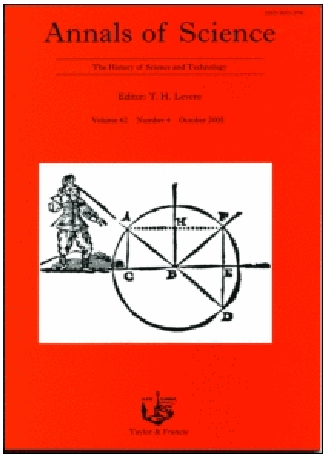

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What We Can Learn from a Diagram: The Case of Aristarchus's **On The Sizes and Distances of the Sun and Moon**

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What We Can Learn from a Diagram: The Case of Aristarchus's *On The Sizes and Distances of the Sun and Moon*

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Summary

By using the example of a single proposition and its diagrams, this paper makes explicit a number of the processes in effect in the textual transmission of works in the exact sciences of the ancient and medieval periods. By examining the diagrams of proposition 13 as they appear in the Greek, Arabic, and Latin traditions of Aristarchus's *On the Sizes and Distances of the Sun and Moon,* we can see a number of ways in which medieval, and early modern, scholars interpreted their sources in an effort to understand and transmit canonical ancient texts. This study highlights the need for modern scholars to take into consideration all aspects of the medieval transmission in our efforts to understand ancient practices.

Contents

1. Introduction

Our knowledge of the theoretical aspects of Greco-Roman mathematical sciences relies almost solely on the medieval manuscript tradition. Although there is material evidence for some of the more social aspects of the exact sciences, particularly astronomy and astrology, few to no mathematical, observational, or experimental instruments are known to have survived.[1] Although the papyri have turned out to be

[1] One could, of course, argue that instruments like sundials are mathematical, but here I refer to instruments that were used for doing exact science, as opposed to resulting from it. For recent appraisals of the material evidence for astronomical and astrological activity, see J. Evans, 'The Material Culture of Greek Astronomy', *Journal for the History of Astronomy* 30 (1999), 237–307 and J. Evans, 'The Astrologer's Apparatus: A Picture of Professional Practice in Greco-Roman Egypt', *Journal for the History of Astronomy* 35 (2004), 1–44. There are a few objects, such as the Keskintos Inscription and the

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a rich source for scientific activity and cultural transmission, very little theoretical writing survives in these sources.[2] Hence, for the theoretical texts, our sources are usually Greek manuscripts compiled many centuries after the originals they are presumed to preserve. In some cases, our best sources are Arabic, or Latin, translations. The reasons for making these medieval copies, as well as the social and intellectual contexts in which this work was carried out, are thus far removed from that of the original composition. Nevertheless, the manuscripts are the key source for the ancient texts, and a study of the manuscript tradition must always form the basis of an edition of the text.

For scientific works, however, the manuscripts usually contain diagrams as well as text. With a few exceptions, it is only recently that the diagrams in the medieval sources for the exact sciences have received the same level of critical scrutiny as the text itself.[3] Indeed, the first critical apparatus for the manuscript diagrams of a major Hellenistic geometer was made only a few years ago.[4] The standard practice, until recently, has been to print revised diagrams based on the editor's idea of what image will best support the text and benefit the reader. In many cases, these printed figures are, in fact, more helpful for understanding mathematical ideas in the text than what we find in the manuscript sources. Manuscript diagrams, however, are more than

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Antikythera Mechanism, that are important evidence for exact sciences and may have formed part of the material culture of these fields; see A. Jones, 'The Keskintos Astronomical Inscription', *SCIAMVS* 7 (2006), 3–42, D.J. Price, 'Gears from the Greek, The Antikythera Mechanism—A Calendar Computer from c.80 B.C., *Transactions of the American Philosophical Society,* New Series 64 (1974), part 7, and T. Freeth, Y. Bitsakis, X. Moussas, J.H. Seiradakis, A. Tselikas, H. Mongou, M. Zafeiropoulou, R. Hadland, D. Bate, A. Ramsey, M. Allen, A. Crawley, P. Hockley, T. Malzbender, D. Gelb, W. Abrisco, and M.G. Edmunds 'Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism', *Nature* 444 (2006), 587–91 (see also supplementary material online).

[2] A few theoretical astronomical papyri are discussed by A. Jones, 'An "Almagest" Before Ptolemy's?' in C. Burnett, J.P. Hogendijk, K. Plofker, and M. Yano, eds., *Studies in the History of the Exact Sciences in Honour of David Pingree* (Leiden, 2004), 129–36. There are very few Greek mathematical papyri that contain diagrams, and these are either problem texts, usually involving area or volume calculations, or fragments related to Euclid's *Elements.* A number of photographs of mathematical papyri containing diagrams are collected in D. Fowler, *The Mathematics of Plato's Academy* (Oxford, 1987), plates 1–3 and 8.

[3] B.S. Eastwood has made a number of valuable studies of diagrams in Latin MSS of medieval exact sciences, of which I cite only two: B.S. Eastwood, 'Characteristics of the Plinian astronomical diagrams in a Bodleian palimpsest, *Ms. D'Orville* 95, ff. 25–38, *Sudoffs Archiv für Geschichte der Medizin und der Naturwissenshaften* 67 (1983), 2–12 and B.S. Eastwood, 'The Diagram "sphera celestis" in the "Hortus deliciarum": A confused amalgam from the astronomies of Pliny and Martianus Capella, *Analli dell' Instituto e Museo di Storia della Scienza di Firenze*, 6 (1983), 177–86. Other papers which make critical studies of manuscript figures are J.G. van der Tak, 'Calcidius' Illustration of the Astronomy of Heraclides of Pontus', *Mnemosyne* 25, Series 4 (1972), 148–56; and M. Decourps-Foulquier, 'Sur les figures du traité des *Coniques* d'Apollonios de Pergé édité par Eutocius d'Ascalon', *Revue d'histoire des mathématiques* 5 (1999), 61–82; G. De Young, 'Diagram in the Arabic Euclidean tradition: a preliminary assessment', *Historia Mathematica* 32 (2005), 129–79; A. Keller, 'Making diagrams speak, in Bhāskara I's commentary on the *Āryabhatya*'. *Historia Mathamatica* 32 (2005), 275–302. See also K. Weitzmann, 'The Greek Sources of Islamic Scientific Illustrations', in G. Miles ed., *Archaeologica Orientalia in Memoriam Ernst Herzfeld* (New York, 1952), 224–66.

[4] R. Netz, *The Works of Archimedes: Volume I. The Two Books On the Sphere and the Cylinder* (Cambridge, 2004). The practice of including a critical apparatus for the manuscript diagrams was begun by A. Rome. See, for example, A. Rome, ed., *Commentaires de Pappus et Théon d'Alexandrie sur l'Almageste,* 3 vols., Biblioteca Apostolica Vaticana, Studi e Testi 72, 54 and 106 (Rome, 1931–1943). Other examples are A. Lejeune, ed. et trad., *L'Optique De Claude Ptolémée* (Leiden, 1989), A. Jones, ed. and trans., *Pappus: Book 7 of the Collection* (New York, 1986), 620–27, F.J. Ragep, *Nasīr al-Dīn al-Ṭūsī's* Memoir on Astronomy (New York, 1993), 569–82, P. Kunitzsch and R. Lorch, *Maslama's Notes on Ptolemy's* Planisphaerium *and Related Texts,* Bayerische Akademie der Wissenshaften, Phil.-Hist. Klasse (Munich, 1994), 73–75, and R. Lorch, *Thābit ibn Qurra, On the Sector Figure and Related Texts* (Frankfurt am Main, 2001), 122–23.

simply aids for understanding text. Indeed, from a historical perspective, this is an inessential and basically irrelevant feature. To the historian, the diagrams contain information about the specific composition of the manuscripts. Both conceptually and paleographically, they are related in interesting ways to the text.[5] In some cases, they may go back essentially unchanged to the ancient sources and tell us something about the conceptions and methods of ancient authors. In other cases, they demonstrably do not, so that they tell us something about the medieval copyists, editors, and translators. In every case, a manuscript figure is historically more valuable than a modern revision.

This paper explores the historical insights that can be derived from diagrams by using an example from a single mathematical proposition as found in a number of different sources. The particular diagram we will investigate is found in *On the Sizes and Distances of the Sun and Moon*, attributed to Aristarchus of Samos (*c*. early third century BCE).[6] The reason for looking at this diagram is that it vividly highlights the problems involved in reconstructing ancient diagrams. As we will see, it is not possible, in this case, to make a single, mathematically coherent diagram. Hence an editor is faced with choices as to which of the mathematical characteristics of the proposition the diagram should feature. A case like this emphasizes the value of a historical assessment of the manuscript figures for our evaluation of both ancient and medieval understandings of the issues at stake.

It should be said at the outset that the diagram in question is not typical of Greek mathematical diagrams. It has a number of unique features that have been problematic for its readers throughout the centuries. The theorem that it accompanies involves unstated assumptions and the objects that the diagram represents are not perfectly coherent. The same features that make it difficult from a mathematical perspective, however, make it useful from a historical perspective. This paper is a case study of an extreme example of the kinds of changes that could occur in the transmission of mathematical texts. Although it is not a sample of general practice, this example allows us a window through which we can see with more clarity certain activities of the medieval scholars that are normally obscured.

2. Remarks on Drawing the Diagrams

In preparing the figures for this paper, I have given particular attention to replicating the diagrams found in the manuscripts.[7] Naturally, barring high-quality colour photographs, only certain graphic elements can be reproduced. The figures in this paper preserve the orientation, shape, internal scale, and label positions of the originals. Features such as colour, line weight, and letter shape are not preserved.[8]

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[5] The interesting conceptual relationship between the diagrams and the text is discussed at length by R. Netz, *The Shaping of Deduction in Greek Mathematics* (Cambridge, 1999), 13–88. Although he based his study on MS figures, this work is made somewhat problematic by the fact that he does not state in each case which MSS he actually used. There is sometimes considerable variation between the figures in different manuscripts. For an example, see note 44, below.

[6] T. Heath, *Aristarchus of Samos: The Ancient Copernicus* (Oxford, 1913). The text and translation are on pp. 352–411.

[7] I am grateful to Ken Saito for introducing me to a number of techniques for doing this and giving me computer programs designed by himself and Paolo Mascellani for facilitating this process.

[8] Because handwriting is more flexible than type, certain, hopefully trivial, features of the label positions may not be preserved.

I have also made no attempt to mark objects that are drawn by hand as opposed to with compass and straight edge, although I have reproduced the actual shape of the lines. Nevertheless, it is often difficult to see with the naked eye which lines are not perfectly straight, or which curves are not perfectly circular. My hope is that the reader may use the figures provided to understand all the features of the manuscript diagrams necessary to the present discussion.

3. The Treatise and Aristarchus

Aristarchus is dated to the beginning of the third century BCE by Ptolemy (second century CE), who credits him with an astronomical observation recorded in 280 BCE. The argument for attributing the Greek text known as *On the Sizes and Distances of the Sun and the Moon* to Aristarchus should be based on three ancient sources.

The earliest of these is unfortunately the most vague. In his *Sand Reckoner*, Archimedes (*c*.280–212 BCE) mentions Aristarchus no fewer than ten times. Indeed, in this one small work, Archimedes refers to Aristarchus more often than he refers to any other predecessor in his whole surviving corpus. The treatise is clearly a gesture to Aristarchus's work.[9] Although Archimedes does not mention a text called *On Sizes,* he tells us, among other things, that Aristarchus found that the Sun is less than twenty and more than eighteen times the size of the Moon.[10] Since, this result is demonstrated in *On Sizes* 9, it has generally been supposed that Archimedes is referring to *On Sizes.* In fact, however, this conclusion does not follow. There are many ways to account for the fact that this theorem is both attributed to Aristarchus and appears in *On Sizes.* Indeed, the *Sand Reckoner* also ascribes certain hypotheses to Aristarchus that are contrary to what we find in *On Sizes.* The most famous of these is the heliocentric hypothesis, whereas *On Sizes* is based on a geocentric cosmos.[11] This means that Archimedes cannot, in fact, be taken as a source for the authorship of *On Sizes.*

The first text we have which explicitly associates Aristarchus with *On Sizes* also associates him with the heliocentric hypothesis. Toward the end of the first century CE, Plutarch mentions Aristarchus three times in his *On the Face Appearing in the Circle of the Moon.*[12] The first of these discusses a criticism that was levelled against Aristarchus for the sacrilege of putting the Earth in motion.[13] The second states that Aristarchus shows, in a book called *On Sizes and Distances,* that the Sun is more than twenty and less than eighteen times the distance of the Moon from us.[14] This is demonstrated in *On Sizes* 7. Moreover, the way Plutarch expresses this is stylistically

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[9] The relationship between these two treatises is discussed in J. Christianidis, D. Dialetis and K. Gavroglu, 'Having a Knack for the Non-Intuitive: Aristarchus's Heliocentrism Through Archimedes's Geocentrism', *History of Science* 40 (2002), 147–68 and B.E. Wall, 'The Historiography of Aristarchos of Samos', *Studies in the History and Philosophy of Science* 6 (1975), 201–28.

[10] J.L. Heiberg, ed. et int., *Archimedis opera omnia cum commentariis Eutocii*, 3 vols (Leipzig, 1973), cor. et ad. E. Stamatis, vol. 2, 220.

[11] The only theorem in *On Sizes* that is indisputably geocentric is *On Sizes* 7 (note 6, pp. 376–81).

[12] H. Cherniss and W.C. Helmbold, *Plutarch's Moralia: XII* (Cambridge, MA, 1957), 1–223. *On the Face Appearing in the Face of the Moon* is dated to sometime after 71 CE by F.R. Stephenson and L.J. Fatoohi, 'The Total Solar Eclipse Described by Plutarch', *HISTOS* 2, 1998 (http://www.dur.ac.uk/Classics/histos/1998/stephenson.html).

[13] Cherniss, *On the Face,* p. 54 (note 12).

[14] Cherniss, *On the Face,* p. 74 (note 12).

similar to the expression of this theorem found in the introductory section of *On Sizes.*[15] Finally, Plutarch asserts that Aristarchus demonstrates that the ratio of the diameter of the Earth to the diameter of the Moon is greater than 108:43 (=2.51) and less than 60:19 (=3.16). This is shown in *On Sizes* 17. Clearly, by the end of the first century of our era, Plutarch believed that Aristarchus both put forward a heliocentric hypothesis and wrote *On Sizes.* The difficulty is that Plutarch lived over four centuries later than Aristarchus; he probably had no special knowledge of the latter's activities.

Near the beginning of the 4th century, Pappus included Aristarchus's *On Sizes* in a course of mathematical astronomy apparently to be studied between Euclid's *Elements* and Ptolemy's *Almagest.*[16] At this time, the text and its author had long since become an established part of the tradition.

It is only relatively recently that anyone has questioned the idea that *On Sizes* was written by Aristarchus. As A. Bowen and B.R. Goldstein have noted, the ancient sources are not sufficient to prove that *On Sizes* was actually composed in the beginning of the third century BCE by Aristarchus.[17] What they do show is that Aristarchus found a result that is also shown in a text called *On Sizes,* and that by at least the first century CE, this text was attributed to Aristarchus.

As historians, we should like to know not only the facts of the authorship of *On Sizes* but also the intellectual conditions that gave rise to its composition. It is important, in this regard, to be clear about just how little we know regarding the context in which *On Sizes* was composed, because our beliefs about the intellectual context will inform our understanding of the authorship of the text.

I believe that Aristarchus wrote *On Sizes* because of similarities between it and works by Archimedes. Moreover, I believe these should be taken together as key examples of a particular style of early Hellenistic exact science. Both *On Sizes* and a number of Archimedes' works make use of hypotheses not because they are strictly true, but because they yield fruitful results. This interpretation, however, relies on readings of these works that are not certain. Bowen and Goldstein believe that *On Sizes* fits better into a late Hellenistic or early Imperial context, where we have clear evidence that others were specifically interested in computations of the sizes and distances of the luminaries.[18] D. Rawlins, on the other hand, does not believe that *On Sizes* fits into the context of highly developed heliocentric astronomy in which he believes Aristarchus worked.[19]

The point is that each of these positions can be advanced because we have no ancient works that contextualize *On Sizes* before the first century CE, by which time the work was already considered well known. The intellectual context in which *On Sizes* first appeared is simply not a matter of historical record. The text itself neither refers to its sources nor mentions research questions it was intended to address. Nor

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[15] Compare the Greek texts of Cherniss, *On the Face,* p. 74 (note 12) with Heath, *Aristarchus,* p. 552 (note 6).

[16] F. Hultsch, ed. et int., *Pappi Alexandrini collectionis quae supersunt*, 3 vols (Berlin, 1876–1878), 474 and 554–60.

[17] A.C. Bowen and B.R. Goldstein, 'Aristarchus, Thales, and Heraclitus on Solar Eclipses: An Astronomical Commentary on P. Oxy. 53.3710 Cols. 2.33–3.19', *Physis* 31 (1994), 689–729.

[18] They do not, however, claim that there is more evidence for this than the older date, they simply point out that this is also a real possibility (note 17, p. 700, n. 20).

[19] D. Rawlins, 'Ptolemy's Planetary Models as Funny Science', *Journal of Hysterical Astronomy*, 1.1 sec. 7 (1991), 69, n. 6.

is it specifically mentioned by contemporary authors as having been a point of departure for their work.

4. The Transmission of the Treatise

For most of what we do know of the history of *On Sizes,* it was included in a somewhat variable group of texts on mathematical astronomy that was supposed to be read between Euclid's *Elements* and Ptolemy's *Almagest.* In late antiquity, this collection was known as the *Little Astronomy*, while in the medieval Arabic tradition it was called the *Middle Books.* The list and order of the treatises were not fixed but were built around a core of Hellenistic works, in which *On Sizes* was generally included. During the medieval period, works in pure geometry and other exact sciences entered the *Middle Books*, and by at least the thirteenth century, it included original works by Arabic authors. It was not until the Italian Renaissance that *On Sizes* began to be presented and read outside the context of this collection of texts.[20] For the late ancient period, our evidence for this collection is Pappus' commentary to it in the compendium of his mathematical writings.[21] For the Arabic translation efforts in Baghdad, we have some mentions of the collection and of at least one Arabic commentary written on it.[22] Moreover, from the medieval period, we have a number of manuscripts in both Greek and Arabic. These, then, are our evidence for the text of *On Sizes* itself.

The oldest source for the Greek text is a well-known ninth century manuscript, *Vat. gr.* 204. It contains a number of treatises of the *Little Astronomy* as well as Eutocius' commentary on Apollonius' *Conics*. There are a few other important copies made from one or more Byzantine manuscripts which are now lost.[23] The manuscript basis for Heath's critical edition was *Vat. gr.* 204.[24] Although the Greek text of *Vat. gr.* 204 is good, the diagrams are sometimes confusing from a mathematical perspective—a fact that is not clear from Heath's edition.[25] At the same time, the codex contains a fair number of mathematical scholia, which sometimes include auxiliary diagrams and were meant to aid the reader in interpreting the text. From a mathematical perspective, the value of these scholia is mediocre at best and often negligible; nonetheless, their presence shows that some readers in the Greek transmission were making a serious attempt to understand the mathematical content of the treatise and to ensure that this understanding not be lost. Nevertheless, when *On Sizes* was translated into Arabic, in the ninth century,

[20] The two Renaissance Latin editions are discussed below (notes 34 and 35).

[21] Hultsch, *Pappi collectionis,* p. 474 ff. (note 16).

[22] In the twelfth century CE, al-Samaw'al gives the title of a commentary that Qusṭā ibn Lūqā (late ninth century) had written on the *Middle Books;* see F. Sezgin, *Geschichte des arabischen Schrifttums: Band VI, Astronomie bid ca. 430 H.* (Leiden, 1978), 65–66. There are other, mostly later, mentions of the translation work on the *Middle Books*, for example Abū Saʿīd al-Ṣafadī (fourteenth century) attributes editorial work on the *Middle Books* to Thābit ibn Qurra; see F. Rosenthal, *The Classical Heritage in Islam* (Berkeley, CA, 1965), 18.

[23] There are five Vatican MSS that can be considered evidence for early medieval and late ancient work. These are *Vat. gr.* 191, 192, 202, 203, and 204. The text history of *On Sizes* has been studied by B. Noack, *Aristarch von Samos: Untersuchungen zur Überlieferungsgeschichte der Schrift* περὶ μεγεθῶν καὶ ἀποστρημάτων ἡλὶου καὶ σελήνης (Wiesbaden, 1992).

[24] Heath, *Aristarchus,* p. 326 (note 6). He also made use of Wallis' critical edition, which includes an apparatus detailing the readings of a number of other MSS (note 36).

[25] In particular, the figures for *On Sizes* 7, 13 and 14 (ff. 113v, 116r and 116v).

and Latin, in the sixteenth, it required considerable work to make sense of the Greek sources.

Arabic scientific researchers almost certainly encountered *On Sizes* in a very small number of Greek MSS that contained some selection of the *Middle Books.* The first evidence that this group of treatises was known in Baghdad is a reference to a lost commentary on the collection made by Qusṭā ibn Lūqā (d. 912).[26] Around the same time that our earliest Greek MS of the text was being copied in Byzantium, Thābit ibn Qurra composed an Arabic edition in Baghdad. This was done as a *revision,* a genre of composition commonly used by the Baghdad mathematicians for the preservation of Greek sources.

A *revision* was a correction along theoretical, or mathematical, lines. The goal of a revision was not to change the text but to preserve what was held to be its essential content from the vagaries of transmission: textual corruption, distorted argumentation, strange translations and, most significantly for our purposes, poorly drawn figures. In other words, strict textual preservation was sacrificed for the sake of conceptual preservation, based, of course, on the Arabic author's idea of the original content. Revisions of mathematical texts are often characterized by redrawn, and relabelled, figures and fleshed out mathematical argumentation.

In order to undertake a revision, a scholar had to have a solid command of the subject. Those who produced revisions of Greek texts also wrote original Arabic treatises on related topics. Revisions were often carried out on the basis of previous translations but this was not necessarily a division of labour between a philologist and a mathematician; for example, Thābit revised the translation of the *Elements* made by Qusṭā ibn Lūqā while he himself produced the translation used by the Banū Mūsā in their revision of the later books of Apollonius' *Conics.* It is not known whose translation Thābit used as the source of his edition of *On Sizes.* Since Thābit was himself a translator, we should admit the possibility that he made the revision directly on the basis of a Greek text. Thābit's version of *On Sizes* is currently known from a single MS made toward the end of the thirteenth century, probably in Cairo.[27] The hand is clear, and although there are a fair number of common errors in the text, the diagrams are very fine.

The most common Arabic version of *On Sizes,* indeed of all the *Middle Books,* is that made by Naṣīr al-Dīn al-Ṭūsī (1201–1274). In the midst of the political and military turmoil through which he lived, al-Ṭūsī carried out a remarkable project of cultural preservation. The core of this work was a series of new *editions* of the foundational works in Greek and Arabic exact sciences, which included the *Middle Books*.

These *editions* were a new genre for the presentation of canonical sources. An *edition* was stylistically closer to its Arabic source than a *revision* to its Greek source. For the most part, Ṭūsī's editions were more concise than his sources and written in a more standardized technical idiom. Ṭūsī reworked individual phases to be more concise, eliminated repetitions and unnecessary elements of proof structure and only

[26] F. Sezgin, *Geschichte des arabischen Schrifttums,* p. 66 (note 22).
[27] E. Kheirandish, *The Arabic Version of Euclid's* Optics, 2 vols. (New York, 1999), xxvii.

occasionally introduced clarifying words or steps in an argument. In some places, he also corrects arguments that had suffered textual corruption.[28] In the case of *On Sizes,* it can be shown that Ṭūsī made his edition on the basis of Thābit's revision.[29] In particular, all the diagrams are based on those of the Thābit version and are identically lettered. The Ṭūsī version of *On Sizes* is extant in a number of MSS, and has been printed in the Hyderabad edition of Ṭūsī's *Middle Books*.[30]

Although there is some evidence that medieval Latin translations of *On Sizes* were made, none of these has been found.[31] The earliest extant Latin editions are two Renaissance translations made on the basis of Greek sources. Neither of these presents the treatise in the same context as the manuscript tradition. In 1488 and again in 1498, the humanist scholar Giorgio Valla (*c*.1430–1499), now best known to scholars for his library of Greek manuscripts, published a Latin version of *On Sizes* in a collection that was mostly made up of translations of Greek treatises. In this book, our treatise appears together with a dizzying array of philosophical, medical, mystical, and mathematical texts.[32] Valla's translation has been little studied and does not seem to have affected the later tradition of reading the text.

On the other hand, the edition of the text published by Frederico Commandino (1506–1575) toward the end of a long career of translating Greek mathematical sources, produced a lasting influence in the European scholarship on the treatise.[33] Commandino worked carefully through all the mathematical arguments supplying a number of key lemmas and redrew many of the diagrams to reflect his understanding of the mathematics.

John Wallis (1616–1703) made good use of Commandino's book when he produced the first critical edition of the Greek text based on Greek MSS and supplied with a critical apparatus.[34] Heath, in turn, used Wallis' apparatus as his

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[28] For examples of his textual practice, see R. Rashed, *Les mathématiques infinitésimales du IXe au XIe siècle*, 2 vols. (London, 1996), 12–27 and J.L. Berggren and N. Sidoli, 'Aristarchus's *On the Sizes and Distances of the Sun and Moon:* Greek and Arabic Texts', *Archive for History of Exact Science,* 61 (2007), 213–254.

[29] This is shown in Berggren and Sidoli, 'Aristarchus's *On the Sizes'* (note 28).

[30] Al-Ṭūsī, *Kitāb Aristarkhus fī jirmay al-nayirayn wa-bu'dayhimā* (Hyderabad, 1940). A facsimile of one of the manuscripts, Tabriz National Library MS 3484, has been printed by J.A. Chavoshī, intro., *Nasir al-Din at-Tusi: Taḥrīr-e Mutawassīṭāt* (Tehran, 2005).

[31] Noack, *Aristarch von Samos,* pp. 45–47 (note 25).

[32] The title page of the 1498 edition reads as follows: *Georgio Valla Placentino Interprete hoc in volumine hec continentur Nicephori logica, Georgii Valle libellus de argumentis, Euclidis quartus decimus elementorum, Hypsiclis interpretatio eiusdem libri Euclidis, Nicephorus de astrolabo, Aristarchi Samii de magnitudinibus et distantiis solis et lunae, Timeus de mundo, Cleonidis musica, Eusebii Pamphili de quibusdam theologicis ambiguitatibus, Cleomedes de mundo, Athenagorae philosophi de resurrectione, Aristotelis de cello, Aristotelis magna ethica, Aristotelis ars poetica, Rhazes de pestilentia, Galenus de inequali distemperantia, Galenus de bono corporis habitu, Galenus de confirmatione corporis humani, Galenus de presagitura, Galenus de presagio, Galeni introductorium, Galenus de succidaneis, Alexander Aphrodiseus de causis febrium, Pselus de victu humano, Impressum Venetiis per Simonem Papiensem dictum Beuilaquam. 1498. Die ultimo Septembris.*

[33] F. Commandino, *Aristarchi De magnitudinibus, et distantiis solis, et lunae, liber cum Pappi Alexandrini explicationibus quibusdam. A Federico Commandino in Latinum conuersus, ac commentarijs illustratus* (Pesaro, 1572),

[34] J. Wallis, *Aristarchi Samii de magnitudinibus & distantiis solis & lunae, liber. Nunc primum Graece editus cum Federici Commandini versione Latina, notisque illius & editoris* (Oxford, 1688). Wallis' text was based on the highest contemporary standards of text criticism and included a number of useful innovations it the apparatus; see M.D. Reeve, 'John Wallis, editor of Greek mathematical texts, in G.W. Most, ed., *Editing Text, Texte edieren,* Aporemata: Kritiche Studien zur Philogiegeschichte, Band 2 (Göttingen, 1998), 77–93.

primary check against *Vat. gr.* 204, which Wallis did not consult.[35] In this way, Commandino's understanding of the mathematics and his diagrams, which were an important aspect of his scholarship, have formed the starting point for all subsequent readings of the Greek text.

We can use the diagrams in the Greek, Arabic, and Latin sources to examine differences in the way this treatise was read during periods of translation, as opposed to its transmission by the Byzantine scribes. We will focus on the diagrams to *On Sizes* 13.

5. The Treatise Itself

On Sizes is a work of structural cosmology or astronomy, which is presented in the style of deductive mathematics. It shows that if we start with a few simple, perhaps crude, assumptions about the visible phenomena of the luminaries, then we can demonstrate certain statements about their relative sizes and distances and their sizes relative to that of the Earth.[36] Hence, it begins with hypotheses, which allow some aspect of the physical world to be depicted in geometric diagrams and these, in turn, to be used in computations. It then proceeds, through established mathematical methods, to derive numerical statements about the things in the world. Although we cannot be certain at what point, prior to the first century, *On Sizes* was composed, it is worth noting that Archimedes' mechanical treatises, *Floating Bodies* and *On the Equilibrium of Planes,* which are of the same type, were written in the third century BCE.[37] It is generally believed that the other Greek treatises of this form, such as Autolycus' *On a Moving Sphere* and *Risings and Settings* and Euclid's *Optics* and *Phenomena,* were also composed during this period.

The basic strategy of the text is to generate two ratios between numbers that serve as bounds on a ratio of the relative distances of the luminaries from earth or the relative sizes of pairs of the Sun, Moon and Earth. Ratios of sizes are conveyed using both diameters and volumes.

A number of propositions are essentially lemmas leading up to an important result. For example, the first six propositions are needed to show *On Sizes* 7, which provides bounds relating the solar distance to the lunar distance, whereas *On Sizes* 12–14 are lemmas for *On Sizes* 15, which bounds the ratio of the solar diameter to the terrestrial diameter.

The treatise as it stands is simply a series of mathematical statements; it makes no attempt to situate these within an interpretive framework. There are probably a number of valid ways to understand the goals of the text.

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[35] Heath also consulted the work of A. Fortia d'Urban, *Histoire d'Aristarque de Samos*, 2 vols (Paris, 1810), which gave him access to some readings of MSS in Paris that d'Urban had noted.

[36] There are a number of discussions of the aim and approach of the text. For example, O. Neugebauer, *A History of Ancient Mathematical Astronomy* (New York, 1975), 634–43, R.R. Newton, *The Crime of Claudius Ptolemy* (Baltimore, MD, 1977), 171–77 and 389–94, D. Panchenko, "Aristarchus of Samos on the apparent sizes of the sun and Moon," *Antike Naturwissenschaft und ihre Rezeption* 11 (2001), 23–29, Berggren and Sidoli, 'Aristarchus's *On the Sizes*' (note 28).

[37] It should be noted, however, that of these, only *Floating Bodies* contains numerical results. On the other hand, Archimedes' *Measurment of a Circle,* although pure mathematics, exhibits a number of key computational similarities with *On Sizes.* And, as already noted, there are considerable similarities between *On Sizes* and *Sand Reckoner*.

At a basic level, it argues that claims about appearances have mathematical implications that may not themselves be immediately apparent. That is, if we think (a) that the Sun illuminates the Moon, (b) that they appear to be the same size, (c) that their angular span is small—say, at most 2°, (d) that half Moon occurs when they are nearly at right angles from one another—or at most, say, 87° apart, then it can be shown that the Sun is, in fact, much larger and much further away than the Moon, that the Moon is quite small relative to its distance from us, and that the Sun is larger than the Earth while the Moon is smaller than it.

It is possible to think of different contexts in which this work would have been relevant. It may be taken as making the general case that mathematics provides powerful tools for examining the phenomena of the heavens.[38] It may address specific philosophical problems raised by Epicurus and his school about the function of sense perception in producing knowledge about the sizes and distances of astronomical objects.[39] Finally, it may address cosmological issues that were relevant to a concentric-sphere model of the cosmos.[40]

It remains the case, however, that we cannot claim to know the specific intellectual context in which *On Sizes* was produced. Nevertheless, it is clear that it was written for a small group of readers who could be assumed to have had a working knowledge of the methods it uses and an appreciation of the relevance of its results.

6. *On Sizes* 13

The thirteenth proposition, along with *On Sizes* 12 & 14, may be taken as prefatory to *On Sizes* 15. That is, while its results may not be themselves inherently worth the work they require, they are necessary for demonstrating *On Sizes* 15, one of the most important theorems in the treatise.

On Sizes 13 proceeds by making a number of implicit simplifications to the geometry of the Earth's shadow and its relation to the orbit of the Moon. In fact, none of these assumptions are strictly true, and it is not actually possible to draw a mathematically coherent diagram of the configuration the proposition demands. These difficulties have only recently been noticed by historians of science, although they are readily apparent when one confronts the evidence of the manuscript diagrams.[41]

In Figure 1, the circle about centre A is the Sun, the circle about centre B the Earth, and circle C the Moon. The line DE is the diameter of the circle that divides the illuminated side of the Moon from its shadow. This line may be called the *dividing line.* Aristarchus tells us that the end-points of DE sweep out a circle, which we may call the *endpoint circle*, and that a chord of this circle, line FE, intersects the Earth's shadow in the two points F and E. The line FE may be called the *endpoint chord.*

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[38] Neugebauer asserts this as the goal of *On Sizes* in *HAMA,* p. 643 (note 38).

[39] Diogenes Laertius tells us that Epicurus, in his *On Nature,* stated that, "The size of the sun and the remaining stars is just as much, relative to us, as it appears." R.D. Hicks, ed. and trans. *Diogenes Laertius: Lives of Eminent Philosophers,* 2 vols (Cambridge, MA, 1972), vol. 2, 618. According to Diogenes, the argument for this concerned the brightness of the Sun and the claim that no object which is so bright could be far enough away to be very much larger than the Sun appears.

[40] We are told by Archimedes, in his *Sand Reckoner,* that most astronomers considered the cosmos to be the same size as the sphere in which the Sun moves. Heiberg, *Archimedis*, vol. 2, 218 (note 10).

[41] Berggren and Sidoli, 'Aristarchus's *On the Sizes*' (note 28).

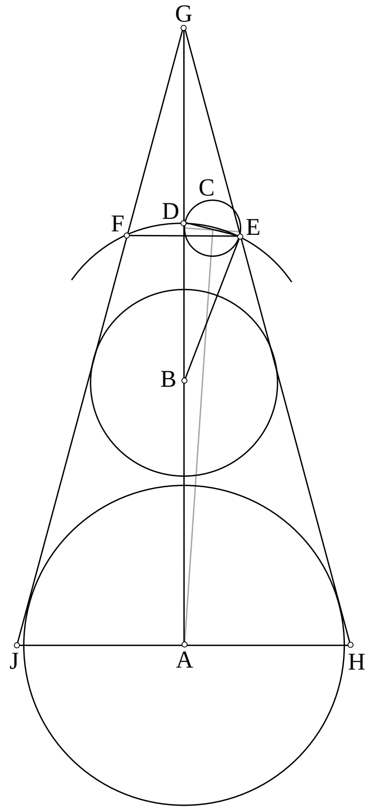

Figure 1. Possible reconstructed diagram for *On Sizes* 13.

Moreover, the proof requires that the objects be set at the first moment when the Moon fully enters the Earth's shadow.

These few statements contain a number of hidden assumptions. In order for the circle EDF to intersect the triangle of the Earth's shadow in the configuration depicted in Figure 1, it must be imagined that the Sun is still relative to the motion of the Moon. Since the duration of an average total lunar eclipse is about an hour, while the Sun is seen to move roughly a degree a day, it is, in fact, fairly reasonable to assume a stationary terrestrial shadow against which to view the motion of the Moon.

In order for the endpoints of DE to sweep out a circle, the triangle BDE must remain congruent as the Moon moves so that it rotates around B. In fact, however, the orientation of the dividing line, DE, is determined by the Moon's position relative to the Sun, not the Earth. Hence, triangle DEB will not remain congruent as the Moon moves around the Earth. Indeed, the endpoints of DE will actually sweep out two curves, the precise shape of which depend on the actual sizes and movements of the Sun and Moon. *On Sizes* 13 approximates these curves with a circle in the vicinity of the terrestrial shadow.

The geometric stipulations of the theorem require that BD is the axis of the terrestrial shadow, that BE is tangent to the Moon, and that the point of tangency, E, falls on the edge of the terrestrial shadow. At the same time, the proof of the proposition demands that the Moon should be entirely within the Earth's shadow.

Clearly, it is not possible for all of these things to be true at the same time. As stated above, the orientation of the dividing line is not determined by the Moon's position relative to the Earth. In fact, the dividing line is perceptibly equivalent to the lunar diameter perpendicular to the line joining the centre of the Sun with the centre of the Moon, shown in grey.[42] Speaking absolutely, the dividing line will be in the hemisphere of the Moon facing away from the Sun, since the Sun is much greater than the Moon.[43] In either case, a tangent drawn from the centre of the Earth to the Moon will not touch the Moon on line GH, the edge of the terrestrial shadow. As shown in Figure 1, when E is on GH, the Moon will protrude from the shadow.

Nevertheless, in order to obtain his computational goals, Aristarchus will assume that there is an endpoint chord and that it meets the line GH at E, the contact point of a tangent to the Moon from the centre of the Earth. These assumptions allow him to use the endpoint line as a key theoretical object for relating the size of the Moon to its distance from the Earth. This, in turn, permits him to determine the size of the Sun relative to the size of the Earth in *On Sizes* 15.

The purpose of *On Sizes* 13 is to develop five numerical statements involving the endpoint chord. These are used in *On Sizes* 14 & 15. The ratio of the endpoint chord to the diameter of the Moon, EF:d_m, is bounded above and below; upper and lower bounds for the ratio of the endpoint chord to the diameter of the Sun, EF:d_s, are provided; and the ratio of the endpoint chord to the extension of the solar diameter to the edges of the cone of the terrestrial shadow, EF:JH, is given a lower bound. These bounds are as follows.

$$\textit{On Sizes}\ \ 13\text{a, b: } (2 =)\ 2{:}1 < \mathrm{EF}{:}d_m < 88{:}45\ (= 1.96)$$
$$\textit{On Sizes}\ \ 13\text{c, d: } (0.11 =)1{:}9 < \mathrm{EF}{:}d_s\ \ 22{:}225\ (= 0.10)$$
$$\textit{On Sizes}\ \ 13\text{e: } (0.10 =)\ 979{:}10,125 < \mathrm{EF{:}JH}$$

One of the most interesting features of *On Sizes* 13 is that the two key geometric objects, the endpoint circle and the endpoint line, are purely theoretical. That is, they cannot be taken as well-defined geometric objects in a strictly accurate depiction of a lunar eclipse. Nevertheless, Aristarchus treats them as perfectly coherent objects, and both his diagram and his proof are constructed around them.

In order to address the particulars of Aristarchus's approach, we should compare the text with a diagram that is as close as we can get to the original. When we consult the manuscripts, however, we find that none of them contains the same diagrams we find in the published edition. In fact, consideration of the variety in the diagrams makes it likely that none of the diagrams in our sources can be taken as reproducing that which Aristarchus drew to accompany his text. Nevertheless, a study of the extant diagrams will tell us a good deal about how the text was read in the medieval and early modern periods.

[42] *On Sizes* 4 shows that the dividing circle is not perceptibly different from a great circle.

[43] *On Sizes* 2 demonstrates that a larger sphere illuminates more than half of a smaller sphere, while *On Sizes* 9 proves that the Sun is much larger than the Moon.

7. The Diagrams for *On Sizes* 13

Figure 1, which accompanies the foregoing discussion of *On Sizes* 13, is not based on any manuscript evidence. As described in the previous section, it has been drawn to meet a number of the requirements of the proof. The dividing line, DE, is nearly a lunar diameter, as shown by *On Sizes* 4. The tangents drawn from B to the Moon touch it at the endpoints of the dividing line, D and E. The endpoint chord, EF, the endpoint circle, EDF, the dividing line, DE, and the edge of the lunar shadow, GH, all meet in a single point, E. This means that the figure fails to meet another requirement of the proof. Since line BE is tangent to the Moon, while a different line, GH, falls on E from the Sun, GEH must intersect the Moon in two points. Hence, DE cannot be the true dividing line. The impossibility of drawing an accurate diagram that depicts all the geometric features of the proposition appears to have been realized by medieval scholars, as shown by the variety of medieval diagrams.

The oldest Greek MSS contain two diagrams for *On Sizes* 13.[44] This is the second proposition that has two diagrams in the manuscript. *On Sizes* 1 also has two diagrams, one for the case of equal spheres and another for the case of unequal spheres. Indeed, in *On Sizes* 1, the same diagram cannot serve for both cases. In fact, in *Vat. gr.* 204, the two parts of *On Sizes* 1 are numbered as separate propositions.

In *On Sizes* 13, however, the choice to include two diagrams seems to have been made on purely visual grounds. The first two parts of the theorem, relating the endpoint chord to the diameter of the Moon, are carried out using Figure 2. This diagram is basically sufficient for this, except that lunar diameter λο appears to be a continuation of ξλ, whereas the text specifies that it be parallel to ξν. In *Vat. gr.* 191, a generally good thirteenth–fourteenth century MS, there are no lines in the Moon [f. 62r]. Almost every one of the older Greek MSS takes a different approach to the lines in the Moon, but none is better than *Vat. gr.* 204.[45]

For the latter parts of *On Sizes* 13, *Vat. gr.* 204 contains Figure 3, coming after the text, as is the common practice in Greek manuscripts. This diagram contains the lines in the Sun that are needed for the proof, but it drops the Moon entirely. In fact, however, line βν is also used in the later parts, so that the reader must refer back to the first diagram, or a line joining β and ν has gone missing.

This is a rare case of a proposition using more than one diagram where it is not mathematically necessary. This can be contrasted with cases where more than one figure is actually required for the proof, as, for example, in *On Sizes* 1 or a number of

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[44] This is true of all of the Greek manuscripts that I have seen, the oldest MSS, all in the Vatican Library, Vat. *gr.* 191, 192, 203, 203, 204, and a number of their descendants.

[45] A number of the diagrams for *On Sizes* 13 are reproduced in Noack, *Aristarch von Samos*, Taf. VI A, XI B, XII B, XIII A and XXV (note 25). A full-colour image of the first diagram for *On Sizes* 13 in *Vat. gr.* 204 can be seen online at http://www.ibiblio.org/expo/vatican.exhibit/exhibit/d-mathematics/Greek_math2.html. This image is part of the exhibit "Rome Reborn: The Vatican Library & Renaissance Culture," edited by A. Grafton (http://www.loc.gov/exhibits/vatican/acknow.html). Netz prints significantly different figures for *On Sizes* 13 & 14 in *Shaping,* pp. 41 and 42 (note 5). That for *On Sizes* 13 is a single, combined figure and, from a mathematical perspective, it contains a number of interesting features. Whoever drew this figure must have had a peculiar understanding of the theorem. Unfortunately, Netz is not able to say where he saw this figure. In correspondence, he tells me it was likely at Trinity College, Cambridge. It is not listed in M.R. James, *Catalogue of Western Manuscripts in the Library of Trinity College, Cambridge* (Cambridge, 1904), and Heath knew of no Aristarchus MSS in Britain (note 5, p. 325). Whatever the case, this is not one of the old MSS which concern us here.

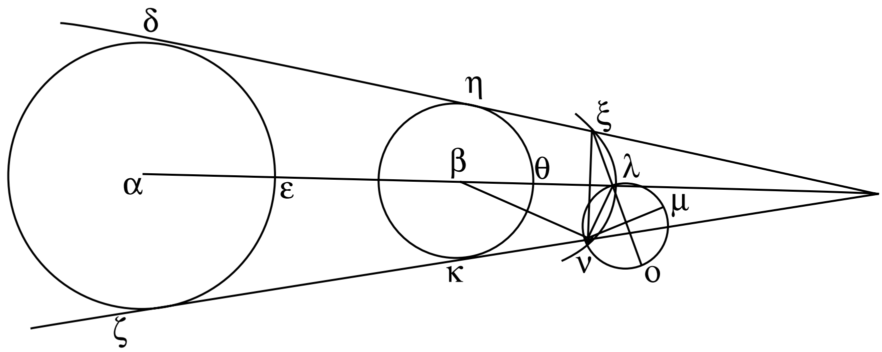

Figure 2. First diagram for *On Sizes* 13 in *Vat. gr.* 204 [f. 116r].

theorems Apollonius' *Conics.* It is clear that there has been at least some change in the diagrams from those that originally accompanied the text. A line has gone missing, and a couple of others are inconsistent with the text. It is not possible, however, to determine if there were originally two diagrams, or only one. The rarity of such double diagrams and their inaccuracy may argue against taking them as original, but these features do not prove anything.

In contrast to the Greek manuscripts, the Arabic MSS all contain a single diagram for *On Sizes* 13. This is presumably because Thābit drew a single diagram for his revision, which was, in turn, the basis for Tūsī's edition. Another possibility is that the Greek text that Thābit used as his source had only one diagram. We do not have Thābit's autograph for this text; nevertheless, a comparison of the three medieval versions of the treatise shows that Thābit drew new figures for his revision that were lettered in the order required by his expression of the proofs.[46] All of the diagrams in the two Arabic versions are similarly lettered, although there is occasionally minor variation in the lines drawn. For the most part, this lettering is different from what one would expect from a standard, or even consistent, transliteration of the lettering in the Greek MSS. In *On Sizes* 13, these variations in the diagrams are complemented by minor changes in the proofs.

The sole evidence for Thābit's revision is usually referred to as the Kraus MS, because it was sold by the book dealer H. P. Kraus.[47] E. Kheirandish has argued that this MS was copied by ʿAlī al-Marrākushī, a thirteenth century CE astronomer of Cairo. The diagrams for *On Sizes* certainly show the touch of an expert.[48] The diagram of *On Sizes* 13, Figure 4, has a number of interesting features. Unfortunately, it is not possible to determine whether all of these features were in Thābit's original or were introduced at some later point in the transmission.

[46] For a detailed comparison of the proofs, see Berggren and Sidoli, 'Aristarchus's *On the Sizes*' (note 28).

[47] H.P. Kraus, *Monumenta codicum manuscriptoirum. An Exhibition Catalogue of Manuscripts of the 6th to the 17th Centuries from the Libraries of the Monasteries of St. Catherine, Mount Sinai, Monte Cassino, Lorsch, Nanantola* (New York, 1974), p. 45, no. 18.

[48] So too, however, do those of *Arch. Selden. A.* 45, which are drawn with remarkable skill inside the small boxes left for them when the text was copied.

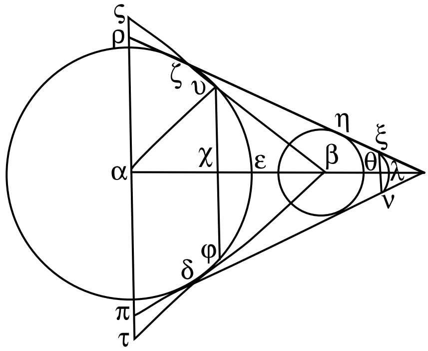

Figure 3. Second diagram for *On Sizes* 13 in *Vat. gr.* 204 [f. 116v].

This diagram is one of the most carefully drawn of any in the medieval sources, with due attention being given to mathematical requirements. One line in the Moon is again a bit off, and there is one missing line. The lunar radius, لط, should be parallel to the endpoint chord, حك, and a solar radius should be joined between points ت and ا. The most interesting feature of this diagram, however, is the placement of the endpoint chord. In fact, the diagram has been drawn to meet a different requirement of the proof than those in the Greek tradition. In Figure 4, the Moon has been set entirely within the Earth's shadow at the very beginning of total eclipse. This is, as mentioned, one of the suppositions of the proof as stated in the text, but adhering to it has certain implications for the rest of the diagram.

At the first moment of total eclipse, the Moon will be tangent to the surface of the shadow cone. This means that the lines drawn tangent to the Moon from the Earth will touch the Moon somewhere inside the Earth's shadow. The diagram in the MS of Thābit's revision is carefully drawn and depicts this situation. Hence, the extremities of the endpoint chord do not fall on the surface of the terrestrial shadow. This slight detail of the figure gives us a key insight. Whoever drew this diagram must have confronted the contradictory prerequisites of the proof and decided that the assumption of total eclipse was more important than the configuration we find depicted in the Greek manuscripts, or later Latin translations.

Thābit's new diagram served as the basis for the later Arabic transmission. The diagrams of the Ṭūsī edition all contain a single figure, similar in many ways to Thābit's and identically lettered. There are a few general differences between the two versions. The Ṭūsī MSS show both the solar radii, lines اس and ات, as is required by the way Ṭūsī develops the proof. In at least one case, a manuscript of Ṭūsī's text depicts the lunar radius, لط, parallel to the endpoint chord, حك, as it should be.[49] The Ṭūsī MSS do not, however, generally show the extremities of the end-point chord entirely inside the shadow, as in Figure 4. This means that the diagram does not represent the first moment of total eclipse. In *Tabriz* 3484, for example, the Moon is

[49] *Arch. Selden. A*. 45, f. 147r.

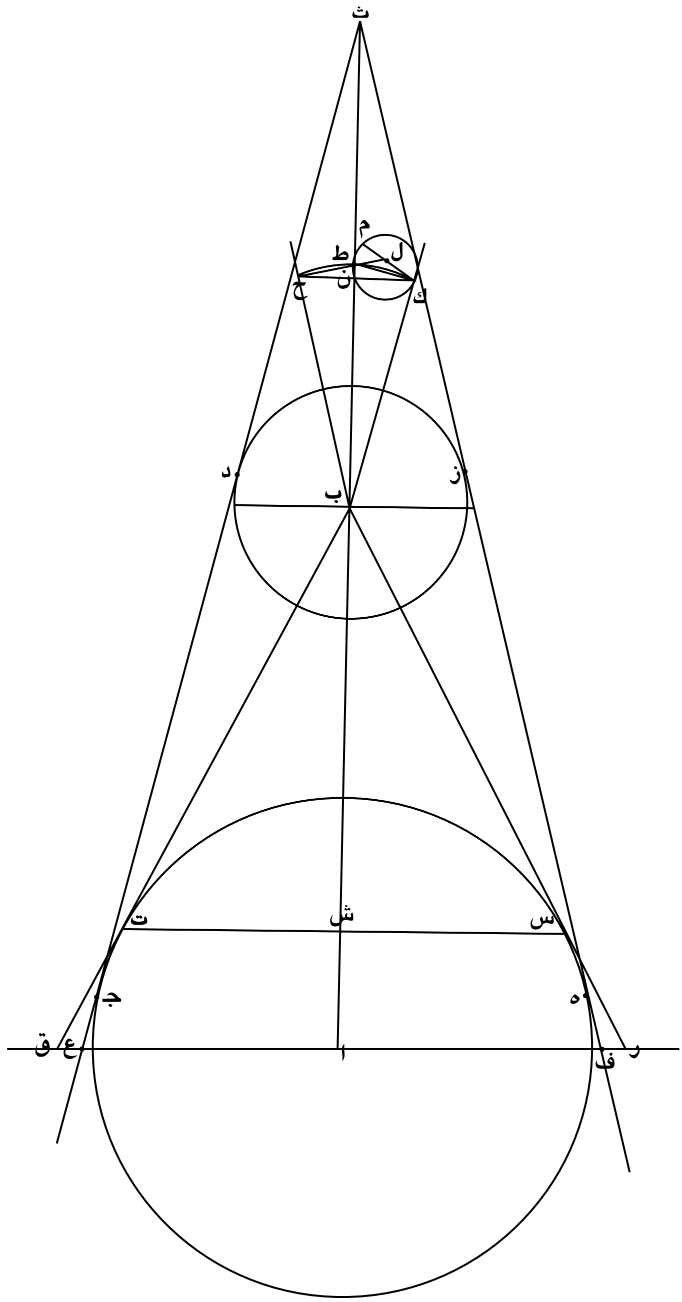

Figure 4. Diagram for *On Sizes* 13 in the Kraus MS of Thābit's revision [131r].

entirely inside the shadow and does not touch any of the lines that are supposed to be tangent to it.[50]

[50] Tabriz National Library MS 3484, p. 179 (note 32).

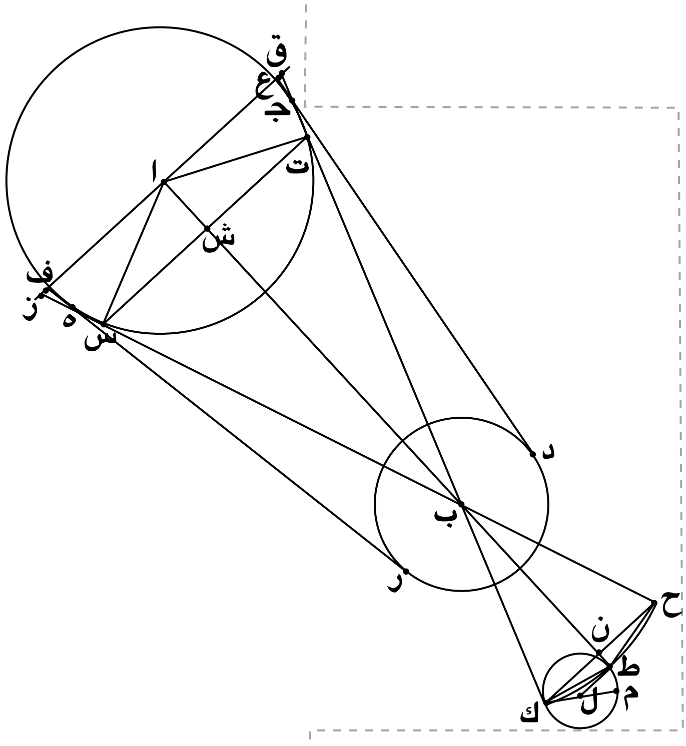

Figure 5. Diagram for *On Sizes* 13 in the Bodlean Library's *Arch. Seld. A.* 45 [147r]. The dashed line shows the edges of the written text.

In *Arch. Selden. A.* 45, the lines of the terrestrial shadow, perversely, do not actually extend beyond the Earth.[51] The copyist has attempted to squeeze the diagram into the box that was left for it, but the Sun protrudes out into the left-hand margin; see Figure 5. The typographical layout of the diagram explains why the terrestrial shadow does not proceed beyond the Moon, since the lines would have been drawn over the text. The fact that no part of the terrestrial shadow is depicted, however, is somewhat harder to understand, since this is the most important object under discussion. In fact, if the shadow had been drawn consistently with the other elements in the figure, both the endpoint chord and the Moon would protrude slightly beyond the shadow's edges. By leaving the terrestrial shadow out of the diagram, the copyist has deftly avoided confronting the fact that there is no way to draw a mathematically accurate diagram which represents all of the assumptions of the proof.

[51] This diagram is similar in a number of ways to that printed by Netz, *Shaping*, p. 41 (see note 44). In Netz's diagram, however, the Moon and the endpoint line are proportionally much larger than in *Arch Selden. A.* 45, and a different diameter has been drawn through the Moon. It is worth noting that the diagram Netz prints for *On Sizes* 14 is completely unlike that found in *Arch. Selden. A.* 45.

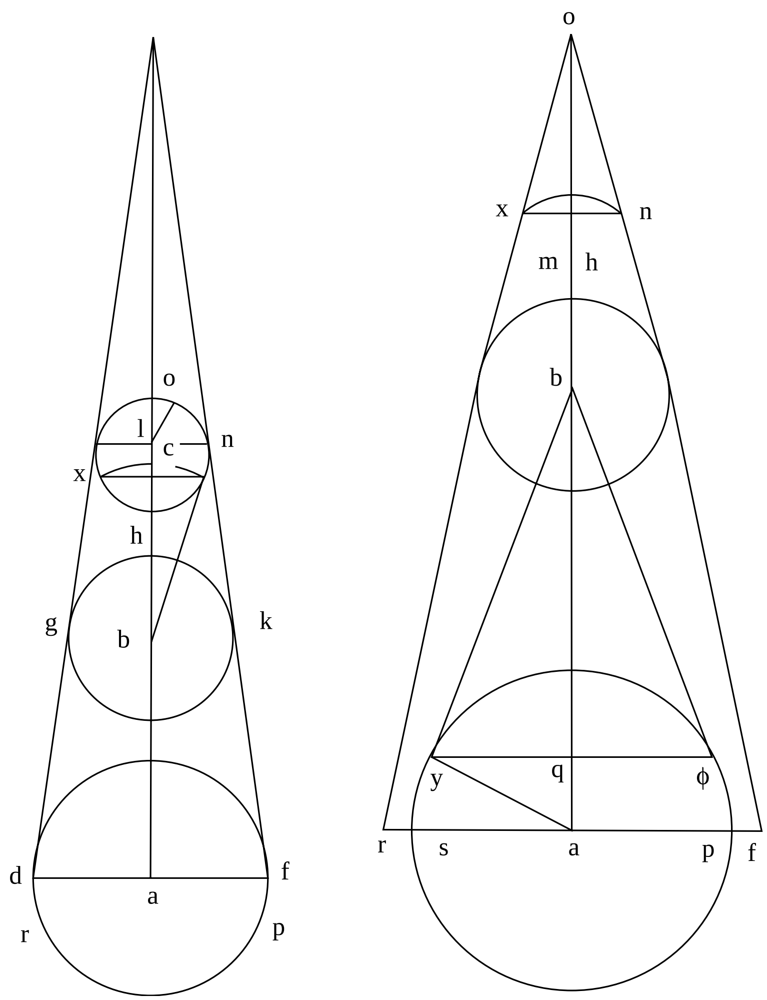

Figure 6. Diagram for *On Sizes* 13 in Valla's Latin edition, fs. fiii_v–fiiii_r.

The first printing of the treatise contains an unfortunate attempt to reproduce both of the Greek manuscript figures. Giorgio Valla used two diagrams, on separate pages, for his Latin translation, each of which contains serious errors; see Figure 6. It is clear that the mathematics of the proof could not be understood on the basis of these diagrams. In the first figure, the Moon fills the whole terrestrial shadow, whereas it should fill no more than half of it. The diameter of the Moon is drawn parallel to the dividing line, whereas an angle with the dividing line is required for the proof. A radius, *lo*, is drawn seemingly at random. There appears to be no endpoint chord, or else the endpoint chord is somehow the same as the dividing line. Labels are missing, so that it is not possible to name either the lunar diameter or the dividing line. In the second diagram, the cone of the terrestrial shadow does not touch the Sun, and its lines bend at the Earth. Moreover, the points *m* and *h* seem to float meaninglessly within the shadow. If one wishes to follow the text, it is simplest to redraw the figures. Indeed, one wonders if these diagrams were composed with Valla's input, or perhaps put together on the basis of a manuscript with no consideration for their mathematical purpose.

Toward the end of his life, Commandino saw to press a competent and beautifully printed translation of *On Sizes.*[52] For the purposes of modern scholarship, Commandino's translation of the text has had a lasting impact. With regard to the diagrams and the mathematics of the proof, Commandino's work has guided all later readings of the text. Published by Camillo Franceschini in 1572, this edition was particularly lavish in its treatment of the figures. The diagrams were all redrawn to adhere to the internal constraints of the mathematics, new ones were included to supplement Commandino's comments, and each was printed as many times as was necessary such that it always appeared on a pair of facing pages whose text referred to it. Because of the difficulty of the proposition and the number of Commandino's comments on it, the single diagram for *On Sizes* 13 was printed on six pages, filling the whole page in each case (Figure 7).[53]

Commandino combined the two diagrams for *On Sizes* 13 into a single figure, much in the same way as Thābit. This should be contrasted, however, with his choice to make two diagrams for *On Sizes* 14, whereas the Greek MSS contain only one. The choice to make two diagrams for *On Sizes* 14 is understandable, given the state of the Greek evidence. The diagrams in the Greek MSS for *On Sizes* 14 are strange and fail to represent a number of important features of the geometric configuration. Commandino provides an initial figure that depicts the whole situation, but which is difficult to read. He overcame this visual difficulty by including another diagram, which shows a crucial part of the figure in greater detail.[54] Hence, this second figure was required for similar reasons as was the second figure to *On Sizes* 13 in the Greek MSS. It was included to fulfil the visual requirement of displaying clearly all the information necessary to understanding the proof.

From a mathematical perspective, Commandino's diagram for *On Sizes* 13 is competently drawn. Unlike the diagram to Thābit's text, however, Commandino's figure, like those in the Greek MSS, depicts the Moon protruding from the terrestrial shadow. This is a reasonable choice, and having made it, Commandino depicts the

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[52] Commandino, *Aristarchi De magnitudinibus* (note 35).

[53] In fact, there are two versions of the figure, although the differences are slight. The most common version, say A, occurs on ff. 23v, 24v, 26v, 28r, and 29r. The other, B, occurs on ff. 22v and 25v. The differences are subtle, and figure 7 is actually a combination of both. In B, the lunar diameter, LO, is perfectly parallel to the endpoint chord, XN, whereas in A, it has a slight slope, upward in the direction of O. In B, the letter Y is found as in figure 7. In A, it is inside the circle, marking the same point. In B, the letter G appears to be represented by a capital theta. (In both A and B, the letter that is indicated by T in Commandino's text, and figure 7, appears to be represented by a capital upsilon.) The most obvious difference is in the ornamentation. In A, we find a pair of three dots on either side of the terrestrial shadow, as in figure 7. In B, these are replaced by slightly larger floral patterns, a different one on either side. The need for two plates for this figure resulted from the physical requirement of printing the text. The book was bound in quires of 8 pages. The printer would have printed all four pages on each side of the sheet in a single press. Since the diagram to *On Sizes* 13 occurs in the quires marked with signatures F, G, and H—and on one of every pair of pages in G—it was necessary to make two different plates for it.

[54] It is worth noting that this same problem is solved in the Kraus MS using a single diagram by cropping the objects to leave out unnecessary elements such that the relevant lines were drawn in greater detail, although one line is missing (f. 131v). The Ṭūsī MSS also contain a single detailed figure, solving the problem of scale in a number of different ways (see, for example, *Arch. Selden. A.* 45, 148r; *Tabriz* 3484, 181).

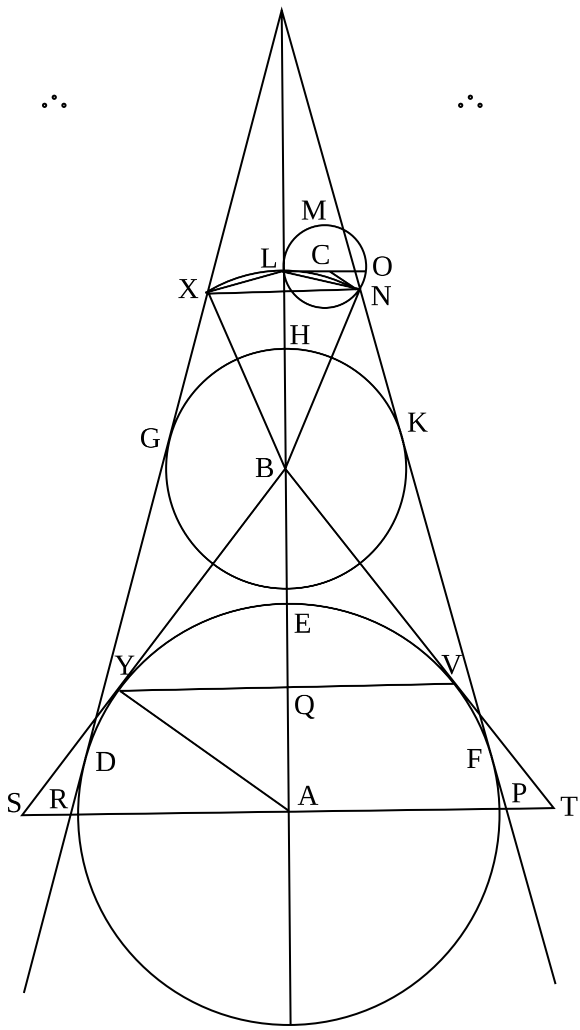

Figure 7. Diagram for *On Sizes* 13 in Commandino's Latin edition, f. 22v, 23v, 24v, 25v, 26v, 28r, and 29r.

other elements in the diagram as the mathematical assumptions of the theorem demand. The conflicting conditions of the proof are then obscured by the geometrical coherence of the diagram, so that only the telltale section of the Moon, peeking out from behind the terrestrial shadow, is left as a hint of the fact that medieval scholars had grappled with these difficulties and resolved them in different ways, with different degrees of success.

Commandino's figure, and hence Commandino's reading of the text, became standard in European scholarship. In Wallis's *editio princeps*, all of the same figures appear and much of the commentary is found in the critical apparatus.[55] Commandino's figures were likewise reproduced by Heath, in the modern critical edition that has formed the starting point of current scholarship on the text.[56] In this way, the historically more useful figures of the manuscript tradition have been overlooked in favour of the mathematically more useful figures made by Commandino.

8. What We Learn from the Diagrams

As Decorps-Foulquier has shown, the diagrams in the late ancient, and medieval, Greek manuscripts were often subject to more intervention than the text itself.[57] In the transmission into Arabic, however, and in the later Arabic edition, the text and the diagrams were both altered; indeed, the intent seems to have been conceptual, as opposed to textual, preservation. In the Italian Renaissance, scholars encountered difficulties in reading the Greek MSS, but whereas the attempt was to stay as close as possible to the Greek text, the diagrams were altered freely.[58] In seventeenth-century England, when Wallis applied the current standards of historical criticism to establishing the Greek text, he apparently did not consider that these same standards should apply to the manuscript diagrams.

He may have believed that, because the diagrams were subject to more variation than the text, there could be no possibility of using the MSS evidence to establish something like the 'true' diagram; that is, the diagram that had originally accompanied the text. In the case of a number of the diagrams in *On Sizes,* this is probably true. Nevertheless, he believed that the diagrams were key to a mathematically coherent understanding of the text. Faced with this dilemma, he chose to print figures that he thought were mathematically sound rather than historically attested. He chose, in a sense, to assume that Aristarchus had an understanding of the text that was mathematically equivalent to his own.

This discussion draws our attention to two fundamental facts: (1) the diagrams in the manuscripts contain information that plays an important role in the way the text is read and understood, and (2) the manuscript diagrams may have undergone significant changes in the transmission from antiquity.

These difficulties are strikingly underscored by the figure to *On Sizes* 13. The contradictions among the assumptions of this theorem mean that there can be no such thing as a single, mathematically coherent figure. As the figure in Thābit's revision shows, it is not possible to simply assume that because the rest of the text demonstrates mathematical competence, the diagram can be reconstructed along mathematical lines. Indeed, the diagram can either be drawn such that the Moon is tangent to the terrestrial shadow or such that the tangent from the centre of the Earth intersects the *endpoint chord* and the surface of the terrestrial shadow in a single point, but it cannot be drawn such that both of these conditions are fulfilled. Another possibility is simply not to draw the terrestrial shadow.

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[55] Wallis, *Aristarchi Samii de magnitudinibus* (note 36).
[56] Heath, *Aristarchus* (note 6).
[57] Decorps-Foulquier, 'Sur les figures', esp. p. 81 (note 3).
[58] Indeed, Commandio's translation was close enough to the Greek that Wallis reprinted it as the translation to his edition of the Greek text (notes 35 and 36).

In many cases when the text was copied, it may not have been realized that anything was at stake, but whenever a scholar grappled with the mathematical issues involved in this proposition, a choice had to be made about how to prioritize the assumptions made in the theorem. In the Arabic tradition, various different choices were made. In the Greek and Latin traditions, the universal choice appears to have been to allow the Moon to protrude from the terrestrial shadow. This seems to be the natural choice; it has never been questioned and has only recently been explicitly discussed as a choice. The fact that this choice appears to us to be obvious, however, is no criterion for deciding whether or not this choice was the one that Aristarchus made. Aristarchus drew the diagram that he actually drew, regardless of whether this was likely, natural or even consistent.

The fact that the original MS figures may have been lost, however, should not prompt us to abandon the MS diagrams that we do have. Faulty as they may be, they are still our only evidence.

In general, when assessing the medieval sources for our texts, the Arabic and Latin evidence should be taken into account.[59] In the case of *On Sizes,* however, it is possible to make a strong argument that the Arabic diagrams were redrawn in such a way as to vitiate them as sources for late ancient activity. For example, the fact that Thābit's revision has a single diagram for *On Sizes* 13 cannot be taken as evidence that his Greek sources had a single diagram. Even here, however, we should be cautious. All of the Greek MSS are copied from a very limited number of ninth- or tenth-century manuscripts—say, between one and three.[60] The vast majority of the Arabic MSS are copies of a thirteenth century edition. At least one, however, is evidence of another medieval Greek MS, which must have existed in Baghdad in the latter part of the ninth century BCE. Perhaps some of the differences between the Greek and early Arabic text, are based in differences between the ninth century Baghdad MS(S) and the ninth-century Constantinople MS(S). The number of Greek MSS that we know existed in the ninth century, and for which we have any evidence, is simply too small to allow us to say with any certainty.

The most obvious lesson is one that is becoming increasingly clear to scholars working in all areas of the ancient and medieval sciences: the diagrams constitute a vital part of our evidence and must be treated with the same degree of critical scrutiny that has long been afforded to the text. It has been shown that in some cases, the text scholia and the diagrams have a different transmission history than the text itself.[61] The diagrams should be presented as they are found in the MSS, accompanied by a critical apparatus. Where this is possible, we should seek to establish the text history of the diagrams and present this in a stemma, as is done for the text and the scholia.

Another key lesson is that we should use the diagrams, as found in the manuscripts, to help make a historically informed reading of the text. In the case of *On Sizes* 13 & 14, the diagrams contain important historical clues to key features of these theorems that long remained unnoticed. Indeed, one might argue that the

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[59] For examples, see W.R. Knorr, 'The Wrong Text of Euclid: On Heiberg's Text and its Alternatives', *Centaurus* 38 (1996), 208–76 and N. Sidoli, 'The Sector Theorem Attributed to Menelaus', *SCIAMVS* 7 (2006), 43–79.

[60] For the manuscript tradition see Noack, *Aristarch von Samos* (note 25). The closely related MS history of Autolycus' work, which was also included in the *Little Astronomy*, has been studied by J. Mogenet, *Autolycus de Pitane* (Louvain, 1950), 51–158.

[61] For the case of scholia, see Noack, *Aristarch von Samos* (note 25), while for diagrams, see Tak, 'Calcidius' Illustration' (note 3).

use of theoretical objects and implicit assumptions for the sake of computational results are some of the most interesting features of *On Sizes*.

From a historical perspective, the diagrams are indications of the way in which the text was read and used. It is significant that in both ninth-century Baghdad and sixteenth-century Italy, the diagrams were redrawn by mathematically competent scholars who were interested in making what they considered to be an improved and more intelligible version of the work available for study. We can use the new diagrams they drew for these texts to study their activity and to shed light on their views of the project of cultural preservation.

Finally, this study should serve as a cautionary tale. We have seen specific cases in which a diagram was considerably revised during periods of cultural appropriation. We must recognize that revisions of this sort may also have been carried out during periods for which we have little or no documentary evidence. We can name other possible junctures or individual editors, but we must generally admit that the figures of canonical texts may have been altered at various times in the centuries that separate us from their original composition. This realization should encourage us both to study the manuscript diagrams with care and to exercise prudence when claiming that any particular feature of these diagrams was intended by the author of the text they accompany.

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