S_T-X, 0) \).
American Call Options: One can exercise at any time up to time \( T \).

Put Option
Written on a stock with price \( S_S \), there is:
- exercise selling price \( X \),
- maturity date \( T \).
European Put Options: At time \( T \), one can “exercise” – buy at \( X \) and sell at \( S_T \). So the payoff will be \( \max(X-S_T, 0) \).
American Put Options: One can exercise at any time up to time \( T \).

Put-Call Parity
If the price of a stock is $S_0$, $c_0$ is the price of a call option, $p_0$ is the price of a put option, and $X$ is the payout of a risk free debt at time $T$, then

$$S_0 + p_0 = c_0 + X e^{-rT}.$$ 

Note that for $0 < t < T$,
- Any asset is a combination of calls, puts, and risk free debts.
- $c_t = p_t + S_t - X e^{-r(T-t)} > p_t + S_t - X$ and $S_t - X e^{-r(T-t)} > S_t - X$, so nobody will exercise a call early.
- $p_t = c_t + X e^{-r(T-t)} - S_t$ and $X e^{-r(T-t)} - S_t < X - S_t$, so for $c_t$ small enough, one might exercise a put early.

**Risk Neutral Probability Pricing of a Call Option**

Consider a stock with price $S_0$ at date 0. Its payout is either $S_T^1 = uS_0$ (state 1) or $S_T^2 = dS_0$ (state 2) at date $T$. Then a call option's payout is $uS_0 - X$ in state 1 and 0 in state 2. Let $1 + r_f = e^{rT}$.

If we assume $d < 1 + r_f < u$ (or else arbitrage) and $dS_0 < X < uS_0$, then

$$c_0 = \frac{\pi^R N(uS_0 - X)}{1 + r_f},$$

where $\pi^R = \frac{1 + r_f - d}{u - d}$.

**Black-Scholes**

The price of a call (European or American) is $c_0 = S_0 N(d_1) - e^{-rT} X N(d_2)$, where

- $N(d_1) = P(z \leq d_1)$, $z \sim N(0,1)$
- $N(d_2) = P(z \leq d_2)$, $z \sim N(0,1)$
- $d_1 = \frac{\ln \left( \frac{S_0}{X} \right) + r_f T}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T}$  ($\sigma = \text{SD} \left( \ln S_t \right)$)
- $d_2 = d_1 - \sigma \sqrt{T}$.

Then by the put-call parity, the price of a put (European) is $p_0 = c_0 - S_0 + X e^{-rT}$.

**Consumption Based Capital Asset Pricing Model**

Every agent/household in the economy is identical. So only study a representative household.

**Two Time Periods, Perfect Certainty**

Suppose the representative household's lifetime utility is $U = U(C_0) + \beta U(C_1)$. Suppose its income in period 0 is $Y_0$ and its income in period 1 is $Y_1$.

Consider a discount bond issued by the government that pays 1 unit of consumption in period 1 and sells for $p_0 = \frac{1}{1 + r_0}$.

The household will choose $q_0$ optimally to maximize its lifetime utility. Then we get

$$p_0 = \beta \frac{U(Y_1 + q_0)}{U'(Y_0 - p_0q_0)}.$$ 

$p_0$ is the equilibrium price if $q_0 = q^*$ the quantity of bonds supplied per household.
**FINITE TIME PERIODS, UNCERTAINTY**

The representative household's lifetime utility (at period 0) is

\[ U_0 = U(C_0) + E_0 \left[ \beta \sum_{t=1}^{\infty} U(C_t) \right] \]

**One Period Security**

Suppose security \( X \) has current price \( P_{X,0} \). It can be redeemed in period 1 for \( x_1 \). Then we get

\[ P_{X,0} = \beta E_0 \left[ x_1 \frac{U'(C_1)}{U'(C_0)} \right] \]

Since \( \frac{x_1}{P_{X,0}} = 1 + r_{X,1} \), can rewrite \( 1 = \beta E_0 \left[ \frac{U'(C_1)}{U'(C_0)} \right] \).

**Risk Free Asset**

For risk-free asset, \( 1 = \beta \left[ 1 + r_{X,1} \right] E_0 \left[ \frac{U'(C_1)}{U'(C_0)} \right] \).

**t Period Security**

Suppose security \( Y \) has current price \( P_{Y,0} \) and pays \( y_t \) in period \( t \) (and nothing prior and after). Then

\[ P_{Y,0} = \beta^t E_0 \left[ y_t \frac{U'(C_t)}{U'(C_0)} \right] \]

**Price of a Stock**

The price of a stock that pays dividend \( \text{div}_t \) in period \( t \) is

\[ S_0 = E_0 \left[ \sum_{t=1}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \right] \]