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Under high stakes and uncertainty the rich should lend the poor a helping hand

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HIGHLIGHTS

- We model the effects of heterogeneity on decisions using a collective-risk dilemma.
- We aim to understand the natural behavior and to infer which strategies are particularly stable in asymmetric collective-risk games.
- Using an evolutionary model with heterogeneity and multiple rounds we analyze contributions and the natural state of these contributions.
- We explore when players contribute the same amount or when the rich players contribute on behalf of the poor.
- Under a certain degree of uncertainty we observe the rich maintain cooperation by assisting the poor.

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ABSTRACT

In social dilemmas, there is tension between individual incentives to optimize personal gain versus social benefits. An additional cause of conflict in such social dilemmas is heterogeneity. Cultural differences or financial inequity often interfere with decision making when a diverse group of individuals interact. We address these issues in situations where individuals are either rich or poor. Often, it is unclear how rich and poor individuals should interact – should the poor invest the same as the rich, or should the rich assist the poor? Which distribution of efforts can be considered as fair? To address the effects of heterogeneity on decisions, we model a collective-risk dilemma where players collectively have to invest more than a certain threshold, with heterogeneity and multiple rounds. We aim to understand the natural behavior and to infer which strategies are particularly stable in such asymmetric collective-risk games. Large scale individual based simulations show that when the poor players have half of the wealth the rich players posses, the poor contribute only when early contributions are made by the rich players. The rich contribute on behalf of the poor only when their own external assets are worth protecting. Under a certain degree of uncertainty we observe the rich maintain cooperation by assisting the poor.
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1. Introduction

Social dilemmas arise when it is costly for the individual to cooperate, but mutual cooperation is beneficial for the group (Gordon, 1954; Hardin, 1968). A typical assumption in social dilemmas is that all individuals have the same potential to contribute. However, social dilemmas in reality often involve diverse individuals with different power to contribute and different risk preferences. Such differences can alter the way decisions are made (McNamara et al., 2004; McNamara, 2013). This kind of diversity is captured by asymmetric social dilemmas that describe a group in which members possess different levels of wealth (Rapoport, 1988; Rapoport and Suleiman, 1993; Ledyard, 1995; Chan et al., 1999; De Cremer, 2007; Wang et al., 2010; Milinski et al., 2011). Such asymmetric interactions can model the behaviors between two individuals of different social status, large and small firms, local municipalities and federal authorities, or among countries of different economic power. The diversity in wealth between these actors causes additional conflict (Chan et al., 1999; Milinski et al., 2011; Tavoni et al., 2011; Jacquet et al., 2013). Differences in wealth can alter decisions because the diverse incentives cause overall uncertainty (Raihani and Aitken, 2011) and result in the inability to coordinate on a solution. For example, additional difficulties in negotiating targets for reducing global green house gas emissions arise because the costs and effects differ between rich and poor countries. Developing (poor) countries are concerned with short term gains since insufficient time has passed to build up assets. On the other hand, developed (rich) countries can consider long term gains since they have sufficient

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capital earned (Landis and Bernauer, 2012). Additionally there is no guarantee that countries can rely on each other and uncertainty can arise when there is a delay between paying the cost and obtaining the benefit of cooperating (Raihani and Aitken, 2011; Abou Chakra and Traulsen, 2012; Barrett and Dannenberg, 2012; Hilbe et al., 2013).

To address the effects of heterogeneity on decisions, a collective-risk dilemma was used in an experimental study between rich and poor individuals (Milinski et al., 2011). Subjects were provided with two types of endowments, a working account and external assets. The incentive behind these two funds is to mimic real life scenarios, individuals can access and use their working money instantaneously, whereas, their external assets can only be obtained in the future (at the end of the game). Rich subjects were given twice the working and external funds with respect to the poor subjects. Subjects had to collaborate and invest towards a certain target amount into a common pool; they had ten consecutive rounds in which they can invest 0€, 2€, or 4€ from their working endowment. If the group failed to reach the target, they risked losing their external funds with 90% probability, while retaining their remaining working endowment. However, if the group's total contributions met the target, then all group members retained their remaining working endowment and their external endowment (Milinski et al., 2008; Dreber and Nowak, 2008; Milinski et al., 2011; Jacquet et al., in press). Additional conflict arises between these two types of individuals, as rich players have more wealth at stake than poor players, but immediate investments hurt poor individuals more by depleting their already low working funds. Both types of subjects have an incentive to protect their wealth, however should the poor invest the same as the rich, or should the rich help and compensate for the poor?

Although several studies have considered heterogeneity between rich and poor individuals, it is unclear how rich and poor individuals should interact: Milinski et al. (2011) compared homogenous groups to heterogeneous groups and found that, indifferent of the type, subjects' total contributions did not differ between these treatments. Several studies showed that the amount of contributions also depends on the implemented risk functions such as linear, nonlinear, or step-level (Ledyard, 1995; Chan et al., 1999; Levati et al., 2007), intermediate targets (Milinski et al., 2011), or communication (Chan et al., 1999; Tavoni et al., 2011). These multifarious results call for a theoretical analysis to help understand the interactions and strategic behaviors between rich and poor subjects.

Previous theoretical models based on evolutionary game dynamics (Hofbauer and Sigmund, 1998; Nowak et al., 2004; Imhof and Nowak, 2006; Nowak, 2006) in collective-risk dilemmas consider either only homogenous groups (Wang et al., 2009; Greenwood, 2011; Abou Chakra and Traulsen, 2012; Barrett and Dannenberg, 2012; Hilbe et al., 2013) or heterogeneous groups with only a single round (Rapoport, 1988; Rapoport and Suleiman, 1993; Wang et al., 2010; Santos and Pacheco, 2011; Santos et al., 2012; Chen et al., 2012; Vasconcelos et al., 2013). However, by allowing for heterogeneity and several rounds, new behaviors can emerge, since players can influence each other and change their own behavior across the rounds (Erev and Rapoport, 1990; Varian, 1994; Abou Chakra and Traulsen, 2012; Hilbe et al., 2013).

Using an evolutionary model with heterogeneity and multiple rounds we analyze the amount contributed and the natural state of these contributions. For instance, our simulations show how the rich or poor players contribute relative to their endowments and whether the players contribute in early rounds or wait and contribute late. This method allows us to infer which strategies are particularly abundant in such asymmetric collective-risk games. We explore when players contribute the same amount or when the rich players compensate and contribute on behalf of the poor.

2. Model

2.1. Individuals

We define an individual strategy using thresholds τ $(0 \le \tau \le T)$ which determines an individual's decision with respect to the collective investments so far (Γ). That is, for every round r a player's strategy is defined as $(\tau_r; a_r, b_r)$, such that the player contributes an amount a_r if Γ is below the threshold τ, otherwise the player contributes b_r. Additionally, individuals are distinguished based on wealth. Individuals' wealth is determined by the amount available for investments and valuable assets they possess. Rich individuals differ in working endowments W and external endowments E from the poor individuals, we assume that $W_{rich} > W_{poor}$.

2.2. Collective-risk dilemma

We model a collective-risk dilemma played among rich and poor individuals. At the beginning of the game, each player receives two types of endowments, a working endowment W and an external endowment E. We assume an asymmetric evolutionary game played among a heterogeneous group of M individuals, half of which are poor players and half of which are rich players selected at random from two mixed populations, a poor one and a rich one. In this game, the external endowment, but not the working endowment, is at stake if the collective target T is not met. Players invest from their working endowment into a common pool, over the course of R rounds. A player i contributes I_i^r in round r resulting in total contribution of $C_i = \sum_{r=1}^{R} I_i^r$. If the group's contribution, $\Gamma = \sum_{i=1}^{M} C_i$, meets the target by the end of the game, then each player i keeps their external endowment and the retained working endowment, receiving $W_i - C_i + E_i$ as payoff, π. However if the target is missed then all players lose their external endowment with some exogenous probability p and thus obtain the payoff $W_i - C_i$. With probability $1-p$, their external endowment is not lost and they receive $W_i - C_i + E_i$ as payoff, π.

2.3. Evolutionary game dynamics

Due to the complexity of the game, we perform individual based simulations instead of working with the replicator dynamics (Taylor and Jonker, 1978; Hofbauer et al., 1979; Zeeman, 1980; Hofbauer and Sigmund, 1998; Nowak, 2006). An asymmetric collective-risk dilemma is played among all individuals chosen from a heterogenous populations. The average payoffs per game (π, average retained endowment) of each player are computed after playing G games in one generation. At the end of a generation, the payoff π_i is translated into a fitness $f_i = \exp[\beta \pi_i]$, where β measures the intensity of selection (Traulsen et al., 2008). The next generation is selected in proportion to the fitness using the Wright–Fisher process (Hartl and Clark, 2007; Imhof and Nowak, 2006; Traulsen et al., 2006). To avoid that strategies that are only beneficial to rich players are adopted by poor players (and vice versa), we assume two different subpopulations, a rich and a poor one. In both of these subpopulations, players are selected in proportion to others of the same wealth type: Rich players compete for the next generation with other rich players. Poor players compete with other poor players. Thus, for each of the two populations there is a separate Wright–Fisher process such that number of rich or poor players remain constant.

Individuals use a single strategy; an offspring inherits the strategy of the selected parent, subject to mutations. We assume

that an offspring's strategy is mutated with probability μ, which occurs independently for the thresholds and the investments. There are many ways to implement mutations. A particularly simple one (which we have also used in related models Abou Chakra and Traulsen, 2012; Hilbe et al., 2013) is the following: when the investments (a_r, b_r) are mutated, a new investment is chosen with uniform probability. When the thresholds are mutated, the new threshold τ has a Gaussian distribution with standard deviation σ around the old threshold.

3. Results

3.1. One round game

We first consider the simplest case where two players make a single decision, which cannot be conditioned: Poor players have a working endowment W_{poor} and an extra endowment E_{poor}. Rich players have a working endowment $W_{rich} = 2 \cdot W_{poor}$ and an extra endowment E_{rich}. The target is $T = 2 \cdot W_{poor}$, and the possible contributions of the rich are 0, W_{poor}, and W_{rich}, whereas the poor can only invest 0 or W_{poor}. In this case, the game can be captured by an asymmetric 3×2 payoff matrix for a rich player versus a poor player given by

(C_{rich}, C_{poor})	0	W_{poor}
0	$(2W_{poor}+(1-p)E_{rich},\ W_{poor}+(1-p)E_{poor})$	$(2W_{poor}+(1-p)E_{rich},\ (1-p)E_{poor})$
W_{poor}	$(W_{poor}+(1-p)E_{rich},\ W_{poor}+(1-p)E_{poor})$	$(W_{poor}+E_{rich},\ E_{poor})$
$2W_{poor}$	$(E_{rich},\ W_{poor}+E_{poor})$	$(E_{rich},\ E_{poor})$

(1)

Meeting the target requires either both the rich player and the poor player contributing the same, $C_{rich} = C_{poor} = W_{poor}$, or the rich player contributing all of her working endowment and the poor player contributing nothing ($C_{rich} = W_{rich}$ and $C_{poor} = 0$). Contributing all of the working endowment is costly, especially if the target is not met and players obtain a payoff of zero with probability p. In this simple case, a full analytical consideration based on the payoff matrix is possible. There are potentially three evolutionarily stable states of the system:

- The situation without any contributions, $(C_{rich}, C_{poor}) = (0, 0)$, is stable if $p < 2W_{poor}/E_{rich}$.
- For $p > 2W_{poor}/E_{rich}$, $(C_{rich}, C_{poor}) = (2W_{poor}, 0)$ become stable.
- The state with $(C_{rich}, C_{poor}) = (W_{poor}, W_{poor})$ is stable if $p > W_{poor}/E_{poor}$ (which implies $p > W_{poor}/E_{rich}$ if $E_{rich} > E_{poor}$).

These stability considerations lead to four possible regimes:

(i) When $p < 2W_{poor}/E_{rich}$ and $p < W_{poor}/E_{poor}$, neither the rich nor the poor contribute, and the system ends up in mutual defection, $C_{rich} = C_{poor} = 0$. Mutual defection is also observed when $p < W_{poor}/E_{rich}$ and the rich have less assets than the poor $E_{rich} < E_{poor}$.
(ii) When $p > 2W_{poor}/E_{rich}$ and $p < W_{poor}/E_{poor}$, the rich players always contribute, $C_{rich} = W_{rich}$, whereas, the poor players always defect, $C_{poor} = 0$.
(iii) When $2W_{poor}/E_{rich} > p > W_{poor}/E_{rich}$ and $p > W_{poor}/E_{poor}$, both players contribute the same amount, either both defect $C_{rich} = C_{poor} = 0$ or both contribute $C_{rich} = C_{poor} = W_{poor}$.
(iv) When $p > 2W_{poor}/E_{rich}$ and $p > W_{poor}/E_{poor}$, either the rich players contribute everything $C_{rich} = W_{rich}$ and the poor players contribute nothing $C_{poor} = 0$ or the poor players contribute everything and the rich players contribute half of their working endowment $C_{rich} = C_{poor} = W_{poor}$.

Thus, there are two major transitions between regimes, cf. Fig. 1b. When p becomes larger than $2W_{poor}/E_{rich}$, rich players always invest. When p becomes smaller than W_{poor}/E_{poor}, poor players never invest. In addition to our analytical considerations, Fig. 1b shows the results of individual based simulations based on the model that is used for more complex setups below.

3.2. Timing of contributions in the two round game

In the single round game, rich players contribute everything when they have more at stake than their working endowment, $E_{rich} > W_{rich}$, and when the poor's endowments fulfill the opposite ranking, $E_{poor} < W_{poor}$. When $E_{rich} > W_{rich}$ and $E_{poor} > W_{poor}$ the rich players contribute only half of their working endowment and the

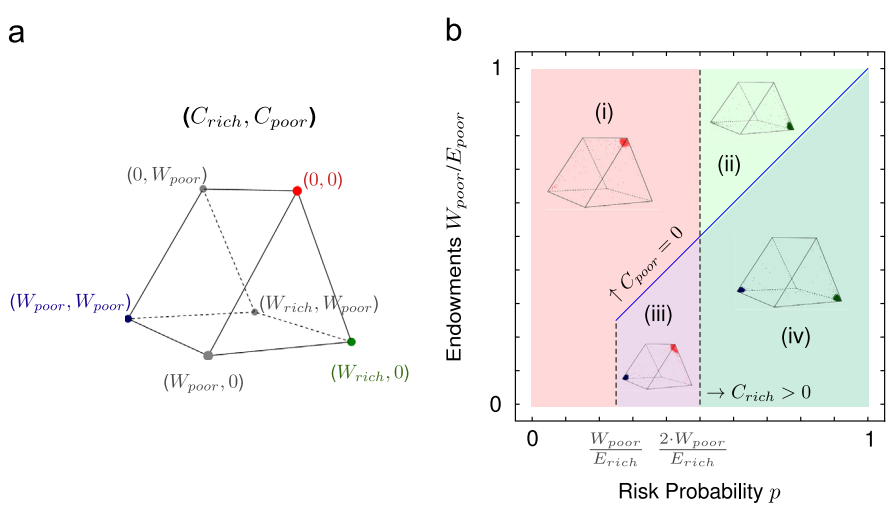

Fig. 1. Summary of the evolutionary dynamics for a pairwise single-round collective-risk game (R=1, M=2). (a) The state space of the rich and poor investment strategies can be represented as a prism. From two poor (0 or W_{poor}) and three rich (0, W_{poor}, and W_{rich}) investment strategies we obtain six possible (C_{rich}, C_{poor}) combinations, the population can contain any mixture of these. (b) Depending on the risk probabilities p and endowments, we can define four regions, under which only certain strategies are evolutionarily stable (Maynard Smith, 1982): (i) If $p < 2W_{poor}/E_{rich}$ and $p < W_{poor}/E_{poor}$, the only stable strategy is full defection, $C_{rich} = C_{poor} = 0$. (ii) If $p > 2W_{poor}/E_{rich}$ and $p < W_{poor}/E_{poor}$, only $C_{rich} = W_{rich}$ and $C_{poor} = 0$ is stable. (iii) If $2W_{poor}/E_{rich} > p > W_{poor}/E_{rich}$ and $p > W_{poor}/E_{poor}$, either both types defect $C_{rich} = C_{poor} = 0$ or both contribute $C_{rich} = C_{poor} = W_{poor}$. (iv) If $p > 2W_{poor}/E_{rich}$ and $p > W_{poor}/E_{poor}$, either only the rich contribute $C_{rich} = W_{rich}$ and $C_{poor} = 0$ or both contribute $C_{rich} = C_{poor} = W_{poor}$. Dots show the result of individual based simulations (In all simulations, we used a game with parameters $W_{rich} = 2$, $W_{poor} = 1$, $E_{rich} = 4$, T=2. For the evolutionary game dynamics, we used N=100, G=1000, β=1, σ=0.15, μ=0.03, averages over $\approx 180{,}000$ realizations and 10^4 time steps per prism. The risk probability p and E_{poor} were varied in the four regions as (i) $E_{poor} = 4$, p=0.2, (ii) $E_{poor} = 1.5$, p=0.6, (iii) $E_{poor} = 4$, p=0.4, and (iv) $E_{poor} = 3$, p=0.7.).

poor all their working endowment. Do such strategies still hold if we allow players to interact over several rounds and influence one another?

To address this question, we first consider a pairwise game where players can interact over $R=2$ rounds: initially, rich players receive $W_{rich}=2\cdot R$ and poor players receive $W_{poor}=R$. In each round, they can contribute 0, 1, or 2 units towards the collective target $T=4$ (but of course the poor can only contribute W_{poor} in total). Unlike the single round game, this time there are several strategies as now players can condition their behavior in the second round based on the total contributions in the first round. We define an individual strategy using thresholds τ $(0\le\tau\le T)$ which directs an individual's decisions with respect to the collective investments so far (Γ). For a two round game, such strategies can be written as $(C_1,\tau;a,b)$, where C_1 determines what is contributed in the first round, and where the player will use τ to decide whether a or b is contributed in the second round.

Similar to the single round game, simulations show that for $p=0.9$, $p<W_{poor}/E_{poor}$ and $p>W_{rich}/E_{rich}$, the poor players do not contribute anything and the rich players contribute all of their working endowment, see Fig. 2a. For a case where $p>W_{poor}/E_{poor}$ and $p>W_{rich}/E_{rich}$, the rich players contribute half of their working endowment and the poor players contribute everything, Fig. 2b, which was also a possible solution in our analytical analysis of the single round case. The other state, where the rich players contribute all of their working endowment and the poor players contribute nothing, is only briefly observed in our numerical example. The limited duration could be due to the vast amount of strategies possible; for a two round game and a discrete set of τ we have 135 possible strategies. For example, in the case of a rich player with the strategy $(C_1,\tau;a,b)$ there are three possible values (0, 1, or 2) for C_1, a and b and 5 possible τ values (0,1, 2, 3, 4). Solving the replicator dynamics of this system in 134 dimensions numerically for specific initial conditions would in each case lead to some fixed point or attractor, but in all cases the result would depend on the initial condition. As we are not interested in specific initial conditions or basins of attraction, we focus on the mutation selection equilibrium of stochastic evolutionary game dynamics instead. We thus perform simulations of the stochastic Wright–Fisher process (Imhof and Nowak, 2006). We conduct large scale computer simulations to understand the effects of timing and the effects of external endowments ($E_{rich}\ge E_{poor}$) on the decisions of the players. For $E_{poor}\le 1$ the poor players do not contribute. In this situation they have very little to gain unless the rich players assist them, which does not occur when $E_{rich}=W_{rich}$, Fig. 3a. The rich compensate missing contributions of the poor when $E_{rich}>W_{rich}$, Fig. 3b. Contributions by the poor players increase when $E_{poor}>1$. Irrespective of E_{rich}, the poor begin to contribute once they have sufficient assets worth protecting, $E_{poor}\ge 3$, at which point the rich only contribute up to half of their working money, while the poor players contribute almost everything, Fig. 3.

These results do not differ qualitatively from the one round game. However, only with the inclusion of another round the timing of contributions can be explored. When $E_{poor}>W_{poor}$, poor players contribute in the first round, whereas the rich players contribute in the second round, Fig. 3. When $E_{poor}\le W_{poor}$ and $E_{rich}>W_{rich}$, the rich contribute in both rounds equally while the poor wait until the second round of the game. To understand the difference in the timing of contributions, we extracted the most abundant strategies used by the two types of players. Neither player have an incentive to contribute when $E_{rich}=W_{rich}$ and $E_{poor}<1$ and thus we find that both rich and poor players are unconditional defectors and never contribute. If we increase E_{rich} to $2W_{rich}$, the rich players use the unconditional 'altruistic' strategy and contribute in both the first and second round, while the poor

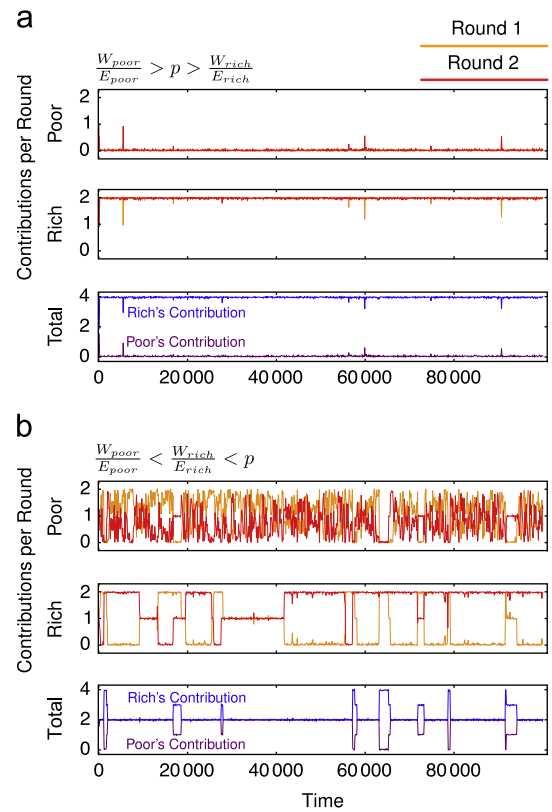

Fig. 2. Time series for a collective-risk dilemma with two rounds and two players ($R=2$, $M=2$). (a) For $E_{poor}=1$ and $p<W_{poor}/E_{poor}$ as well as $p>W_{rich}/E_{rich}$, the poor players contribute nothing while the rich players contribute their full working endowment, 2 in each round. (b) For $E_{poor}=4$, $p>W_{poor}/E_{poor}$, and $p>W_{rich}/E_{rich}$, both players contribute. Most often, the poor players contribute their full working endowment whereas the rich players only contribute half of their working endowment. (game parameters $p=0.9$, $W_{rich}=4$, $W_{poor}=2$, $E_{rich}=6$, $T=2$; parameters of the evolutionary game dynamics $N=100$, $G=1000$, $\beta=1$, $\sigma=0.15$, $\mu=0.03$ and 10^5 time steps).

players maintain the unconditional defector strategy. The rich players maintain their altruistic strategy for high external endowment as long as $E_{poor}<3$.

In some cases, the rich players appear to contribute in the first round to entice contributions from the poor players in the second round. This is observed when $E_{rich}=W_{rich}$ and $1<E_{poor}<3$ where we find that both types of player condition their strategy based on contributions in the first round, Fig. 4a. The most abundant strategy observed for the rich players was a strategy where the rich will contribute 2 in the first round and 1 in the second, this strategy is $(2:\tau;1,1)$. Under these conditions the poor player will use the conditional strategy $(0:\tau;0,1)$, contributing 1 in the second round if the rich players have contributed in the first round. Thus, for moderate extra endowment, the poor are assisted by the compensation from the rich players. However, these expectations are lifted when $E_{poor}\ge 3$, under such case the poor players use unconditional 'early' strategies of the form $(2:\tau;a,b)$ and consequently the rich use the conditional 'late' strategy $(0:\tau;a,0)$ since they have no reason to compensate. Now when $E_{rich}>W_{rich}$ the rich and poor player will swap strategies depending on E_{poor}, Fig. 4b. For $E_{poor}<3$ the rich players will use $(2:\tau;2,2)$ most frequently and contribute everything while the poor players will defect and contribute nothing. However, when $E_{poor}\ge 3$ the rich players use the 'late' contribution strategy $(0:\tau;2,0)$ since the poor will contribute everything in the first round using strategy $(2:\tau;\cdot,\cdot)$ (contributing everything in the first round, they cannot contribute in the second round).

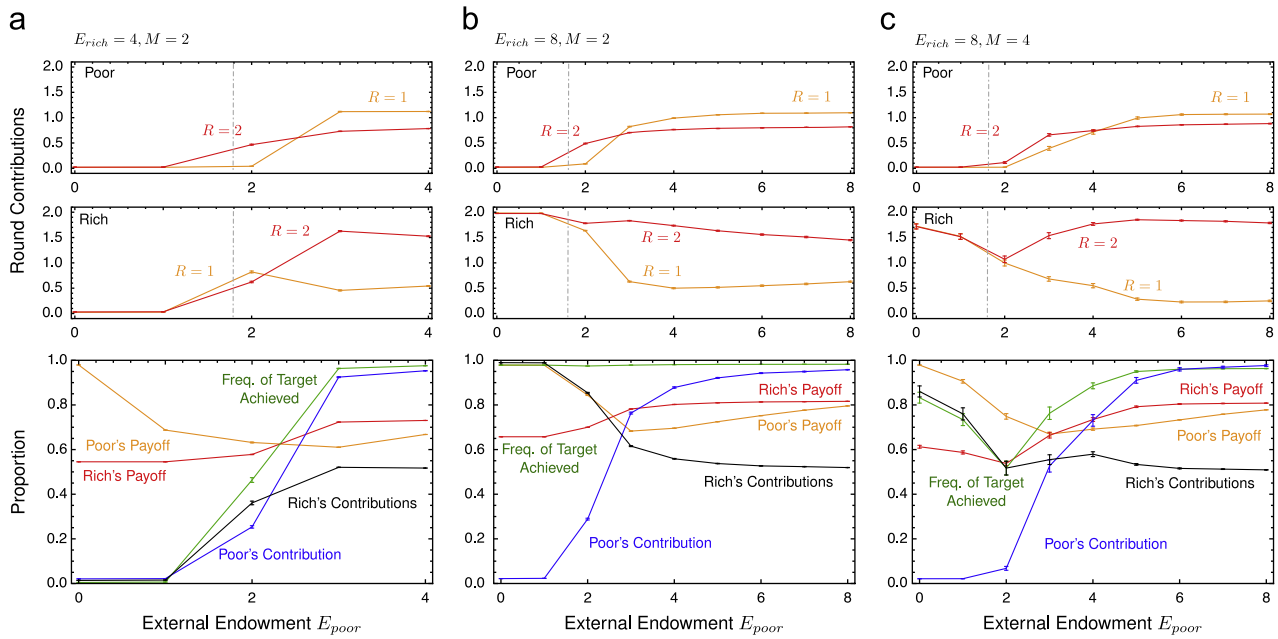

Fig. 3. Timing of contributions in a collective-risk game with two rounds, $R=2$. Simulations show total contributions and their relative proportions for both rich and poor players. Total contributions are measured relative to their working endowment W, payoffs relative to the total endowment $(W+E)$. In addition, the probability to reach the target is shown (approximated by its frequency). (a) For two players, $M=2$ and $E_{rich}=4$, on an average, when $E_{poor} > W_{poor}$ the rich players contribute up to half of their working funds while the poor contribute almost everything. (b) For two players, $M=2$ and $E_{rich}=8$, the rich contribute all of their working endowment when $E_{poor} < 2$. The poor players contribute more than half of W_{poor} once $E_{poor} > W_{poor}$. At this point, the rich players start to contribute only half of W_{rich}. (c) For four players, $M=4$, we observe similar patterns as in (b). However, the probability the target is met decreases for low E_{poor}. Even under such conditions, the rich still contribute more than half of W_{rich} (game parameters $p=0.9$, $T=MR$, $W_{rich}=4$, $W_{poor}=2$ (vertical gray line), $E_{poor} \in (0, E_{rich})$; parameters of evolutionary game dynamics $G=1000$, $\beta=1$, $\sigma=0.15$, $\mu=0.03$; averages over 10^5 time steps and $\approx 12,000$ independent realizations, error-bars=(standard deviation)/realizations).

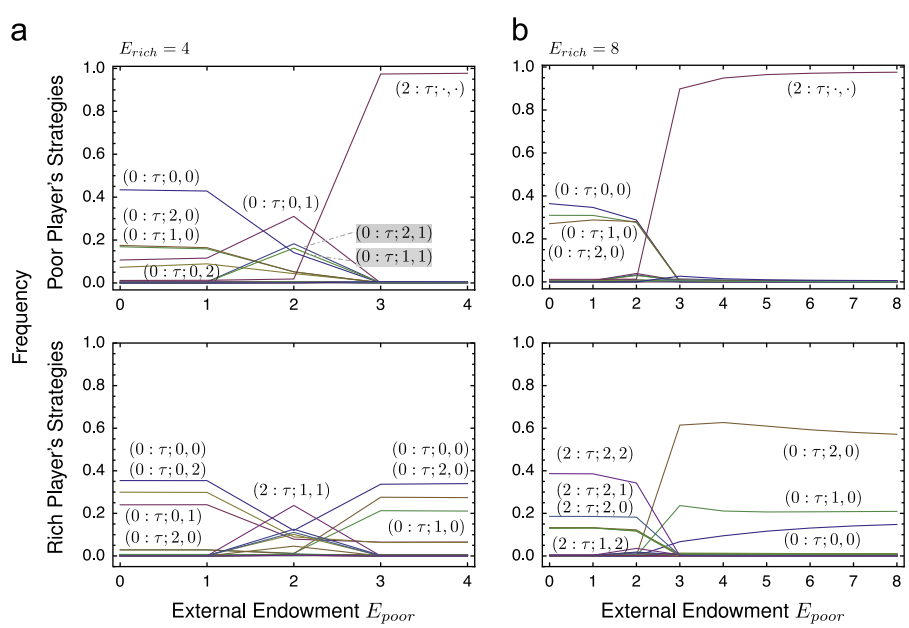

Fig. 4. Strategies in a collective-risk game with two player and two rounds ($M=2$, $R=2$). Rich individuals have a working endowment $W_{rich}=4$ and poor individuals have $W_{poor}=2$. (a) For $E_{rich}=4$, on an average, when $E_{poor} < W_{poor}$ the most abundant strategy for both rich and poor players is defection with the form of $(0:\tau;0,0)$. When $E_{poor} > W_{poor}$ the poor players contribute everything in the first round with most abundant strategy being $(2:\tau;\cdot,\cdot)$. In contrast, the rich players contribute on an average half of their contributions in the second round. When $E_{poor} = W_{poor}$ the rich contribute in the first round half of their endowment and only if the poor contributed would they contribute further in the second round; the rich player's most abundant strategy here is $(2:\tau;1,1)$ while the poor player's most abundant strategy is $(0:\tau;0,1)$. (b) For $E_{rich}=8$, the rich and poor players use opposite strategies depending on E_{poor}; the rich players contribute all of their working endowment when $E_{poor} \leq W_{poor}$ while the poor players contribute all of their working endowment once $E_{poor} > W_{poor}$ (game parameters $p=0.9$, $R=2$, $T=4$, $W_{rich}=4$, $W_{poor}=2$, $E_{poor} \in (0, E_{rich})$; parameters of evolutionary game dynamics $N=100$, $G=1000$, $\beta=1$, $\sigma=0.15$, $\mu=0.03$; averages over 10^6 time steps and ≈ 5000 independent realizations).

In summary, when the poor players have half of the wealth of the rich players, they do not contribute unless some early signal is given by the rich players. The rich signal and contribute on behalf of the poor when the rich's own external endowment is worth protecting ($E_{rich} > W_{rich}$). As the poor's external endowment increases ($E_{poor} \geq 3$), they contribute all of their working

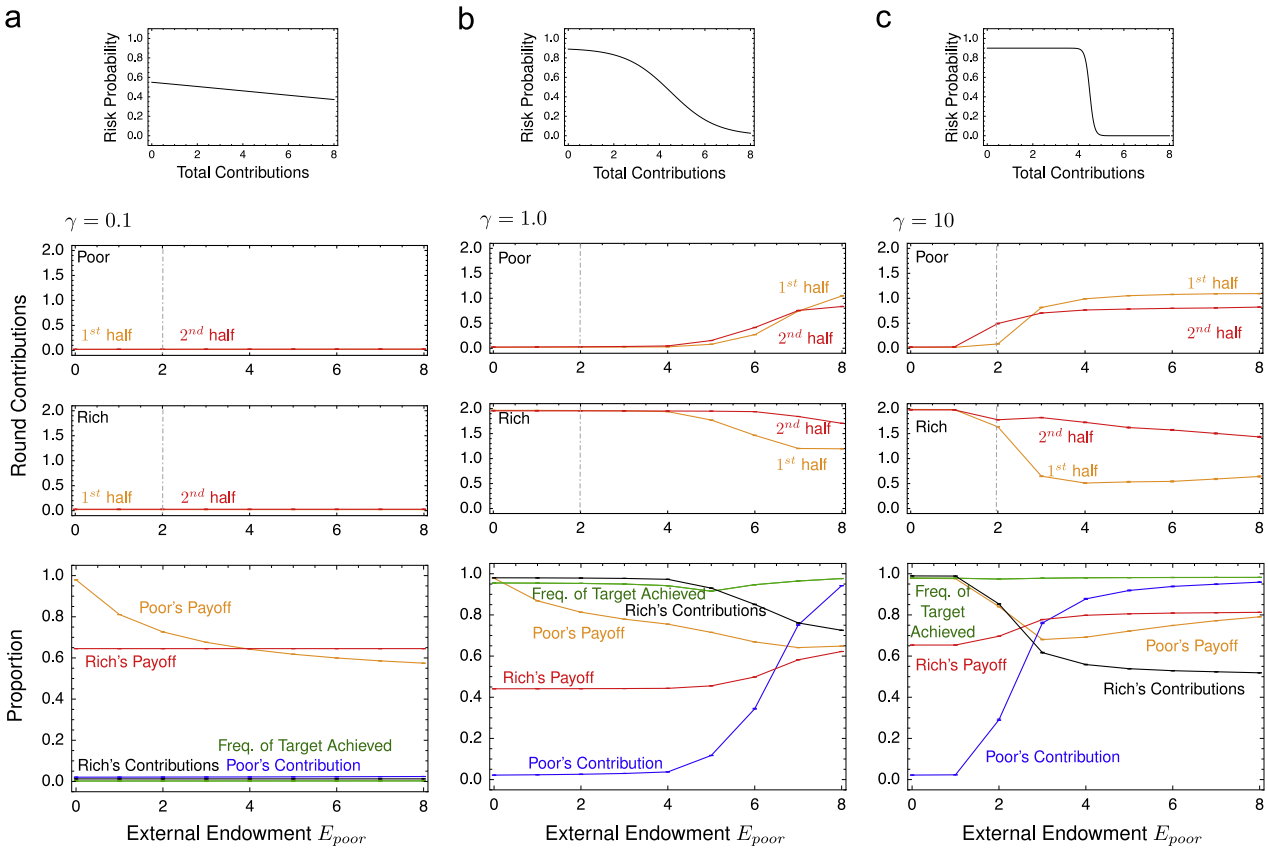

Fig. 5. Collective-risk game with two rounds and two players: exploring the effects of various risk curves based on the Fermi function. (a) With an approximately linearly decreasing risk curve (γ=0.1), the target is not met, as neither rich nor poor players contribute to the common pot. (b) With a sigmoidal curve (γ=1.0) poor players contribute nothing as long as $E_{poor}<4$, while the rich players contribute everything. As E_{poor} increases both types of player contribute to the common pool. (c) As we approach a step function (γ=10.0), the rich players contribute everything only when $E_{poor}<2$. (game parameters $W_{rich}=4$, $W_{poor}=2$, $E_{rich}=4$, T=4, $p=0.9/(1+\exp(\gamma(\Gamma-3.5)))$, where the total contribution after the last round is Γ, parameters of evolutionary game dynamics N=100, G=1000, $\beta=1$, $\sigma=0.15$, $\mu=0.03$; averages over 10^5 time steps and $\approx$25000 independent realizations, error-bars=(standard deviation)/realizations).

endowment in the first round; such a strategy implies that the rich can now guarantee that the target will be reached if they contribute just half of their working endowment.

3.3. Risk probability in the two player, two-round game

Uncertainty can also be the result of probable risk, and thus, we explore the effects of various risk curves especially since experimental outcomes differed based on the curves implemented (Chan et al., 1999; Ledyard, 1995). We vary the risk probability such that higher contributions continuously decrease the risk, $p=p_{max}/(1+\exp(\gamma(\Gamma-(T-\varepsilon)))$. Here, Γ is the collective pool after the last round, p_{max} is the maximum risk, and the small number ε ensures that in the limit of $\gamma\to\infty$ we recover the initial risk probability curves. When γ=0.1 the risk curve decreases almost linearly, in such a case we lose cooperation as neither rich nor poor players contribute, Fig. 5a. When we implement a sigmoidal curve (γ=1.0), the rich players contribute all of their working endowment, but contributions by the poor players are low as long as $E_{poor}<4$. An increase in E_{poor} results in higher contributions from the poor players and now the rich players use the late strategy. Interestingly, the rich players maintain their high contributions (more than 80% of their working money) for all E_{poor}, Fig. 5b. Finally, when we increase γ even further (γ=10) we recover the typical scenario where the poor players contribute more than rich players in the first half of the game and the rich players contributing in the second half of the game, for all $E_{poor}>W_{poor}$, Fig. 5c. Thus, rich players invest and contribute on behalf of the poor players when the risk probability curve describes a function with a gradual rather than sharp decrease.

3.4. Increased number of players

So far, we have only considered pairwise interactions, however, real-world situations often involve a large group of individuals (multiple players) negotiating or interacting over two rounds. Thus, we further expand our analysis to games with multiple players and multiple rounds and explore their effects on contributions. Individual strategies $(\tau_r;a_r,b_r)$ are defined as before, but now different parameters can be chosen for each round. Increasing group size causes uncertainty as it is harder to coordinate on a good equilibrium, while increasing the number of rounds alleviates uncertainty by increasing the time for coordination (Hilbe et al., 2013). As expected, increasing group size increases uncertainty. Simulations for M=4 show a decrease in the probability by which the target is met for all $E_{poor}<4$, Fig. 3c. However, although uncertainty increases, the rich players still contribute more than half of their working endowment when $E_{poor}<2$. Once $E_{poor}>W_{poor}$, the rich players use the late contributor strategies, contributing only if the poor players have contributed.

4. Discussion

So far, most theoretical models of cooperation have addressed situations in which all actors have identical properties. Heterogeneity is an additional complication, because actors may have

different incentives to cooperate and different abilities to do so. Although heterogeneity tends to be associated with negative effects in public goods games (Ledyard, 1995), some experiments have shown that it can have positive effects on the outcome. For instance, subjects endowed with more wealth than they feel they have earned tend to equalize their profits with the unfortunate subjects (De Cremer, 2007; van Dijk and Grodzka, 1992). In contrast, when subjects felt that the endowments were earned, no efforts were made to equalize, but contributions were made proportional to the endowments (van Dijk and Wilke, 1994). It seems that human subjects have their own notion of 'fair share' (an equal distribution or a proportional allocation) which depends on how the heterogeneity was obtained. In a homogeneous group we allocate fairness as equal share, however what is 'fair' for a heterogeneous group?

In our model, players contribute in similar fashion under low heterogeneity and payoffs obtained were proportional to an individual's wealth. In contrast, with increasing heterogeneity, rich players increase their contributions and compensate for the poor player's inability to contribute. Rich players contribute more in the case where they are pivotal to reach the target. Otherwise, they contribute the minimum required or the same as the poor players. Poor players evolve conditional strategies when their endowment is low. They contribute in the second half of the game only if rich players contribute in the first half. To ensure robustness of our numerical results, we have also considered other parameter values not reported here. For example, we have varied the number of rounds from 1 to 10, analyzed group size from 2 to 6. In the case of two players, we have considered strategies that condition on the other player's move rather than on the collective pool. We have also decreased the steps between possible contributions. This analysis suggests that the results we have reported are qualitatively robust: the rich compensate missing contributions of the poor when they have a greater incentive (high external endowment) in order to ensure that the target is met. As a rule of thumb this occurs when $E_{rich} > E_{poor}$ and $p > 2 \cdot W_{poor}/E_{rich}$.

Resource division seems to be evolutionarily imprinted in us, and we have a sense of 'fairness' for allocating resources (Proctor et al., 2013). In our model we show different timing and investment allocation between the rich and poor, but how does uncertainty and risk affect our decisions? In line with experimental evidence (Rapoport, 1988; Rapoport and Suleiman, 1993; Ledyard, 1995; Chan et al., 1999; De Cremer, 2007; Milinski et al., 2011), we find that our simulated outcomes also depend strongly on risk probability functions, as expected. For instance, we find a loss of cooperation for linearly decreasing risks, as neither rich nor poor players contribute, Fig. 5a. In this well-studied case, the only equilibrium is full defection. On the other hand, full cooperation is an equilibrium when risk is a sigmoidal function. In such a case the target is met with a higher probability especially with increasing external endowment, Fig. 5c. These results reflect experimental evidence which shows that under non-linear payoff functions the free-riding strategy is no longer dominant (Ledyard, 1995; Chan et al., 1999; Levati et al., 2007). Interestingly, as contributions increase to the point where the risk is also high, we find the rich players increasing their contributions and thereby compensating for the poor players, see Fig. 5b. This result may be counterintuitive and unexpected given that we have uncertainty and our individuals do not communicate. However, for a certain degree of risk there is an equilibrium where the best strategy for rich players is to maintain cooperation by assisting the poor. This help is not altruistic, but in the pure self interest of rich players – they fare better by paying to prevent the loss of their assets. This implies that in the face of risk, a fair share may be very different from our usual intuition of equal distribution of costs or a distribution proportional to the endowments. In the collective risk dilemma considered here, it can be in the best interest of the poor not to invest anything, whereas the rich have incentives to go beyond the equal share and compensate the missing contributions of the poor.

Although we may feel doomed in the face of uncertainty, it can also bring unexpected help from others. According to our results, under high stakes and uncertainty, the best response is for the rich player to lend the poor player a helping hand.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jtbi.2013.10.004.

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