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On the use of the term *diastēma* in ancient Greek constructions

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Abstract

I examine the use of the term *diastēma* by Greek geometers in both plane and spherical constructions. I show that while *diastēma* may be translated as *radius* in plane constructions, this will not work on the sphere. These investigations have some implications for how we think of construction in Greek mathematics in general.

Résumé

J'examine l'usage du terme *diastēma* chez les géometres grecques dans le contexte des constructions géometriques planes et sphériques. Je démontre que bien qu'il soit possible de traduire le terme *diastēma* par *rayon* en géometrie plane, ce n'est pas le cas pour la géometrie sphérique. Ces recherches ont des portées sur nos conceptions des constructions dans la mathématique grecque.

MSC: 10A20

Keywords: Radius; *Diastēma*; Construction; Euclid; Archimedes; Autolycus; Theodosius

A recent paper in this journal explored the relation of the word διάστημα, *diastēma*, "distance, interval, opening," to the expression ἡ ἐκ τοῦ κέντρου, *hē ek tou kentrou*, "the [line] from the center."[1] It was found that in constructions in the plane the term *diastēma* could be taken as a technical term for radius, as in fact it is most often translated.[2] No consideration was given, however, to constructions on the sphere, where the term is also used and where it cannot mean radius. This note attempts to formulate a definition of the term *diastēma* that works in both plane and spherical constructions. These considerations have implications for how we think about constructions in ancient Greek geometry in general.

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[1] Fowler and Taisbak [1999].

[2] An exception is B. Vitrac who prefers *intervalle* [Vitrac, 1990–2001, Vol. 1, 169 ff]. He gives his reasons for using *intervalle* as opposed to *distance* in Vitrac [1990–2001, Vol. 1, 169 n. 8].

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1. The relationship of the term *diastēma* to our concept of radius

Making no attempt to be exhaustive, and without going into the details of any mathematical theory, we use the term radius in three ways:

(1) an actual line drawn from the center of a circle, or a sphere, to its circumference, or surface;
(2) the distance from the center of a circle, or a sphere, to its circumference, or surface, regardless of whether an actual line is so drawn; and finally
(3) a certain real number, r, which is directly related to other real numbers associated with the circle or the sphere, such as the circumference, C, or the surface, S. For example, we say $C = 2\pi r$, or $S = 4\pi r^2$.

The Greek geometers had a special expression for our use of the term *radius* to denote an actual line joining the center of a circle or sphere, and its circumference or surface. They used the expression ἡ ἐκ τοῦ κέντρου τοῦ κύκλου (τῆς σφαίσρας), "the [line] from the center of the circle (of the sphere)," which through ellipsis often became simply ἡ ἐκ τοῦ κέντρου, "the [line] from the center." As Fowler and Taisbak, and Mugler before them, point out, the term *diastēma* is used in its dative form whenever a circle is to be drawn in the plane with a particular radius.[3] Fowler and Taisbak noticed the crucial difference between the two expressions for radius. The radius is only referred to as "the [line] from the center" if it is already found in the figure. Whenever a circle is to be drawn, however, the term *diastēma* is used.[4] In this sense, it corresponds to our use of the term *radius* to denote the geometric distance between the center of a circle and its circumference. When a circle is drawn on a sphere, however, it is also drawn with a *diastēma*, and here the term cannot mean *radius*. Both of these constructions will be discussed below. The Greek geometers seem to have had no concept corresponding to our abstract notion of the term radius as a real number, r, which we use in such expressions as $C = 2\pi r$. This is not to say that they had no way of relating a circle's radius to its circumference, or a sphere's surface to other areas.[5]

2. Uses of *diastēma* in the geometric corpus

The basic meaning of *diastēma* is "distance," and this is how Mugler defines it in his dictionary.[6] We find it used in this basic sense in a number of passages in the mathematical literature; one from Archimedes will suffice to make the point. In *On Conoids and Spheroids* 9, we read (see Fig. 1) ἁ δὴ ἑτέρα διάμετρος τᾶς τοῦ ὀξυγωνίου κώνου τομᾶς ἤτοι ἴσα ἐντὶ τῷ διατήματι τᾶν AZ, BH ἢ μείζων ἢ ἐλάσσων, "clearly the other diameter of the ellipse is either equal to the distance (*diastēma*) of [the lines] AZ, BH, or is greater, or is less."[7] Here, *diastēma* simply denotes the distance between two geometric

[3] Fowler and Taisbak [1999, 361, 363] and Mugler [1959, 136].
[4] Fowler and Taisbak [1999, 363].
[5] Archimedes demonstrates various relationships between a circle's diameter, circumference, and area in *Measurement of a Circle* [Heiberg, 1972, Vol. 1, 232–243]. He proves that the surface of a sphere is four times the area of a great circle in *On the Sphere and the Cylinder* I 33 [Heiberg, 1972, Vol. 1, 120–124].
[6] Mugler [1959, 136].
[7] Heiberg [1972, Vol. 1, 296–298].

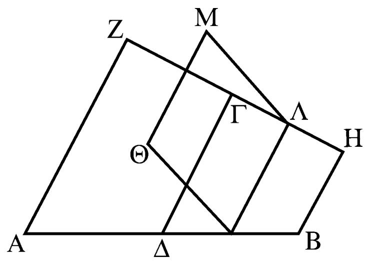

Fig. 1. Archimedes, *On Conoids and Spheroids* 9.

objects, in this case lines. In fact, in the diagram a line, ZH, has been joined perpendicular to the two parallels AZ and BH, but Archimedes prefers to make the more general statement involving the distance than a particular statement about the line ZH.

2.1. Diastēma *in plane constructions*

In the construction of a circle, *diastēma* has a more specific meaning, the distance between the center of a circle and its circumference. Euclid postulates the construction of circles with any center and any *diastēma*.[8] As Mugler points out, when a *diastēma* is used for drawing a circle it appears as an instrumental dative; thus a *diastēma*, in this locution, is always something with which a circle is drawn.[9] The *diastēma* itself—or in the case of Archimedes, a line which is equal to the *diastēma*—is denoted by apposition.

The way this functions in the *Elements* is quite consistent. We may take *Elem.* I 12 as an example (see Fig. 2), κέντρῳ μὲν τῷ Γ διαστήματι δὲ τῷ ΓΔ κύκλος γεγράφθω ὁ EZH, "With the center, Γ, and with the *diastēma*, ΓΔ, let the circle EZH have been drawn."[10] In general, in the *Elements*, circles are drawn with a *diastēma* which is equal to a line which is already in place, with one point lying at the circle's center and the other on its circumference. This text, however, illustrates an interesting point which can be made about Euclid's use of the term *diastēma*. Since there is no line ΓΔ, it is clear that τῷ ΓΔ, "the [*diastēma*] ΓΔ," refers to a property that the two points A and B have regardless of whether or not a line is drawn between them. Thus, the *diastēma* ΓΔ is the distance between Γ and Δ. Here, τῷ ΓΔ, "the [*diastēma*] ΓΔ," signifies the *diastēma* denoted by ΓΔ not the line denoted by ΓΔ. In the Euclidean text, circles are always drawn with a *diastēma* which is itself designated by two letters.

I make this point because Archimedes often draws circles with a line as the *diastēma*. We may take, as an example, *On Spirals* 16, γεγράφθω κύκλος ὁ ΔΤΝ κέντρῳ μὲν τῷ Α διαστήματι δὲ τᾷ ΑΔ, "Let the circle ΔTN have been drawn with the center, A, and with the *diastēma*, the [line] AΔ."[11] The expression τᾷ ΑΔ cannot mean "the [*diastēma*] AΔ" because the article τᾷ is feminine whereas the noun διαστήμα is neuter. The use of the feminine article followed by two letters is the common idiom for a line in Greek geometric texts. Archimedes' expression διαστήματι δὲ τᾷ ΑΔ is probably ellipsis for

8 Heiberg [1969, Vol. 1, 4–5].
9 Mugler [1959, 136].
10 Heiberg [1969, Vol. 1, 20].
11 Heiberg [1972, Vol. 2, 56].

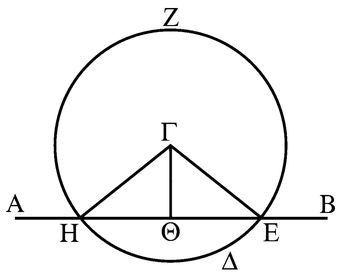

Fig. 2. Euclid, *Elements* I 12.

"with the *diastēma* equal to the line ΑΔ."[12] And in fact, we find this full expression in the *Method* 9: γεγράφθω δὲ καὶ κύκλος ἐν τῷ ἐπιπέδῳ τῷ ἀποτέμνοντι τὸ τμᾶμα κέντρῳ μὲν τῷ Η, διαστήματι δὲ τῷ ἴσῳ τῇ ΑΗ, "And, in the plane cutting the section, let a circle have been drawn with center H and with the *diastēma* equal to the [line] AH."[13] This is not to say that the *Method* text preserves a more pristine version of Archimedes' expression, merely that it spells out the complete thought.[14]

2.2. Diastēma *in spherical constructions*

The term *diastēma* is also used by Greek geometers to draw a circle on a sphere. This construction first appears in a systematic treatise in Autolycus, *On the Moving Sphere* 6.[15] The construction is only used twice in the two works of Autolycus and both of these instances are in *On the Moving Sphere* 6. Autolycus' work is not strictly geometrical and there is no attempt to derive constructions from first principles such as we find in the *Elements*; nevertheless, it will be useful to look at this text. *On the Moving Sphere* 6 is a strange blend of geometry and astronomy. The theorem states that if a great circle is inclined to the axis of a sphere then it will be tangent to two equal and parallel circles, and that, of these circles, the one near the visible pole will always be visible and the one near the invisible pole, always invisible. Astronomically, this means that the local horizon will be tangent to two equal and parallel circles which divide those stars which are always visible or always invisible from those stars which are seen to rise and set. Geometrically, the theorem states that if a great circle is inclined toward poles of a sphere, it will be tangent to two equal and parallel circles which will be situated on opposite sides of the original great circle with respect to the stated poles. Following the enunciation and the setting-out, the construction begins as follows (see Fig. 3): Ἔστω γὰρ ὁ πόλος τῆς σφαίρας ὁ φανερὸς ὁ Δ, καὶ διὰ τοῦ Δ καὶ τῶν τοῦ ΑΒΓ κύκλου πόλων μέγιστος κύκλος γεγράφθω ὁ ΑΔΕ, καὶ κείσθω τῇ ΑΔ περιφερείᾳ ἴση ἡ ΓΕ καὶ πόλῳ τῷ Δ διαστήματι δὲ τῷ ΑΔ κύκλος γεγράφθω ὁ ΑΖΗ, "For let the visible pole of the sphere be Δ, and, through Δ and the pole of the great circle ΑΒΓ, let the circle, ΑΔΕ, have been drawn, and let the arc ΓΕ have been laid out equal to ΑΔ, and with the pole Δ and the *diastēma* ΑΔ let the circle ΑΖΗ have been drawn."[16]

[12] See Netz's discussion of the way ellipsis functions in Greek mathematical expressions [Netz, 1999, 152–153].

[13] Heiberg [1972, Vol. 2, 476].

[14] Notice, in particular, that someone has replaced Archimedes' Doric τᾷ with the common ῃ̃.

[15] The construction of a circle on a sphere using a *diastēma* is also found in the Aristotelian *Meteorology* III 5, 276b 8, but this text, although certainly early, is of uncertain provenance. See Jones [1994] and Vitrac [2002] for recent discussions.

[16] Mogenet [1950, 203].

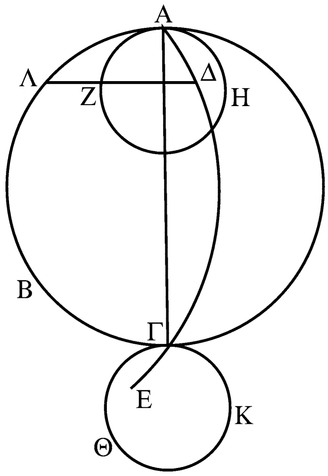

Fig. 3. Autolycus, *On the Moving Sphere* 6. In the figures that have come down to us with the text of Autolycus, no attempt is made to preserve visual perspective. All circles are simply folded down into the plane of the drawing, preserving their essential mathematical properties; for example, circle ΑΖΗ is equal to circle ΓΘΚ and they are tangent to the circle ΑΒΓ at points Α and Γ.

In Autolycus' text, none of the steps in this construction are either postulated or established through propositions. We do not know how to find the pole of a given circle, how to draw a great circle through two given points, nor even how to lay out one arc of a great circle equal to another.[17] Moreover, within the scope of this text, it is hard to determine exactly what is meant by *diastēma*. Autolycus uses the neuter article τῷ to designate the *diastēma* ΑΔ, but we do not know if he means the rectilinear distance or the distance as defined by arc ΑΔ; after all no chord ΑΔ, has been drawn. If, however, as in *Elements* I 12, a *diastēma* is a property that two points have whether or not a line is drawn between them, then it should make no difference whether or not a chord ΑΔ has been drawn. Perhaps there was some systematic treatise, available to Autolycus, which demonstrated some of these constructions and clarified the use of *diastēma*, but if there was, we no longer posses it.

The first systematic treatise on spherical geometry that contains constructions is the *Spherics* of Theodosius. Although the *Spherics*, like the *Elements*, begins with a construction, there are no postulates; so the most basic constructions must be assumed. In the *Spherics*, the use of a *diastēma* to draw a circle with a given pole first appears, appropriately enough, in a construction. *Spherics* I 19 demonstrates how to set out (ἐκθέσθαι) the diameter of a given sphere. The construction begins immediately following the enunciation (see Fig. 4): νενοήσθω[18] γάρ ἡ σφαῖρας, ἧ δεῖ τὴν διάμετρον ἐκθέσθαι, καὶ εἰλήφθω ἐπὶ τῆς ἐπιφανείας τῆς σφαίρας δύο τυχόντα σημεῖα τὰ Α, Β, καὶ πόλῳ μὲν τῷ Α, διαστήματι δὲ τῷ ΑΒ, κύκλος γεγράφθω ὁ ΒΓΔ, "For let the sphere have been imagined, the diameter of which it is

[17] The first of these constructions, *to find the poles of a given circle*, is established in Theodosius' *Spherics* I 21; the second, *to draw a great circle through two given points*, in *Spherics* I 20; the third, *to lay out the arc of a great circle equal to a given arc of a great circle*, is assumed in Spherics III 6, but never postulated [Heiberg, 1927, 36–40 & 134].

[18] This verb is the standard term used in the case of a solid construction that cannot be fully or accurately represented by the plane figure. See for examples Euclid's *Elements* XII 13–14, 17–18; Apollonius, *Conics* I 52, 54, 56; Ptolemy's *Analemma* 6, and *Geography* I 24; Heiberg [1969, Vol. 4, 120–122, 126–136]; Heiberg [1891–1893, 158–162, 166–170, 174–180]; Heiberg [1907, 137]; and Nobbe [1966, 53]. The expression is also found hundreds of times in Archimedes' corpus [Heiberg, 1972].

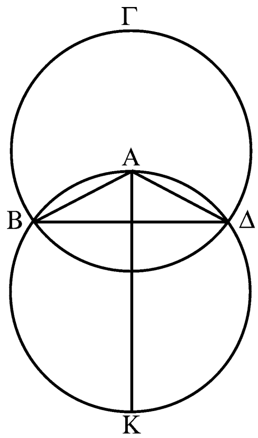

Fig. 4. Theodosius, *Spherics* I 19.

necessary to set out, and let two random points A and B have been taken on the surface of the sphere, and, with the pole A and the *diastēma* AB, let the circle ΒΓΔ have been drawn."[19] There are a couple of things to notice about this passage. The first is that the ability to draw a circle with a given pole and a given *diastēma* is assumed without being postulated. The second is that the line and the arc AB are not actually drawn until later in the proposition. At the point at which circle ΒΓΔ is drawn, the *diastēma* is taken to be something that we can directly apprehend, and which we denote with the names of its end points, just as we would denote a line or an arc. Since no arc AB has been mentioned, it seems likely that the *diastēma* AB is the rectilinear distance AB.

The next proposition makes it perfectly clear that this is the case. *Spherics* I 20 shows how a great circle is drawn through two points given on a spherical surface. There are two cases: (1) either the points lie on the end points of a diameter, or (2) they do not. The first case is summarily dismissed with the statement that if the two points lie on the diameter of the sphere, then an indefinite number of great circles will be drawn through them. The second case begins as follows (see Fig. 5): μὴ ἔστω δὴ τὰ A, B σημεῖα κατὰ διάμετρον τῆς σφαίρας, καὶ πόλῳ μὲν τῷ A, διαστήματι δὲ τῇ τοῦ τετραγώνου πλευρᾷ τοῦ εἰς τὸν μέγιστον κύκλον ἐγγραφομένου, κύκλος γεγράφθω ὁ ΓΔΕ, "Let the points A and B not be on the diameter of the sphere, and let the circle ΓΔΕ have been drawn with the pole A and with a *diastēma* [equal to] the side of the square inscribed in a great circle."[20] Here, it is clear that the *diastēma* is set equal to a chord which runs from the circle's pole to its circumference and there is no reason not to assume that it is so in all cases.[21] The *diastēma*, then, is a sort of generating chord, the rectilinear distance between the pole and the circumference. In a circle drawn on the sphere the *diastēma* cannot be equal to the circle's radius.

The analogy with the circle on the plane is clear; in both cases the *diastēma* is the generating rectilinear distance between the generating point and the circle itself. We saw that in the plane there was no problem translating *diastēma* with *radius* since in the plane our concept of radius encompasses this generating

[19] Heiberg [1927, 34].

[20] Heiberg [1927, 36].

[21] This particular *diastēma* will produce a great circle because *Spherics* I 16 proves that the chord joining the pole of a great circle with its circumference is equal to the side of a square inscribed in it.

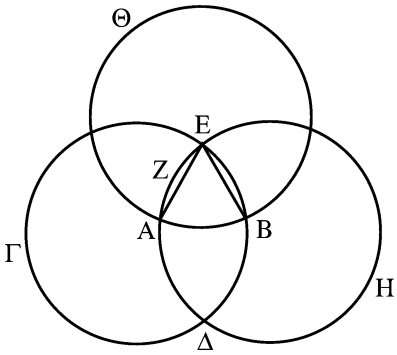

Fig. 5. Theodosius, *Spherics* I 20.

distance; however, in the spherical case *radius* cannot be used and *distance* should be preferred.[22] We may simply want to translate with *distance* in all cases.

3. What is a *diastēma* in Greek geometry?

One answer to this question is simple: a *diastēma* is a distance with which a circle is drawn in a plane or on a sphere. But what does it mean to draw a circle in the context of a Greek geometric text? Fowler and Taisbak maintain that the term *diastēma* "always means the opening of a (notional) compass."[23] It seems likely that the term *diastēma* originates from the use of the compass as a geometric instrument and finds its way into the systematic treatises because of its usefulness in allowing the definition of a circle to function as an axiom.[24]

Schmidt was of the opinion that the figures of ancient spherics were meant to be drawn on solid spheres and he thought that this could be demonstrated from some of Theodosius's constructions.[25] Indeed, a brief look at the figures in the texts of ancient spherics makes it clear that it would have been very difficult to develop an intuitive grasp of spherical geometry using these as the only reference. Here again, we may have a case of the constructions of the systematic treatises being an abstraction from mathematical practice. Although it is a simple matter to draw a circle on a sphere which is in the relative vicinity of its pole with a compass, when we try to draw a great circle we need to take more care. In this case, the compass must be large in relation to the sphere so that the points will meet the surface at a great enough angle to fix and draw.[26]

The fact that Theodosius never postulates the ability to draw a circle on a sphere might lend credence to the idea that these circles are held to be drawn by the same instrument or notional instrument as circles

22 Heiberg translates *diastēma* with *radius* throughout his Latin translation of Theodosius' *Spherics*, as does Toomer in a number of places in book II of his English translation of Ptolemy's *Almagest*, Heiberg [1927, 34 ff.], and Toomer [1984, 106 ff.].

23 Fowler and Taisbak [1999, 363].

24 Mueller provides a discussion of the way that definitions function as axioms in Greek mathematics [Mueller, 1991].

25 Schmidt [1938, 13–14].

26 In fiddling with a compass and some spheres, I found that it was quite easy to draw great circles on a sphere using a compass with legs about three times the radius of the sphere.

on the plane because then Theodosius could consider the spherical situation to be essentially covered by the Euclidean postulate. On the other hand, Theodosius also assumes the ability to lay out (κεῖσθαι) an arc equal to another arc in *Spherics* III 6 without ever bothering to show how this would be done with a proposition along the lines of Euclid's proof that a line can be set out (θέθαι) equal to another line in *Elements* I 2.[27] In general, it does not seem that Theodosius is as concerned with the first principles of constructions as Euclid.

It is not at all obvious what Theodosius is doing with some of his constructions. Berggren has argued that some of Theodosius' constructions need to be read as existence proofs.[28] The example that Berggren gives is the first case of *Spherics* I 20, which we saw was dismissed almost out of hand. Theodosius says, εί μὲν οὖν τὰ A, B κατὰ διάμετρόν ἐστι τῆς σφαίρας, φανερόν, ὅτι μέγιστοι κύκλοι ἄπειροι διὰ τῶν A, B σημείων γραγήσονται, which Berggren reads as, "When A and B lie diametrically opposite it is clear that arbitrarily many great circles can be drawn through A and B."[29] From this reading, Berggren takes the proof to be about the possibility of constructing a great circle though A and B. On the other hand, we might read the same text as, "If, now, A and B are on the diameter of the sphere, it is clear that indefinitely many great circles will be drawn through A and B." Under this reading, one could argue that the reason Theodosius is so dismissive of this case is not that it is obvious that an indefinite number of great circles *can* be so drawn, because one still has the problem of actually drawing one, but rather that an indefinite number *will* be so drawn and yet none of these will be determinate or in any way privileged. The problem is dismissed not because it is obvious but because it does not allow of a determinate solution. It is analogous to the problem of drawing a line through a given point on the plane. Such "problems" were probably considered outside the scope of the geometers interest because, having an indefinite number of solutions, they provided the geometer with no new insight or tools.

There are two propositions that argue strongly against reading Theodosius' constructions as existence proofs. *Spherics* I 19 shows us how to set out (ἐκθέσθαι) the diameter of a given sphere; and *Spherics* I 21 shows us how us how to find the poles of a given circle. In *Spherics* I 1, on the other hand, in the process of finding a given circle's center, the circle's diameter and a particular circle's poles are constructed, moreover, they are constructed in different ways than in *Spherics* I 19 and 21. Here, as often in Greek geometry, the manner in which a construction is carried out is as important as the fact that it can be carried out.[30] Theodosius' constructions deserve to be studied at greater length.

The systematic geometric treatises make no mention of practical constructions through the use of tools; however, we have other texts in the geometric corpus that make it clear that the Greek geometers were concerned with accurate drawings and designed special tools to accomplish them.[31] The postulates and constructions of the systematic treatises seem principally to perform a logical function but they are sometimes modeled around actual techniques of drawing.[32] The postulates and constructions allow

[27] Heiberg [1927, 134] and Heiberg [1969, Vol. 1, 8].

[28] Berggren [1991, 246].

[29] Heiberg [1927, 36] and Berggren [1991, 246].

[30] See Netz for a similar stance on the role of geometrical analysis in Greek mathematical texts [Netz, 2002, especially 152].

[31] Some examples are Diocles' use of a bone ruler to draw a parabola [Toomer, 1976, 63–67]; Eratosthenes' mechanical solution to the problem of finding two mean proportionals to two given lengths [Hultsch, 1976–1978, Vol. 3, 90–96]; and the *neusis* constructions carried out by unknown mathematicians who manipulate a ruler according to the conditions of the construction until the right fit is found [Hultsch, 1976–1978, Vol. 1, 249–250].

[32] Schmidt's arguments make this clear in the case of some of Theodosius' spherical constructions [Schmidt, 1938, 13–14].

geometric objects to be constructed in ways that then allow geometers to write proofs about the objects so constructed because their manner of construction is determinate. Using a *diastēma* to draw a circle tells us how the circle has been drawn in a way that introduces necessity into the construction itself. The use of a *diastēma* allows us to then say that certain lines are equal because that is how the circle was drawn. In fact, in constructions that employ a circle we find that this is often how the circle is used. Theodosius' text, like Euclid's, gives the geometer all the information necessary both to follow the logical development of the material and to reconstruct the figures.[33] The use of a *diastēma* to draw a circle probably performs a dual function of satisfying the logical needs of the systematic treatise and modeling the actual practice of geometers making figures on the plane or the sphere.

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[33] As Schmidt points out there does not seem to be any other motivation for *Spherics* I 19 [Schmidt, 1938, 114, n. 7].