Research on Ancient Greek Mathematical Sciences, 1998–2012

Nathan Sidoli

Introduction

This survey deals with research in ancient Greek mathematics and, to a lesser extent, mathematical sciences in the long first decade of the 21st century. It is modeled on the two foregoing surveys by J.L. Berggren and K. Saito and gives my personal appraisal of the most important work and trends of the last years, with no attempt to be exhaustive.

My overview of the work in this period is divided into two main sections: the "Methods," which researchers have applied, and the "Topics," which have garnered significant research interest. Many of the methods and topics that have been addressed in this period are not new, but the emphasis has often changed in response to a changing historiography. Of course, many of the individual works I discuss utilize more than one method and address more than one topic. Hence, the placement in my paper of any particular study is necessarily somewhat arbitrary.

I should mention some restrictions that must necessarily be made in such a survey. I deal only with mathematics, and some of the exact sciences insofar as they relate to mathematics, mathematical practice and mathematicians. This is a fairly arbitrary division. Ptolemy and Heron regarded themselves as mathematicians; astrologers of the Hellenistic periods were called mathematicians. A similar survey for astronomy, however, would be as long as that for mathematics, and those for mechanics and astrology would be nearly as long again. I only consider work in major European languages. In general, I restrict myself to the first decade of this century, give or take a few years on either side. I do, however, make a few exceptions to this to mention works published in the previous periods that were not discussed in either of the foregoing surveys by Berggren or Saito.

It may be useful if I explain my general views on the changing historiography of the period, which led me to organize this paper as I have. The majority of scholars working in the field now share certain methodological and historiographical commitments. There have been no polemics along disciplinary lines such as erupted in the 1970s (see pages 5–6 and 20, above). Of course, there have been disputes about historiographic methods, but these have largely focused around questions about the best way to rewrite the history of Greek mathematics, not about whether or not this needs to be done at all. The majority of scholars in the field now appear to accept the idea that Greek mathematics is *not* our mathematics, and that it needs to be understood in the context of its own historical setting. It appears that the old historiography has been overcome. There are very few who still believe in such historiographical artifacts as Pythagoras' deductive mathematics, geometric algebra, or a crisis precipitated by the discovery of incommensurability.

In general, the historiography of Greek mathematics has responded to changing trends in the historiography of science and of mathematics, but only slowly and tentatively. This is partly because the methodologies and approaches of these various fields are still necessarily different. Historians of Greek mathematics cannot directly employ many of the methodological approaches, such as studies of institutional practice or instrumentation, used by historians of later periods; and they are compelled to use methods, such as philology and reconstruction, which historians of later periods have no interest in, or find distasteful. Nevertheless, in the period under consideration, historians of Greek mathematics have followed earlier trends in the history of science, such as focusing on local topics for which there is good evidence, exploring the implications of material culture, and fleshing out the influence of cultural, religious and intellectual contexts on mathematical activity. Most importantly, historians of Greek mathematics, like other historians of science, are now no longer primarily interested chronicling a series of results, which are arbitrarily assumed to be important, but are rather focused on practices of knowledge production and discourse in the mathematical sciences and how these were related to such practices in other areas of ancient life.

With the passage of time, there have been changes in the group of researchers focusing on ancient Greek mathematical sciences. A number of senior scholars have moved on to other interests, or reduced their output; but the most significant change, in this regard, has been the untimely loss of the late W.R. Knorr, the most prolific and prominent historian of Greek mathematics in the second half of the twentieth century [Fowler 1998]. At the same time, a number of new scholars have entered the field, to the extent that it now appears to be more active than it had been for decades. Of the new scholars, I will mention only the two most active and influential, R. Netz and F. Acerbi.

A snapshot of changing trends in the field can be captured by considering three books that have been published during the period covered by this survey. The most influential book during this period has been R. Netz's *The Shaping of Deduction in Greek Mathematics* [1999a], which is perhaps the only book on Greek mathematics to have received a wide audience and recognition among non-specialists. This book has done much to open new approaches to deduction in Greek mathematics by focusing on cognitive, linguistic and diagrammatic practices. S. Cuomo's *Greek Mathematics* [2001] takes a broad scope of mathematical activity in the ancient Mediterranean world and gives as much attention to the historical evidence as to the topics treated. Although this book provides almost no coverage of the technical accomplishments of the Greek mathematical sciences, it situates mathematical practices in the broader social and political context and addresses the fact that mathematics was used by many professional practitioners in the course of their work. F. Acerbi's *Il silenzio delle sirene* [2010a] is a compact, technical work that will probably not attract much attention outside of the circle of specialists. But for them it will be essential. The book deals with historiographical issues concerning methods and topics, treats the techniques that Greek mathematicians used to approach their subject matter, and ends with a full appendix on the Greek manuscript traditions of the major sources. Although the arguments are often brief, this work shows how the old historiography has been left behind and argues for a reevaluation of many suggestions and assumptions of previous historians that have often been accepted as fact.

Methods

Many of the methods employed by scholars during this period have been used for decades, even centuries — such as manuscript studies and mathematical formalization — while others are techniques emphasized by historians of science working on more modern periods — such as studies of material culture, or contextualization in various social and religious milieus.

Textual Studies

Those who study the history of the ancient mathematical sciences face a conspicuous scarcity of evidence. For Greek work, although there is some papyrological evidence, there is almost no other material evidence and the descriptions of mathematical activity in literary and philosophical texts are often difficult to reconcile and interpret. Hence, for the history of Greek mathematics, our principal sources are the mathematical texts preserved in the medieval manuscript tradition.

For these reasons, the foundation of scholarship in the Greek mathematical sciences has been, is, and will remain, textual studies, in which all serious scholars of Greek mathematics engage to one degree or another. Textual studies, however, encompass a wide range of activity including codicological studies of the papyri and manuscripts themselves, producing critical editions on the basis of these, grammatical and philological analysis and description of the text, translation, and mathematical and scientific commentary.

Ancient Texts

Our most direct textual evidence for Greek mathematical activity comes from papyri and other written sources, of which there are very few. Arithmetical tables, and publications and studies of these, have been catalogued by Fowler [1988, 1995]. A couple of papyri dealing with "algebraic" type problems have been published and studied by Sesiano [1999]. Bülow-Jacosen and Taisbak [2003] have edited, and studied, a papyrus containing "practical" geometrical problems of the kind that we find in the Heronian corpus, and Jones [2012], relying on new readings of certain abbreviations, has reedited it, and produced a new technical study.

For the history of Greek astronomy, the largest new source of documents has been published and studied by Jones [1999], in his *Astronomical Papyri from Oxyrhynchus*. This work shows that the tabular methods of Babylonian mathematical astronomy were circulating in Greek by at least the early centuries of the common era and that these were in use at the same time as tables based on pre-Ptolemaic geometric modeling, and Ptolemy's *Handy Tables*.

There is still no comprehensive study of mathematics and mathematical astronomy in Greek papyri, but Friberg [2005, 193–267] has made a study of the mathematical methods contained in Greek papyri from Egypt, and shown that the technical methods used in these texts go back to Old Babylonian methods.

Medieval Manuscripts

The vast majority of work on Greek mathematics deals, not with the relatively few ancient texts that have survived in various forms, but with the much greater quantity of texts that are preserved in medieval manuscripts. There have been a number of new studies of the medieval manuscript traditions of Greek mathematical works. Along with older work by Mogenet [1950] and Noack [1992] on Autolycus and Aristarchus, respectively, a new study of the manuscripts of Theodosius' *Spherics*, by Czinczenheim [2000], gives us a good sense for the overall manuscript tradition of the so-called *Little Astronomy*. Decorps-Foulquier [1999a] has produced a thorough study of the manuscripts of the scholarship on Apollonius' *Conics* carried out by Eutocius.

There is no work that covers all of the manuscript evidence for Greek mathematics, but Acerbi [2010a, 269–375] gives a treatment of the Greek traditions of a number of major works. This will be invaluable for experts and can help others get a sense for the real basis of our current editions and appreciate how unreliable this sometimes is. This means that we now have overview coverage of the major manuscript sources in both Greek and Arabic.

The Direct and Indirect Traditions

Historians of Greek mathematics divide the manuscript sources into direct and indirect traditions. The direct tradition is constituted by source texts, in Greek, found in manuscripts, or papyri, while the indirect traditions are commentaries and summaries of these in Greek along with translations, and their commentaries and summaries, largely in Arabic and Latin. A useful overview of the medieval transmission of mathematical texts is provided by Lorch [2001].

Although it might seem that only manuscripts in the direct tradition should be treated by historians as primary sources, even in the direct tradition the mathematical texts were subjected to numerous revisions over the centuries. The texts were often edited by scholars who were themselves practitioners, or teachers, of the fields that the texts transmitted, and who took the scope of their role to include a correction of the words of text based on their own understanding of the ideas that these words conveyed. Hence, in order to understand the nature of our source documents, we are often in a position of having to reconstruct a lost context of mathematical activity on the basis of both the direct and indirect traditions.

The question of the relative importance of the direct and indirect traditions has been reopened by Knorr [1996, 2001], who revisited a late nineteenth century debate between the classical philologist J.L. Heiberg and the Arabist M. Klamroth. The debate concerned the importance of the indirect tradition for determining the original form of that most canonical of Greek mathematical works, Euclid's *Elements*. At the time, Heiberg had argued that the Arabic sources were not reliable and could be ignored, but Knorr looked at propositions from Book XII in Greek, Arabic and Latin versions and came to the conclusion that certain Arabic traditions were more valuable than the direct tradition. This question was taken up again by Rommevaux, Djebbar and Vitrac [2001], who enlarged the scope by examining Book X, and concluded that the Greek sources were considerably more varied than Heiberg acknowledged, but that the relationships between the various versions were too complex to simply assert that the Arabic witnesses were more valuable. Acerbi [2003a] has studied the importance of the Arabo-Latin tradition for studying the contexts of Book V. The manuscript tradition has also been studied by looking at the scholia in Greek and Arabic manuscripts [Vitrac 2003; Djebbar 2003]. This work has culminated in a detailed study by Vitrac [2012] of the issues involved, including an introduction to the entire textual history of the *Elements*, which will be of great use to both experts and non-experts alike. He concludes that the indirect tradition is also very diverse, that differences between the various traditions are rarely global, and that, in our present state of knowledge, it would more useful to produce editions of the various versions of the indirect tradition than a new edition from the Greek manuscripts.

Because of the fidelity of William of Moerbeke's Latin translations of Archimedes, and the loss of a number of the medieval Greek manuscripts, the indirect Latin tradition has long been an important resource for studying his work. Recently, d'Alessandro and Napolitani [2012] have edited the fourteenth century Latin translations of Archimedes' works by Jacob of San Cassiano. They conclude that Jacob had access to a Greek manuscript which is not otherwise known to us, and which sometimes contains better readings than those which are still extant. This reconfirms the importance of the Latin tradition for assessing Archimedes' mathematics.

For some topics and authors, the indirect tradition is essential. For example, Hogendijk [1999/2000] has found traces of a lost book by Menelaus on the foundations of geometry in works by al-Sijzī, and I have argued that an examination of various Arabic and Latin versions of Menelaus' *Spherics* indicates that the so-called Menelaus Theorem was not his discovery but was adopted by him from previous work [Sidoli 2006]. Details of scholarship on the indirect tradition in Arabic are discussed by Van Brummelen in his contribution to this book (see below, pages 102–107).

New Editions

There have been a number of new critical editions, in both the direct and indirect traditions. Some of these are based on a reassessment of the manuscript evidence, but many of them make a critical edition available for the first time. The bulk of these are editions of versions in the indirect traditions. Arabic editions are treated in detail by Van Brummelen (pages 102–107), and I will only mention a few of them here.

Czinczenheim [2000] has produced a new edition of Theodosius' *Spherics*, along with a French translation. Tihon [1999] has completed the project, begun by Mogenet, of editing Theon of Alexandria's larger commentary on Ptolemy's *Handy Tables*. Tihon [2011] and Mercier [2011] have also begun the project of editing Ptolemy's *Handy Tables*, along with an English translation and technical commentaries. Stückelberger and Grasshoff [2006] have produced a new edition and German translation of Ptolemy's *Geography*. This includes extensive commentaries and a CD-ROM containing all the location data.

In Arabic sources, Berggren and I have produced an edition of the Arabic text, along with English translation and commentary, of Ptolemy's *Planisphere*, previously only published in a later Latin version [Sidoli and Berggren 2007]. Rashed and Bellosta [2010] have produced the first critical edition of Apollonius' *Cutting off a Ratio*, previously only available in Halley's Latin translation.

In Latin sources, Jones [2001] has reedited, along with an English translation and commentary, the Latin of William of Moerbeke's translation of a Greek work on catoptrics, erroneously ascribed to Ptolemy and usually thought to be due to Heron. Takahashi [2001] has provided the first edition, along with English translation and commentary, of a medieval Latin translation of Euclid's *Catoptrics*.

A welcome, and important, new trend has been the publication of editions that include multiple versions of a text. Kunitzsch and Lorch [2010, 2011] have edited Arabic versions of Theodosius' *Spherics* and *Habitations* together with Latin translations based on these. Vitrac and Djebbar [2011] have produced a new Greek edition of the so-called Book XIV of the *Elements*, actually due to Hypsicles of Alexandria, along with the first edition of an Arabic translation of this text. Rashed, Decorps-Foulquier and Federspiel [2008–2010] have carried out the massive task of providing a full edition of the Greek and Arabic texts for Apollonius' *Conics*, along with a French translation and commentaries. This is the first time that the first four books of the Arabic text have been edited.

F. Acerbi and B. Vitrac have started a new series of editions, including translation and commentary, of Greek works, focusing especially on less well-known treatises. Acerbi [2012] has produced the first volume of the series, a new edition of Diophantus' *Polygonal Numbers*, with Italian translation and technical commentary. The anonymous introduction to Ptolemy's *Almagest*, the first three parts of which have already appeared, will be published in this series [Acerbi, Vinel and Vitrac 2010].

The Language of Greek Mathematics

Following previous work by Mugler [1959] and Aujac [1984], Federspiel has carried out a series of studies on the technical language of Greek mathematical works. Of these, I will mention a paper on the usage of the definite and indefinite article, which introduces the concept of the "anaphor" as a part of a proposition in which the mathematician explicitly draws inferences that result from the construction [Federspiel 1995], and a series of papers on the language of Apollonius' *Conics*, including one on the problems, which is relevant to the way we understand constructions in Greek geometry [Federspiel 1994, 1999–2000, 2002, 2008].

Chapters 4 and 5 of Netz's *Shaping of Deduction in Greek Mathematics*, on "Formulae" and "The shaping of necessity," deal with the linguistic structure of Greek mathematical prose [Netz 1999a, 127–239]. Netz shows how Greek mathematical prose is structured from the bottom up — from simple objects, such as points and lines, to clusters of assertions, including previous theorems or aspects of the diagram, into arguments. Acerbi [2011a], in his monograph study *La sintassi logica della matematica greca*, gives extensive coverage of the linguistic choices that Greek mathematicians used to express the logical development of their arguments.

There have been a number of other papers that treat the language of Greek mathematics, of which I will mention only two. Vitrac [2002] gives a careful analysis of the language of a locus theorem in Aristotle's *Meteorology* III.5, which he argues, against the majority opinion, may have been due to Aristotle himself. Acerbi [2011b] provides a careful study of the "givens" terminology of Greek mathematicians, arguing that they used the concept of "given" to deal with the determinate existence of mathematical objects and as a way to give a deductive framework to constructions and calculations.

New Translations

There have been many new translations of source texts during this period. Most of the new editions are accompanied with modern language translations, and a number of the translations are discussed in the next section, on canonical authors. Here, I will mention only a few works that do not fit into these other categories.

Solomon [2000] has made a new translation, with commentary, of Ptolemy's *Harmonics*, and Berggren and Jones [2000] have translated the theoretical chapters of Ptolemy's *Geography*, adding an introduction and detailed annotations. Bowen and Todd [2004] have produced an English translation of the short treatise by Cleomedes, the *Heavens*, which they argue was part of a general exposition of Stoicism. The translation is accompanied by critical notes and many diagrams. Evans and Berggren [2006] have translated Geminus' *Introduction to the Phenomena*, and included a thorough introduction, commentary and translations of a number of related texts. Winter [2007] has made a new translation of the Aristotelian *Mechanical Problems*, which he argues was written by Archytas, based on the rather dubious assumption of a linear progression in the history of early mechanics. Sefrin-Weis [2010] has made a new translation, with detailed commentary, of Pappus' *Collection* IV, which although providing the first English translation includes a number of peculiar translation choices. This book also includes an edition of the Greek text, which was made by comparing two previous critical editions with the only manuscript source. The diagrams, however, are not an accurate reflection of what we find in the manuscript, but have been reconstructed from the mathematical argument.

Textual Studies on Canonical Authors

For Greek mathematics, the canonical authors have always been the three great geometers of the Hellenistic period: Euclid, Archimedes and Apollonius. Since our understanding of ancient Greek mathematical sciences centers around these three figures, they have been the focus of considerable textual work.

Euclid

Euclid is probably the most canonical author in the history of mathematics, and most studies of Greek mathematics begin, and unfortunately often end, with an investigation of his texts.

There have been a number of new translations of his works. Vitrac [1990–2001] has finished his project of making a French translation of the *Elements* with commentaries, and Acerbi [2007] has translated the complete works of Euclid into Italian, along with a fair number of related texts and provided the whole with commentaries. Acerbi's translation has Greek on the facing pages. Taisbak [2003] has made a detailed study of the much neglected *Data*, including commentaries and an English translation facing a reprinting of the Greek text.

The work referred to above on the direct and indirect traditions of ancient Greek works has highlighted the importance of the medieval versions of Euclid's *Elements*, not just for understanding the medieval period in its own right, but also for our understanding of Euclid's own work, with the core text being the *Elements*. There has been considerable research on the medieval tradition of Euclid's works, especially by S. Brentjes, G. De Young, A. Djebbar, E. Kheirandish, and A. Lo Bello. The details of this work are discussed by Van Brummelen, below (see pages 102–106). This work constitutes the greatest recent addition to our knowledge of the textual sources for Euclid's work, but it should be stressed that although we have a Greek edition and a number of Latin editions of his *Elements*, there is still no complete edition of any of the Arabic versions of this essential text.

Archimedes and the Archimedes Palimpsest

Arguably the greatest mathematician of the Hellenistic period, Archimedes had a different approach from Euclid. In place of systematic, foundational treatises, he focused on the measuration of curvilinear figures, and the mathematization of mechanics. Netz has begun a project of producing the first English translation of his works, with linguistic and literary commentaries, of which the first volume has appeared [Netz 2004a].

The most important development in textual studies of the Archimedes corpus has been the rediscovery, and sale at auction, of the so-called Archimedes Palimpsest. This manuscript, which contains some unique treatises by Archimedes, was read by Heiberg in the first decade of the twentieth century and then subsequently lost. Wilson [1999] gives a description of the codex, and the fascinating story of this object is told by Netz and Wilson [2009]. Netz, Noel, Wilson and Tchernetska [2011] have now produced a transcription of the text with facing pages of various images of the palimpsest,[1] accompanied with papers on the history, conservation and imaging of the manuscript and The Archimedes Palimpsest Project.

Since Heiberg was able to read the majority of the Archimedes text, including some parts that have since been lost, this work on the palimpsest has not revolutionized our knowledge of Archimedes, as some researchers had initially anticipated. Nevertheless, this textual work has lead to some new readings, such as a new interpretations of *Method* 14 [Netz, Saito and Tchernetska 2001–2002], and a new reconstruction of the final propositions, based on considerations of the physical conditions of the original Archimedes codex [Saito and Napolitani, below]. It seems certain that further study of this important source will continue to yield insight into Archimedes' mathematics.

Apollonius

The textual work on Apollonius during this period has been extensive. As well as Decorps-Foulquier's work on the Greek Apollonius manuscript tradition, and Federspiel's work on prose usage in the Greek text, discussed above, they have collaborated with R. Rashed in producing a new edition of the Greek and Arabic sources of Apollonius *Conics*, along with a French translation [Rashed, Decorps-Foulquier and Federspiel 2008–2010]. Taking this work together with the new edition of the Arabic, and oldest surviving, edition of Apollonius' *Cutting off a Ratio* [Rashed and Bellosta 2010], we now have a solid textual basis on which to form an assessment of Apollonius' contribution.

At the end of their lengthy study of Apollonius' *Conics*, Fried and Unguru [2001] have appended the first English translation, by Fried, of *Conics* IV. It has also appeared in a separate printing, with an introduction discussing the contents of the book [Fried 2002].

Diagrams

Another major area of research interest related to textual studies has been on the diagrams preserved in the manuscript sources, treated as both material object and text. The recent interest in this area was largely motivated by Chapter 1 of Netz's *Shaping of Deduction in Greek Mathematics*, "The lettered diagram" [Netz 1999a, 12-67]. Saito [2009] has given an overall discussion of the importance of textual studies for understanding Greek mathematics, which situates diagram studies within the scope of textual studies.

With the exception of the Belgian school, earlier editors, particularly Heiberg, paid little attention to manuscript diagrams, so that the critical editions of the canonical works do not contain the diagrams that were transmitted in the ancient or medieval periods. This situation is now beginning to be rectified. Netz's translation of Archimedes' *Sphere and Cylinder* includes a critical edition of the manuscript figures

[1] The palimpsest has been imaged using different spectra of light, including ultraviolet and x-ray fluorescence, to try to bring out the erased text.

[Netz 2004a]. K. Saito has undertaken a major project of examining and editing the manuscript images, focusing first on the work of Euclid. The diagrams of *Elements* I–IV and VI have been published, along with those of a number of Euclid's other works, such as the *Phenomena* and *Optics* [Saito 2006b, 2008].[2] The diagrams in the Euclidean tradition in Arabic have been investigated by De Young [2005].

The overall characteristics of the manuscript diagrams have been described by Saito [2012], with respect to diagrams in Euclid, and by Saito and myself, with respect to diagrams in Euclid, Archimedes, Apollonius, Theodosius, and others [Saito and Sidoli 2012]. The main results of this work shows that diagrams in ancient Greek works functioned as a kind of schematic, characterized by oversimplification and indifference to visual accuracy. In many cases, especially in solid geometry, the diagrams were meant to be redrawn on the basis of the text and the manuscript diagrams. This implies that it is the construction, rather than the diagram, that acts as a new source of necessity in the argument.

There have already been a number of results from this interest in diagrams. Decorps-Foulquier [1999b] has used the diagrams in Apollonius' *Conics* to explore the editorial work that Eutocius performed in preparing the Greek edition of the *Conics* that has come down to us. I have used the variant diagrams of a proposition from Aristarchus' *On the Sizes and Distances of the Sun and Moon* to show that the text was read in different ways by different readers in the medieval period, and to point out some technical issues that had previously gone unnoticed [Sidoli 2007]. Malpangotto [2010] has used the diagrams in ancient, medieval and early modern version of Theodosius' *Spherics* to show that different authors and editors used diagrams as a way of organizing their approach to spherical geometry and their conceptualization of the objects on the sphere.

During this period, there have also been a number of philosophical and logical studies on diagrams and diagrammatic reasoning. These are discussed below.

Material Culture

The material culture of science became a major area of interest and research for historians of science in the 1970s and 80s. Probably due to a scarcity of evidence, and the difficulties involved in interpreting the little evidence that there is, it took a long time for this trend to influence historians of the ancient exact sciences. During the period under consideration, however, a number of scholars have attempted to bridge this divide through various techniques.

In the first place, there have been a number of new discussions of the material evidence we do have. In her general overview, *Greek Mathematics*, Cuomo [2001] discusses some of the material evidence for the use of mathematics by various types of professionals, such as builders and acountants. Lewis [2001] has provided a textual, archeological and reconstructive study of the surveying instruments that were used by engineers and builders. This study includes translations of a number of related texts. Evans [1999] gives an important discussion of the material evidence for astronomy in the Greco-Roman world. In this regard, Jones has studied the meridian lines on a group of three sundials used to indicate the time of year (see below). Evans [2004] also discusses the material objects that would have made up the practice of professional astrologers, who were often called mathematicians in the ancient period.

There have been a number of treatments of astronomical works that were presented in the form of monuments, or public installations. Jones [2005b] gives a new edition, with translation and commentaries, of Ptolemy's *Canobic Inscription*. The original installation is not known, but a transcription of it is preserved in the manuscript tradition. Jones [2006] has also made a study of the Keskintos Astronomical Inscription, based on the actual artifact. This includes a text, translation and a reconstruction of an epicycle model that could produce the values found on the inscription. Lehoux [2007a] has made a complete study of the Greco-Roman material, both artifacts and text, that relate to parapegmata — monuments that were constructed in public spaces to keep track of a local position in some cyclical, usually astronomical, phenomena.

[2] See www.greekmath.org/diagrams/diagrams_index.html for the latest results of this project.

An important area of prolific activity has been research on the so-called Antikythera Mechanism — that is, the remains of a system of gears used to produce analog calculations of astronomical positions. There has been so much work done in this area that I cannot hope to cover it all in this context. I will mention only the main projects and individuals.

Since the mid 90s, M.T. Wright, and his collaborators, have been working on the Antikythera Mechanism, using new x-ray imaging and physical reconstruction. Wright [2007] has summarized the fruits of this work, and also included a bibliography of over ten of his papers that have resulted from this project. In the early years of this century, a new research project, The Antikythera Mechanism Research Project, has brought together researchers from a variety of backgrounds to study the device. This project has produced computed tomographic data of the internal structure of the device as well as new surface images. The project has led to a number of new results, of which I will mention only a few. Freeth, et al. [2006] set out the preliminary results of the project and proposed a new structure for the internal gearing. Freeth, Jones, Steele, and Bitsakis [2008] describe much more detail of the chronological and eclipse prediction dials on the surface of the object, and Freeth and Jones [2012] give an overview of their current view of the workings of the device and its relationship to ancient astronomy and cosmology. J. Evans, C.C. Carman and A.S. Thorndike have also been working on the device using the new images from the Antikythera Mechanism Project, comparison with ancient geometric models, and physical reconstruction. They have made a number of interesting findings, and offer a somewhat different interpretation of the gearing [Evans, Carman and Thorndike 2010; Carman, Thorndike and Evans 2012]. In their contribution to this volume, Evans and Carman argue that the Antikythera Mechanism can be used to help us reconsider the material culture of mathematical astronomy in the ancient world [Evans and Carman, below].

There have also been a number of studies which attempt to get a picture of the material conditions of Greek mathematicians by seeing what we can learn about this from their written works. For example, Tybjerg [2004] has argued that Heron makes mechanical devices tools of demonstration, and applies geometry to the mechanical objects. K. Saito and I have argued that the presentation of Theodosius' *Spherics* indicates that mathematicians were drawing diagrams on real globes in the course of studying spherical geometry [Sidoli and Saito 2009]. In harmonics, Creese [2010] has given a study of the texts related to the monochord as a scientific instrument, and Barker [2000, 192–229] has analyzed Ptolemy's treatment of scientific instruments in his *Harmonics*. This work makes it clear that Heron and Ptolemy were working in a similar tradition of using mechanical devices in their work, and applying mathematics to the study of instruments.

Social and Institutional Context

Another research methodology which has slowly affected the history of the ancient exact sciences from history of science has been research into the social and intellectual contexts in which ancient practitioners worked. Again, the primary reason why this trend has so weakly effected research on the ancient period is the almost total lack of evidence. For even the most famous of the Greek mathematicians, we often know with certainty nothing about the actual circumstances of their lives.

Nevertheless, there have been some studies that try to bridge this evidential gap in roundabout ways. Netz [2002a] has used loose demographic techniques to argue that there were very few mathematicians throughout the whole period of Greco-Roman antiquity, and a broad survey of the textual evidence to assert that those whose lives we know anything about came from the upper classes and did not generally earn a living through their mathematical activities. Asper [2003] has argued that theoretical mathematics arose out of a background of the practical mathematics practiced by professionals, and that mathematicians developed an interest in impersonalization, proof, and diagrams as way to distinguish themselves from other elite intellectuals, such as philosophers and sophists. Asper [2009] has also argued that theoretical mathematics, at least in the early period, was characterized by having no institutional setting, whereas the practical mathematics of professionals was loosely institutionalized in the educational settings of oral instruction where professionals learned their trades. Cuomo [2000, 9–56] has provided a discussion of the textual evidence we have for the social role of professionals who used mathematics, such builders, accountants and astrologers, and for the general, albeit vague, praise that intellectuals of all types had for a mathematical education.

If we take a strict sense of context, we still know little of the life circumstances of most mathematicians — how they made their living, where they learned mathematics, if they had any students, or how many people they might expect to read and understand their work. In fact, however, if we take a slightly broader concept of context, there has been a fair bit of work done to contextualize Greek mathematicians. In later sections, I will discuss work that broadens our understanding of the philosophical and intellectual contexts in which mathematicians worked, and the context of mathematical practice in which they produced their results.

Mathematical and Logical Studies

Historians of Greek mathematics now put less focus on producing the kinds of formal descriptions — in the logical or mathematical sense — of Greek mathematical works that were common in the past. Nevertheless, philosophers, logicians and mathematicians continue to have an interest in Greek mathematics, particularly Euclid's *Elements*. Moreover, there have been a number of developments along these lines which are important for our understanding of Greek geometry. In particular, it is now widely recognized that the predicate logic at the basis of Hilbert's program of critically reevaluating the foundations of geometry is insufficient to grapple with constructive practices and diagrammatic inferences. Hence, the constructive nature of Greek geometry was never properly understood under this program. Recently, a number of researchers have been working from various perspectives to try to develop the logical apparatus to handle the constructive, diagrammatic aspects of Greek geometry.

The authors of the following works are largely concerned with aspects of their projects that will be of interest to working mathematicians and logicians. In the following summaries, however, I only highlight aspects of their work to which I think historians of Greek mathematics should pay heed.

Using Martin-Löf's intuitionistic type theory, Mäenpää and von Plato [1990] give a formal treatment of Euclid's construction postulates (Posts. 1–3) and provide deduction rules that show how theorems and problems can be described in a single logical structure. Mäenpää [1997] gives a type-theoretic reading of Greek mathematical analysis which shows that predicate logic does not give a natural treatment of the fact that geometrical analysis treats a configuration of objects, and cannot explain the fundamental role of auxiliary constructions. He also discusses the overall logical form of an analysis-synthesis pair, which gives a new reading of the sense in which analysis is the reverse of synthesis.

There has been some work put into formalizing Euclidean geometry. Avigad, Dean and Mumma [2009] have developed a formal system that gives a model of Euclid's geometric practice. This includes attention to diagrammatic reasoning, which is provided with its own system of axioms — such as axioms of betweenness or intersection. In this model, constructions are performed by rules that guarantee the existence of certain objects, given certain conditions. This is not, however, fully constructive in the sense that construction rules are "built-in" theorems which assert that given some conditions certain objects exist that have various properties. Beeson [2010], however, provides a formal model of Euclidean constructive geometry as a intuitionistic geometry, and shows that if the geometry proves an existence theorem, the object can be constructed using the constructive methods provided by the theory. He argues that Euclid's constructions are meant to be well-formed algorithms in the strict sense. As well as developing formal models, specific questions can also be addressed using such methods. For example, Alvarez [2003] has given a logical analysis, in terms of construction, of the proof structure of those propositions in the *Elements* that rely on superposition, I.4, I.8 and III.24.

Pambuccian [2008] gives an overall treatment — historical, conceptual and technical — of work devoted to axiomatizing geometric constructions throughout the twentieth century. He is only briefly concerned with Greek geometry, but points out that constructive axiomatics preserves the distinction attributed to Geminus by Proclus between postulates, which ask for the production of something not yet given, and axioms, which refer to relationships that obtain between givens. He defines constructive approaches as quantifier-free, and points out that the intuitionistic approach mentioned above fails to meet this criterion.

Logical and Philosophical Studies of Diagram-Based Reasoning

Closely related to this material are a number of philosophical and logical studies that deal with diagrammatic practice, and the use of diagrams to draw inferences in Euclidean geometry.

K. Manders has published a work on Euclidean diagrammatic practice that has circulating widely among philosophers of mathematics in preprint form since 1995, introduced by a shorter overview of his ideas and how they have changed [Manders 2008a,b]. In these papers, he asserts that diagrams introduce concepts and starting points into the Euclidean argument and claims that Euclidean geometry is a practice of making inferences on diagrams and statements, but that it does not depend on geometric objects. Miller [2007] has created a formal system for diagrammatic inference and proof. He argues that the idea that diagrammatic methods are inherently informal is incorrect, and that the Euclidean practice can be understood as an informal implementation of a potentially rigorous approach. Panza [2012] provides a new study of the diagrammatic practices of Euclid's plane geometry, incorporating the new approaches that have been developed by logicians and philosophers, but arguing that Euclidean geometry is fundamentally about geometric objects and that the diagrams represent these. He argues that diagrams allow Euclid to deal with issues of identity, construction, existence and being "given," and that the geometric objects under study in the text inherent some properties from their diagrams.

Unfortunately, all this work focuses strictly on Euclid's *Elements*, and usually only the first book. Hence, a number of the conclusions reached are incompatible with the broader context of constructive and diagrammatic practices in which the *Elements* should be situated.

Balanced Reconstruction

For good reasons, elaborate reconstructions of ancient mathematical results and theories have fallen out of favor. In particular, historians of ancient mathematics no longer try to recreate the historical development of periods for which the texts have been completely lost, or never existed. The primary reason for this is that our reconstructions must always be plausible and linear whereas we know that the history of mathematics and science, for periods in which we have ample evidence, does not always proceed in such an idealized way. Moreover, the number of plausible, reconstructed histories is often equal to the number of scholars working along such lines.

Nevertheless, since so many of the Greek texts have been lost, historians still engage in a more balanced form of reconstruction, in which they try to show that certain results were possible, that certain methods could have been used, or how a certain result may have been shown, relying strictly on methods that are consistent with the available textual evidence. The scope for such reconstructions is more limited than the reconstruction of historical developments. For example, Masià-Fornos [2010] has shown that a putative lacuna in Archimedes' *Sphere and Cylinder* can be supplied rather naturally using concepts and methods readily available to Archimedes. In a similar vein, I have shown that mathematical methods evidenced in Ptolemy can be used to reconstruct the full argument for an obscure chapter in Heron's *Dioptra* [Sidoli 2005]. Acerbi [2011c] has given a reconstruction of the proof for the final, incomplete proposition of Diophantus' *Polygonal Numbers*. He has also given a proof in five theorems, using ancient methods, of the ancient claim that there are only thee homeomeric lines [Acerbi 2010b]. Such an argument does not prove that this proof was actually made in antiquity, but does show that a proof of its kind could have been produced, so that there is no reason to doubt the ancient claim that there was a proof of this fact.

New Readings

As the study of Greek mathematical sciences has come more strongly under the influence of the history of science, it has also come under the general tendency of scholars in the humanities to produce new readings of canonical texts. Of course, in some sense all historical work on this period involves the production of new readings of familiar texts; nevertheless a number of influential tendencies can be discerned.

Ancient Mathematical Contexts

In a sense, the most pervasive tendency of the new historiography has been the continuing practice of reading Greek mathematical texts in the context of ancient mathematical methods: that is, using the often terse technical sources to try to understand how Greek mathematicians actually thought about, and did, mathematics. Hence, many studies in the period under consideration could be placed in this section, but I present here only a selection of significant examples.

Probably the most complete project in this vein is a monograph study of Apollonius' *Conics* by Fried and Unguru [2001]. This book presents a new reading of the complete extant text and argues that Apollonius' mathematics should be understood and explained in the geometrical form in which it is presented. It also takes a polemical attitude towards the old historiographical methods and presents a case study for the success of the new historiography advocated by Unguru in the 1970s. Fried [2002] has studied the role of analogy in Apollonius' introduction and the use, in *Conics* VII, of the so-called *homologue*, a line which is defined by cutting the transverse axis in the same ratio that the transverse axis has to the latus rectum. He argues that proportion, as *analogia*, was an organizing principle in Apollonius' theory.

There have been new readings of the theory of ratio developed in *Elements* V. Acerbi [2003a] gives a new reading of both the Greek and Arabo-Latin traditions of *Elements* V, in which he argues that by the time the theory was composed, the concept of generality and the linguistic tools used to express it were fully developed. Looking at the same material, but reading it in a different context, Saito [2003] argues that the pre-Eudoxean theory of proportion reconstructed by Becker is unnecessary to explain the usage of proportion theory in the ancient mathematical context. The proportion theory put forward in *Elements* V can be understood better as a ratio theory that is useful for doing elementary geometry than as a complete, and general, treatment of ratio. Hence, any pre-Eudoxean theory should also be understood as having been some loose set of theorems and techniques for ratios that are useful in geometry.

There are also a number of specific results, or new readings of texts that have been produced in this way. By showing that Heron's *Dioptra* 35 can be read in the context of mathematical methods found in Ptolemy's works, I have argued that ancient gnomonics can be applied to the solution of problems in spherical astronomy [Sidoli 2005]. Saito [2006a] examines the treatment of quadrature in a range of Archimedean works and shows that the texts can be read more naturally when we realize that Archimedes did not work with a general concept of quantity but rather investigated well-defined figures and compared their various attributes with one another.

This approach has also been used to help us understand the goal and approach of lost works. For example, Acerbi [2008] provides a background for understanding the goal of Euclid's *Fallacies* by discussing the role of false proof in a wide range of philosophical and mathematical sources, and identifying a number of false proofs as possibly having an origin in this lost text. While we still do not know about the detailed contexts of this work, we can now develop a better sense of the questions it may have addressed and the approaches it may have followed.

Intellectual and Philosophical Contexts

Another type of scholarship which is not new, but which has certainly be influenced by tendencies in history of science and other areas of humanities scholarship, has been the practice of reading ancient mathematical authors in the contexts of other philosophical and intellectual fields with which they were in contact and with which they competed.

Mansfeld [1998] has examined the introductions to mathematical texts from Apollonius' *Conics* to Theon's commentaries. This examination of the preliminary material, in contrast to the austere and often strictly technical core texts, shows that Greek mathematicians situated their work in the intellectual and philosophical currents of their time. Cuomo [2000], in her book on Pappus, spends considerable effort situating his work in the general intellectual context of his time.

Bernard [2003b] gives a programmatic argument that categories of ancient rhetoric can be used to analyze mathematical texts. He also provides a detailed example of this program, arguing that an obscure passage in Pappus's *Collection* can be understood in terms of the techniques developed by orators and rhetoricians in late antiquity [Bernard 2003a].

Feke and Jones [2010] give a general overview of Ptolemy's philosophical approach, which shows both that he conceived of his project in the terms of the philosophy of his time and that his works had an internal philosophical consistency. Bernard [2010] makes a reading of the ethical and practical aspects of Ptolemy's mathematical project and argues that many aspects of its presentation may have been addressed towards the discourse of professional astrologers. Feke [2012] shows that Ptolemy situated his work in the philosophical currents of his time, while maintaining that the best way to pursue true philosophy is through mathematical investigations of the natural world.

Literary Readings

A relatively new trend has been a handful of studies that introduce non-mathematical, almost literary readings of Greek mathematical texts.

Netz [1999a], in passages of his *Shaping of Deduction in Greek Mathematics*, argues that there is a similarity between the use of formulaic language in Greek mathematical texts and in Homeric epic poetry. In his *Ludic Proof* [2009], he argues that the Hellenistic geometers can be read as using literary techniques that are comparable to what we find in the literature of this period — such as compositional variation between themes and methods, an element of surprise in the narrative structure, and experimentation with various genres. Wagner [2009] has given a Deleuzian reading of his own speculative reconstruction of Greek mathematical practice, and argues that Deleuze's analysis of Francis Bacon's approach to painting may be of use in understanding diagrammatic practice in Greek mathematics.

Unguru and Fried [2007] have gently ribbed the idea of highly postmodern readings by producing a fictional, purely literary, reading of Apollonius's *Conics* as a work on sexual politics and comparing this with a H.G. Zeuthen's highly mathematical and their own more historical readings.

Topics

In this section, I discuss a number of topics that have been addressed by historians over the long first decade of this century. Some of these topics have been perennial favorites — such as the origins of Greek mathematics, or geometrical analysis — while others are fairly new, such as late ancient mathematicians and commentators, or combinatorics.

Origins and Early Evidence

The origins of Greek mathematics, although poorly documented and highly speculative, was once a favorite topic for historians of the subject. Despite the fact that most historians have grown wary of such origin stories, it still receives some attention.

The relationship between Greek mathematics and Mesopotamian, that is Old Babylonian, mathematics can now be reappraised in the light of the many new studies of the latter that have been made since the early 1990s. Friberg [2007], in a study of various similarities between the two, has tried to revise the old view of a strong and direct influence of Babylonian mathematics on the development of theoretical Greek mathematics. It should be noted, however, that these similarities are only found in the subjects studied — which are relatively obvious objects of elementary mathematical investigation — not in the concepts or methods. Robson [2005], on the other hand, expresses the opposite opinion, namely, that given the rather stark differences of conception and approach, and the fact that Old Babylonian mathematics had already died out over 1,000 years before Greek mathematics began to be practiced more than 1,000 miles away, it is unlikely that the one greatly influenced the other. Given the current state of evidence, it seems unlikely that consensus can be reached on this issue. Although there certainly seems to be some continuity from Old Babylonian and Selucid mathematics to the "practical" material found in Greek papyri and the Heronian corpus, as shown by Friberg [2005, 193–267], the more advanced material of the Old Babylonian period shows such divergence from the approaches and methods of the Greek geometers that it is possible to argue that the later is independent of the former.

Although most historians of Greek mathematics are now highly doubtful that Pythagoras and the early Pythagoreans played a significant or unique role in the origins of Greek mathematical science, Zhmud [2006] has presented an updated version of his views on this in the new English translation of his book on the early Pythagoreans. In particular, he has strengthened his arguments by references to his work on Eudemus' histories of the mathematical sciences [Zhmud 2012]. It should be pointed out, however, that for the earliest history he must still rely on reconstructions, which are based on the assumption that the mathematical sciences must develop in a linear, cumulative way. Also on the subject of Pythagorean science, Borzacchini [2007] takes what he calls a cognitive approach to reconstructing an origin for the idea of incommensurability in Pythagorean music theory. Most of his concrete evidence, however, is drawn from the work of Archytas, which is rather late for an origins story. Mueller [2003] reconstructs a more intuitive and less rigorous proof of *Elements* I.32, that the sum of the angles of a triangle are two right angles, and argues that this could have been written as early as the fifth century, possibly by Pythagoreans. The majority opinion, however, now seems to be that the early Pythagoreans did not play a unique role in the development of the mathematical sciences. For example, Asper [2003] argues that theoretical mathematics, which arose first among the sixth century Ionians, crystalized in the Athenian milieu in the attempts of mathematicians to differentiate themselves from other elite intellectuals, such as philosophers and sophists. Netz [2004b], however, associates the origins of theoretical mathematical work with the production of mathematical texts, and hence studies the earliest source we have of for written proofs, Simplicius' account of Hippocrates' quadrature of lunes, which was taken from Eudemus' history.

There have also been a number of studies of the mathematics of Plato's time. Zhmud [1998] has made a skeptical study of the common idea that Plato was an "architect of science" by showing that this idea originated in the early Academy, which suggests that Plato was more influenced by mathematicians than they by him. Huffman [2005] has produced a monograph study of Archytas, a late Pythagorean contemporary of Plato's, who everyone agrees did important work in theoretical arithmetic and harmonics. Acerbi [2000, 2005] has done some work on the often cryptic references to mathematics in the writings of Plato himself.

Late Ancient Mathematicians and Commentators

Most of what we think we know about the origins of Greek mathematics comes from texts written many centuries later, in the late ancient period. This material used to be primarily of interest to historians of Greek mathematics because of its usefulness for studying earlier periods. A welcome change in this recent period of historical scholarship is that historians have developed an interest in the mathematics and scholarship of late antiquity for its own sake, and not merely for the historical material it contains.

Cuomo [2000] has produced a monograph study of Pappus, the most creative mathematician of late antiquity, and shown that his treatment of the mathematics of his predecessors was both an essential part of his mathematical project and explicable in the context of other late-ancient cultural and intellectual practices. Bernard [2003a] has explored the rhetorical aspects of Pappus' work, and linked it with the rhetoric of the period. He has also shown that this can be used as an example to argue for the general utility of sophistical and rhetorical categories as a context for understanding mathematical work [Bernard 2003b].

There has also been new work on Eutocius' commentaries on Archimedes and Apollonius. Netz [1999c] has argued that Eutocius' treatment of a problem due to Archimedes actually transforms the mathematical concepts in important ways, and he later situated this study between a study of Archimedes' approach and that taken by the medieval mathematician ʿUmar Khayyām [Netz 2004c]. Decorps-Foulquier [1998, 1999a,b] has made a number of studies of Eutocius as an editor of, and commentator on, Apollonius' *Conics*.

Netz [1999b] has studied the famous divisions of a proposition set out by Proclus in his commentary on Euclid's *Elements* I. He shows that this division was probably a description produced by Proclus that affected the later transmission of the Euclidean text. Hence, it should not be used as a universal model for all Greek mathematical texts.

Netz [1998] has written a general description of the commentary tradition and its effect on our image of Greek mathematics. Chemla [1999] has responded to this by pointing out the difficulties often involved in distinguishing between source text and commentary, and Bernard [2003c] has responded by highlighting the didactic aspects of this tradition.

Numeracy

Although there has been much study of literacy in the Greco-Roman world, our understanding of the state of numeracy in this period is still very slight. Netz [2002b] introduced a program of studying Greek numeracy, defined as counting with physical counters. More recently, Cuomo [2012] has announced a project to study Greco-Roman numeracy more broadly by focusing on textual and archeological evidence. This work in still in a preliminary stage.

Greek Combinatorics

One of the most interesting developments in the period covered by this paper has been some work that demonstrates that combinatorics, in a fairly developed sense, was practiced by Greek mathematicians. This refutes the previously prevailing view that the Greeks did no substantial work in combinatorics.

The first significant step in this work was made by mathematicians who recognized the relationship between two numbers attributed to Hipparchus by Plutarch and Schröder numbers in modern combinatorics [Stanley 1997; Habsieger, Kazarian and Lando 1998]. This was then built upon by Acerbi [2003b], in a historical paper in which he shows how Hipparchus' work in combinatorics could have been related to technical features in Stoic logic, and gives a discussion of many of the passages by Greek mathematicians and philosophers that have a combinatorial significance. He argues that this work on combinatorics probably took place in the context of technical studies of logic, the source texts of which were lost as interest in the technical study of logic declined.

Another argument for the use of combinatorics in Greek works concerns Archimedes' *Stomachion*, which only survives in two separate Greek and Arabic fragments. Netz, Acerbi and Wilson [2005] argue that Archimedes used combinatorial reasoning in his analysis of the stomachion puzzle-game. There is no direct evidence of combinatorics in the fragments that survive, however, and Morelli [2009] has argued that a combinatorial interpretation is not supported by any of the other ancient evidence for the game.

Computation and Algebra

There are still relatively few studies of the practical mathematics that was used by professionals, such as builders, accountants and astrologers. J. Sesiano has made editions and new studies of a number of texts related to these problem solving techniques, which clearly had relations to the mathematical practice of other Mediterranean and Near Eastern cultures [Sesiano 1998, 1999]. The last chapter of J. Friberg's book on the mathematics of Babylon and Egypt is also a key study of this material [Friberg 2005, 193–268].

Diophantus

A significant achievement of the last ten years has been a reevaluation of the work on Diophantus from the perspective of the new historiography. Although there has been some recent work on Diophantus along the lines of the old historiography [Meskens 2009, 2010], most work during this period has moved away from an algebraic interpretation of his work. That is, it seeks to understand the positive characteristics of his mathematical practice, without simply situating him as a precursor to the development of algebra.

A number of recent studies have shown that, although Diophantus is not doing algebra in the sense of exploring methods of solving certain types of equations, he does present general and systematic methods for solving arithmetic problems, that is, producing rational numbers that satisfy various conditions. For example, Thomaidis [2005] has shown that Diophantus transforms arithmetical problems into the special terms of his arithmetic theory in such a way as to finally produce an equation that can be solved. In a later paper, he identifies two types of equations in Diophantus' approach, what he calls equalities, which are statements of equality that are not to be solved, and the equation proper, which is the end result of the transformation into the arithmetic theory and which is then solved using the standard techniques of ancient and medieval algebra [Thomaidis 2011]. Christianidis [2007] has helped elucidate the general approach (*hodos*) by showing that the introduction can be read as an explanation of how to transform a problem into the arithmetical theory, and then solve the resulting equation. He then applies this analysis to a famous problem, *Arithmetica* II.8, to divide a proposed square into two squares. Bernard and Christianidis [2012] present a new framework for the first three books of *Arithmetica*, in which they attempt to catalog all the types of methods that are used to transform the conditions of the problem into an equation, and to explain the enigmatic role of "positions" (*hypostaseis*). Acerbi [2009] also supports an essentially non-algebraic reading by arguing that the technical term *plasmatikon* is a late interpolation and cannot be used to support an algebraic reading of the text, as had been done by some scholars.

The result of this research is that we will probably soon be in a position to characterize Diophantus' work — which has so far seemed idiosyncratic at best and cryptic at worst — using the terms and methods in which he expresses himself, and, in this way, develop a historically sound reading of his text.

Analysis

Discussions of geometrical analysis remain a regular pastime of historians of Greek mathematics. As well as the articles discussed above, in relation to logical studies, there have been a number of studies of the goals, language and techniques of analysis.

In a collection of essays in memory of W.R. Knorr, Berggren and Van Brummelen [2000] provide a clear overview of the structure of an analysis-synthesis pair and give examples from both Greek and Arabic texts. In the same collection, Netz [2000] argues that analysis was not heuristic in a meaningful sense, but that a Greek mathematician would publish an analysis in order to reveal the key idea behind the solution to a particular problem.

There has been new research into the terminology of "givens," which is fundamental to Greek mathematics. A given object is one that (1) is present at the beginning of a piece of mathematical discourse, (2) assumed by the mathematician, or (3) determined on the basis of either of these. Fournarakis and Christianidis [2006] make a linguistic study of the terminology in geometrical analysis and conclude that different grammatical forms of the base verb *didōmi* are used to make philosophical distinctions regarding the direction of the inference, which they relate to Pappus' discussion of analysis. It is not clear, however, that these distinctions have any meaning in mathematical practice. Acerbi [2011b] has studied the use of this terminology in all its forms and in the full range of mathematical texts and shows that the concept of "given" was used to discuss the concepts of uniqueness and existence, and to create a general framework within which mathematicians could treat different types of argumentative steps, such as constructions, calculations and operations.

K. Saito and I have made two studies of geometrical analysis that seek to revive the idea that analysis was a heuristic technique that could be used to help solve problems and work out theorems. We have argued that the *diorisms* that are found in problematic analysis are remnants of a practice of both exploring the possibility of solutions and enumerating all possible solutions [Saito and Sidoli 2010]. We have also explored a form of theoretic analysis not previously noticed by historians, and argued that this can be taken as evidence that Greek mathematicians actually did do theoretic analysis, contrary to the claims of some historians [Sidoli and Saito 2012].

Greek Foundations of Mathematics

There has always been interest in the foundations of Greek mathematics, but the influence of the new historiography has produced a focus on what Greek mathematicians themselves thought about the logical foundations of their own activity. Going back to the work of Stenius [1978] and others, there has been a recognition that Greek mathematicians approached foundations by doing mathematics, not meta-mathematics.

Acerbi [2010c] has studied what remains of the foundational work of Apollonius and Geminus and shown that for Greek mathematicians, foundations were approached in directly mathematical ways by reworking the elements — that is, invoking new definitions and construction postulates, justifying constructions by superpositions, rewriting the proofs for accepted theorems and trying to prove assumptions such as common notions and postulates. In this sense, we can see that most Greek mathematicians were involved in foundational issues. For example, Borzacchini [2006] argues that Archimedes introduced the axiom that was subsequently named after him in order to deal with the quadrature of curvilinear figures, to which the Eudoxian and Euclidean approach may not have been readily adaptable.

One mathematician of the Imperial period who was especially interested in foundations was Menelaus. He was proud of the fact that he had used no indirect arguments in his *Spherics* and that he had been able to avoid constructing lines inside the sphere, for both of which he criticized Theodosius. He also wrote an *Elements*, which has been lost, but which was almost certainly meant to be read as work on the foundations of mathematics. Hogendijk [1999/2000] has recovered a number of fragments of this text from al-Sijzī's geometry.

Another way in which Greek mathematicians did foundational work was by paying attention to the structure of their arguments, both at the local and global level. Into this category fall debates about the relative importance and position of problems and theorems, the pros and cons of indirect argument, the use of double indirect arguments to show equalities that were established by various means, the suppression or multiplication of cases, and so on. K. Saito has argued that Greek mathematicians used such structures to address patterns of argument, such that an individual proof often served as a paradigm, in the sense of a model [Chemla 2012, 30–31]. Mendell [2007] has shown that a paradigmatic type of two-step argument, involving showing that a proportion holds first for commensurable and next for incommensurable magnitudes, can be found already in Aristotle's writings.

Constructions

Greek mathematicians seem to have regarded constructions and problems as closely bound up with foundations, as evidenced by the work of Euclid, Apollonius and Menelaus. For much of the twentieth century Zeuthen's thesis that problems served as proofs of existence dominated thinking about Greek constructive practice [Thiel 2005]. Especially, following Knorr [1983], however, scholars came to doubt this view.

Harari [2003] argues that constructions in the *Elements* serve to exhibit spacial relations between objects and that, furthermore, Aristotle's ontology is not compatible with construing constructions as existence proofs. K. Saito and I have used a study of the constructions in Theodosius' *Spherics* to show that ancient geometers distinguished between constructions for the sake of a proof and constructions that can be used to solve problems [Sidoli and Saito 2009]. The former can be counterfactual, while the later are abstractions of operations that can be carried out in practice. This makes it clear that Greek geometers clearly distinguished between their diagrams and the objects the diagrams modeled. This is made clear in their use of the word "to imagine" (*noein*), which is used to mark situations when the diagram may not fully model the objects under consideration. Netz [2010] has studied all the occurrences of this word in the mathematical literature and determined that it primarily denotes three types of situations: three dimensional figures, models of physical objects, and disparities between the diagram and the objects it models.

The Exact Sciences

The exact sciences were regarded by Greek practitioners, not as applications, but as essential branches, of mathematics. From Archytas to Ptolemy we read of the division of mathematics into geometry, arithmetic, astronomy and harmonics, with each individual practitioner setting the significance and relationships of these fields according to his own research interests. In authors like Geminus, we find more elaborate divisions of the mathematical sciences.

There has been considerable work in each area of the exact sciences and I will not be able to do justice to many of them here. I have already discussed a number of editions and translations of works in the exact sciences, but here I will describe a number of important studies in the main fields of the exact sciences.

For the early history of mathematics, the most important science was always astronomy. There has been considerable work on Greek astronomy during the period under consideration, of which I will mention only a few examples. J. Evans' *History and Practice of Ancient Astronomy* [1998] gives a wonderful introduction to many aspects of ancient Mediterranean, and to a lesser extent, Near Eastern, astronomy, with an emphasis on practical methods and concepts. Although this book was designed as a textbook, it is also of use to scholars. Jones [2005a] has made a study of the relationship between mathematical models and the physical reality that they represented in Ptolemy's work. This paper explores the function of mathematical modeling in all of Ptolemy's major writings. Berggren and I have studied Aristarchus' *On the Sizes and Distances of the Sun and the Moon* as a work of mathematics and shown that physical hypotheses are used in this work simply to show how mathematics can be applied to them to produce new knowledge [Berggren and Sidoli 2007]. Van Brummelen [2009], in his book on the early history of trigonometry, has devoted two chapters to Greek trigonometry and its relationship to astronomy (Chaps. 1 and 2).

Spherics, which was the combined study of spherical geometry and spherical astronomy, was a science that bridged our modern conceptual distinction between mathematics and astronomy. I have studied the mathematics involved in Heron's *Dioptra* 35, which shows how to use simultaneous lunar eclipse observations to determine the distance between two terrestrial locations, and shown that this can be interpreted as allowing an exact calculation using the so-called analemma methods [Sidoli 2005]. Malpangotto [2003] has studied Pappus' treatment of, and commentary to, Theodosius' *Spherics*. K. Saito and I have shown that Theodosius' *Spherics* provides evidence that ancient spherics was often practiced by producing drawings on actual globes [Sidoli and Saito 2009].

Another essential branch of the exact sciences was harmonics, which became a field of the quadrivium and was developed by mathematicians from Archytas to Ptolemy. A number of recent studies have fully embraced harmonics as an ancient science. Barker [2000] has investigated Ptolemy's scientific method as it functions in his *Harmonics*, with due attention to conceptual, mathematical and instrumental approaches. He has also produced a monograph study of harmonics in the period from the late Pythagoreans to Theophrastus [Barker 2007]. He treats harmonics as a science and sets out the details of the various methodologies, with their different approaches to mathematization and quantification. Finally, Creese [2010] has produced a thorough study of the monochord, which became the key scientific instrument of ancient harmonics. This work helps us understand the relationship between mathematization and instrumentation in harmonics.

Optics was another area developed by a number of Greek mathematicians. Smith [1999] has made a sort of textbook for studying Greek optics based on original sources in English translation, with an emphasis on Ptolemy's *Optics*. Lehoux [2007b] discusses Galen and Ptolemy's accounts of vision and argues that Ptolemy was deeply concerned to construct a sound epistemological grounding for vision, which he regarded as the most mathematical of the senses.

Another important area of mathematical work was mechanics. Tybjerg [2004] has studied Heron's mathematical approach to mechanics and argues that he integrates mechanics and geometry and presents mechanics as a theoretical discipline based on mathematical demonstration. Berryman [2009] studies the history of Greek mechanics and argues that mechanical conceptions of processes and objects were a vital part of ancient natural philosophy. J. Evans and C. Carman have argued, using the Antikythera Mechanism, that mechanics and the mechanical hypothesis may have been important in the development of astronomical modeling in the Hellenistic period [Evans and Carman, below].

Conclusion

I hope it is clear from this often brief survey that the first decade of the 21st century has seen considerable work done on the history of the ancient Greek mathematical sciences. More researchers have entered the field than have left, and it has been invigorated by new approaches and ideas.

It is clear that the old historiography has been overcome. The new historiographic approach that was so hotly debated in the 1970s has become mainstream. There are now almost no serious scholars of the subject trying to determine how Greek mathematics must have originated based on what seems likely from some mathematical or logical perspective, or trying to understand the motivation for methods found in Apollonius or Diophantus using mathematical theories and concepts developed many centuries after these mathematicians lived. At the same time, Greek mathematics is *mathematics* and attempts to read these works using literary or postmodern approaches have not gained much traction.

There is still no book that does for the new historiography what T.L. Heath's monumental *A History of Greek Mathematics* [1921] did for the old historiography. Hence, scholars of the subject will still make use of Heath's work, and they will still suggest that their students read it. So, while the old historiography has been overcome, it has not been left behind. There is still value in making a mathematical reading of an ancient text, as a number of the studies surveyed above have shown.

The wealth of new studies on both familiar and novel topics means that we will soon be in a position to produce a new synthesis, which will describe the practices of Greek mathematicians as mathematical activity, which can be related to other intellectual and cultural activities of the period, and compared with the mathematical activities of other ancient cultures.

References

Acerbi, F., 2000. Plato: Parmenides 149a7–c3. A proof by complete induction? Archive for History of Exact Sciences 55, 57–76.

——— 2003a. Drowning by multiples. Remarks on the fifth book of Euclid's Elements, with special emphasis on prop. 8. Archive for History of Exact Sciences 57, 175–242.

——— 2003b. On the shoulders of Hipparchus: A reappraisal of ancient Greek combinatorics. Archive for History of Exact Sciences 57, 465–502.

——— 2005. A reference to perfect numbers in Plato's Theaetetus. Archive for History of Exact Sciences 59, 319–348.

——— 2007. Euclide: Tutte le opere. Bompiani, Milan.

——— 2008. Euclid's Pseudaria. Archive for History of Exact Sciences 62, 511–551.

——— 2009. The meaning of πλασματικόν in Diophantus' Arithmetica. Archive for History of Exact Sciences 63, 5–31.

——— 2010a. Il silenzio delle sirene. Carocci Editore, Rome.

——— 2010b. Homeomeric lines in Greek mathematics. Science in Context 23, 1–37.

——— 2010c. Two approaches to foundations in Greek mathematics: Apollonius and Geminus. Science in Context 23, 151–186.

——— 2011a. La sintassi logica della matematica greca. Archives-ouvertes.fr, Sciences de l'Homme et de la Société, Histoire, Philosophie et Sociologie des sciences. hal.archives-ouvertes.fr:hal-00727063.

——— 2011b. The language of the "givens": Its form and its use as a deductive tool in Greek mathematics. Archive for History of Exact Sciences 65, 119–153.

——— 2011c. Completing Diophantus, De polygonis numeris, prop. 5. Historia Mathematica 38, 548–560.

——— 2012. Diofanto: De polygonis numberis. Fabrizio Serra Editore, Pisa.

Acerbi, F., Vinel, N., Vitrac, B., 2010. Les Prolégomènes á l'Almageste. Une édition á partir des manuscrits les plus anciens : Introduction générale — Parties I–III. SCIAMVS 11, 53–210.

Alvarez, C., 2003. Two ways of reasoning and two ways of arguing in geometry. Some remarks concerning the application of figures in Euclidean geometry. Synthese 134, 289–323.

Asper, M. 2003. Mathematik, Milieu, Text. Sudhoffs Archiv 87, 1–31.

——— 2009. The two cultures of mathematics in ancient Greece. In: Robson, E., Stedall, J. (eds.), The Oxford Handbook of the History of Mathematics, Oxford University Press, Oxford, pp. 107–132.

Aujac, G., 1984. Le langage formulaire dans la géométrie grecque. Revue d'histoire des sciences 37, 97–109.

Avigad, J., Dean, E., Mumma, J., 2009. A formal system for Euclid's Elements. The Review of Symbolic Logic 2, 700–768.

Barker, A., 2000. Scientific Method in Ptolemy's Harmonics. Cambridge University Press, Cambridge.

——— 2007. The Science of Harmonics in Classical Greece. Cambridge University Press, Cambridge.

Beeson, M., 2010. Constructive geometry. In: Arai, T., Brendle, J., Kikyo, H., Chong, C.T., Downey, R., Feng, Q., Ono, H. (eds.), Proceedings of the 10th Asian Logic Conference. World Scientific, Hackensack, New Jersey, pp. 19–84.

Berggren, J.L., Jones, A., 2000. Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters. Princeton University Press, Princeton.

Berggren, J.L., Sidoli, N., 2007. Aristarchus's On the Sizes and Distances of the Sun and the Moon: Greek and Arabic texts. Archive for History of Exact Sciences 61, 213–254.

Berggren, J.L., Van Brummelen, G., 2000. The role and development of geometric analysis and synthesis in ancient Greece and medieval Islam. In: Suppes, P., Moravcsik, J., Mendell, H. (eds.), Ancient & Medieval Traditions in the Exact Sciences. CSLI Publications, Stanford, pp. 1–31.

Bernard, A., 2003a. Sophistic aspects of Pappus's Collection. Archive for History of Exact Sciences 57, 93–150.

——— 2003b. Ancient rhetoric and Greek mathematics: A response to a modern historiographical dilemma. Science in Context 16, 391–412.

——— 2003c. Comment définir la nature des textes mathématiques de l'antiquité grecque tardive? Proposition de réforme de la notion de 'texts deutéronomiques.' Revue d'histoire des mathématiques 9, 131–173.

——— 2010. The significance of Ptolemy's *Almagest* for its early readers. Revue de synthèse 131, 495–521.

Bernard, A., Christianidis, J., 2012. A new analytical framework for the understanding of Diophantus's Arithmetica I-III. Archive for History of Exact Sciences 66, 1–69.

Berryman, S., 2009. The Mechanical Hypothesis in Ancient Greek Natural Philosophy. Cambridge University Press, Cambridge.

Borzacchini, L., 2006. Why rob Archimedes of his lemma? Mediterranean Journal of Mathematics 3, 433–448.

——— 2007. Incommensurability, music and continuum: a cognitive approach. Archive for History of Exact Sciences 61, 273–302.

Bowen, A.C., Todd, R.B., 2004. Cleomedes' Lectures on Astronomy. University of California Press, Berkeley.

Bülow-Jacosen, A., Taisbak, C.M., 2003. P.Cornell inv. 69. Fragment of a handbook in geometry. In: Pilz, A., et al. (eds.), For Particular Reasons: Studies in Honour of Jerker Blomqvist. Nordic Academic Press, Lund, pp. 55–70.

Carman, C.C., Thorndike, A.S., Evans, J., 2012. On the pin-and-slot device of the Antikythera Mechanism, with a new application to the superior planets. Journal for the History of Astronomy 43, 93–116.

Chemla, K., 1999. Commentaires, éditions et autres textes seconds: quel enjeu pour l'histoire des mathématiques? Réflections inspirées par la note de Reviel Netz. Revue d'histoire des mathématiques 5, 127–148.

——— (ed.), 2012. The History of Mathematical Proof in Ancient Traditions. Cambridge University Press, Cambridge.

Christianidis, J., 2007. The way of Diophantus: Some clarifications on Diophantus' method of solution. Historia Mathematica 34, 289–305.

Creese, D., 2010. The Monochord in Ancient Greek Harmonic Science. Cambridge University Press, Cambridge.

Cuomo, S., 2000. Pappus of Alexandria and the Mathematics of Late Antiquity. Cambridge University Press, Cambridge.

——— 2001. Ancient Mathematics. Routledge, London.

——— 2012. Exploring ancient Greek and Roman numeracy. BSHM Bulletin 27, 1–12.

Czinczenheim, C., 2000. Edition, traduction et commentaire des Sphériques de Théodose. These de docteur de l'Universite Paris IV. Printed: Atelier national de reproduction des thèses, Lille.

d'Alessandro, P., Napolitani, P.D., 2012. Archimede Latino, Iacopo da San Casiano e il Corpvs Archimedeo alla metà del quatrtrocento. Les Belles Lettres, Paris.

De Young, G., 2005. Diagrams in the Arabic Euclidean tradition: A preliminary assessment. Historia Mathematica 32, 129–179.

Decorps-Foulquier, M., 1998. Eutocius d'Ascalon éditeur du traité des Coniques d'Apollonius de Pergé et l'exigence de clarté: un exemple des pratiques exégétiques et critiques des héritiers de la science alexandrine. Dans: Argoud, G., Guillaumin, J.Y. (réd.), Sciences exactes et appliquées à Alexandrie. Publications de l'université de Saint-Étienne, Saint-Étienne, pp. 87–101.
——— 1999a. Recherches sur les Coniques d'Apollonios de Pergé et leurs commentateurs grecs: Histoire de la transmission des Livres I–IV. Klincksieck, Paris.
——— 1999b. Sur les figures du traité des Coniques d'Apollonios de Pergé édité par Eutocius d'Ascalon. Revue d'histoire des mathématiques 5, 61–82.
Djebbar, A., 2003. Quelques exemples de scholies dans la tradition arabe des Éléments d'Euclide. Revue d'histoire des sciences 56, 293–321.
Evans, J., 1998. The History and Practice of Ancient Astronomy. Oxford University Press, New York.
——— 1999. The material culture of Greek astronomy. Journal for the History of Astronomy 30, 237–307.
——— 2004. The astrologer's apparatus: A picture of professional practice in Greco-Roman Egypt. Journal for the History of Astronomy 35, 1-44.
Evans, J., Berggren, J.L., 2006. Geminos's Introduction to the Phenomena. Princeton University Press, Princeton.
Evans, J., Carman, C.C., Thorndike, A.S., 2010. Solar anomaly and planetary displays in the Antikythera Mechanism. Journal for History of Astronomy 41, 1–39.
Federspiel, M., 1994. Notes critiques sur le livre I des Coniques d'Apollonius de Pergè. Revue de études greques 107, 203–218.
——— 1995. Sur l'opposition défini/indéfini dans la langue des mathématiques grecques. Les études classiques 63, 249–293
——— 1999–2000. Notes critiques sur le livre II des Coniques d'Apollonius de Pergè. (Première partie, Seconde partie.) Revue de études greques 112, 409–443; 113, 359–391.
——— 2002. Notes critiques sur le livre III des Coniques d'Apollonius de Pergè. Première partie. Revue de études greques 115, 110–148.
——— 2008. Les Problèmes des livres Grecs des Coniques d'Apollonius de Pergè. Les études classiques 76, 321–360.
Feke, J. 2012. Ptolemy's defense of theoretical philosophy. Apeiron 45, 61–90.
Feke, J., Jones, A. 2010. Ptolemy. In: Gerson, L.P. (ed.), The Cambridge History of Philosophy in Late Antiquity. Cambridge University Press, Cambridge, vol. 1, pp. 197–209.
Fournarakis, P., Christianidis, J., 2007. Greek geometrical analysis: A new interpretation through the "givens"-terminology. Bollettino di Storia delle Scienze Matematiche 26, 3–56.
Fowler, D.H., 1998. In memoriam: Wilbur Richard Knorr (1945–1997): An appreciation. Historia Mathematica 25, 123–132.
——— 1988. A catalogue of tables. Zeitschrift für Papyrologie und Epigraphik 75, 273-280.
——— 1995. Further arithmetical tables. Zeitschrift für Papyrologie und Epigraphik 105, 225-228.
Freeth, T., Bitsakis, Y., Moussas, X., Seiradakis, J.H., Tselikas, A., Mangou, H., Zafeiropoulou, M., Hadland, R., Bate, D., Ramsey, A., Allen, M., Crawley, A., Hockley., P., Malzbender., T., Gelb, D., Ambrisco, W., Edmunds, M.G., 2006. Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism. Nature 444, 587–591.
Freeth, T., Jones, A., 2012. The Cosmos in the Antikythera Mechanism. ISAW Papers 4. http://dlib.nyu.edu/awdl/isaw/isaw-papers/4.
Freeth, T., Jones, A., Steele, J., Bitsakis, Y., 2008. Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism. Nature 454, 614–617.
Friberg, J., 2005. Unexpected Links between Egyptian and Babylonian Mathematics. World Scientific: Singapore.
Friberg, J., 2007. Amazing Traces of a Babylonian Origin in Greek Mathematics. World Scientific: Singapore.
Fried, M., 2002. Apollonius of Perga: Conics Book IV. Green Lion Press, Santa Fe.
——— 2003. The use of analogy in Book VII of Apollonius' Conica. Science in Context 16, 349–365.

Fried, M., Unguru, S., 2001. Apollonius of Perga's Conica: Text, Context, Subtext. Brill, Leiden.
Habsieger, L., Kazarian, M., Lando, S., 1998. On the second number of Plutarch. The American Mathematical Monthly 105, 446.
Harari, O., 2003. The concept of existence and the role of constructions in Euclid's Elements. Archive for History of Exact Sciences 57, 1–23.
Heath, T., 1921. A History of Greek Mathematics. Clarendon Press, Oxford.
Hogendijk, J.P., 1999/2000. Traces of the lost Geometrical Elements of Menelaus in two texts of al-Sijzī. Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften 13, 129–164.
Huffman, C.A., 2005. Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King. Cambridge University Press, Cambridge.
Jones, A., 1999. Astronomical Papyri from Oxyrhynchus. P. Oxy. 4133–4300a. Memoirs of the American Philosophical Society, 233. American Philosophical Society, Philadelphia, PA.
——— 2001. Pseudo-Ptolemy, De speculis. SCIAMVS 2, 145–186.
——— 2005a. Ptolemy's mathematical models and their meaning. In: Kinyon, M., Van Brummelen, G. (eds.), Mathematics and the Historian's Craft. Springer, New York, pp. 23–42.
——— 2005b. Ptolemy's Canobic Inscription and Heliodorus' observation reports. SCIAMVS 6, 53–98.
——— 2006. The Keskintos Astronomical Inscription, text and interpretations. SCIAMVS 7, 3–41.
——— 2012. P.Cornell inv. 69 revisited: A collection of geometrical problems. In: Ast, R., Cuvigny, H., Hickey, T.M., Lougovaya, J. (eds.), Papyrological Texts in Honor of Roger S. Bagnall. The American Society of Papyrologists, Durham, pp. 159–175.
Kheirandish, E., 1999. The Arabic Version of Euclid's Optics. Vol. I, II. (Kitāb Uqlīdis fī Ikhtilāf al-manāẓir). Springer, New York.
Knorr, W.R., 1983. Construction as existence proof in ancient geometry. Ancient Philosophy 3, 125–148.
——— 1996. The wrong text of Euclid: On Heiberg's text and its alternatives, Centaurus 38, 208–276.
——— 2001. On Heiberg's Euclid. Intercultural transmission of scientific knowledge in the middle ages: Graeco-Arabic-Latin (Berlin, 1996). Science in Context 14, 133–143.
Kunitzsch, P., Lorch, R., 2010. Theodosius Sphaerica: Arabic and Medieval Latin Translations. Franz Steiner Verlag, Stuttgart.
——— 2011. Theodosius, De habitationibus: Arabic and Medieval Latin Translations. Verlag der Bayerischen Akademie der Wissenschaften, Munich.
Lehoux, D., 2007a. Astronomy, Weather, and Calendars: Parapegmata and Related Texts in Classical and Near-Eastern Societies. Cambridge University Press, Cambridge.
——— 2007b. Observers, objects, and the embedded eye; or, seeing and knowing in Ptolemy and Galen. Isis 98, 447–467.
Lewis, M.J.T., 2001. Surveying Instruments of Greece and Rome. Cambridge University Press, Cambridge.
Lorch, R.P., 2001. Greek-Arabic-Latin: The transmission of mathematical texts in the Middle Ages. Science in Context 14, 313–331.
Mäenpää, P., 1997. From backwards reduction to configurational analysis. In: Otte, M., Panza, M. (eds.), Analysis and Synthesis in Mathematics. Kluwer, Netherlands, pp. 201–226.
Mäenpää, P., von Plato, J., 1990. The logic of Euclidean construction procedures. Acta Philosophica Fennica 49, 275–293.
Malpangotto, M., 2003. Sul commento di Pappo d'Alessandria alle «Sferiche» di Teodosio. Bollettino di Storia delle Scienze Matematiche 23, 121–148.
Malpangotto, M., 2010. Graphical choices and geometrical thought in the transmission of Theodosius' Spherics from antiquity to the Renaissance. Archive for History of Exact Sciences 64, 75–112.
Manders, K., 2008a. Diagram-based geometric practice. In: Mancosu, P. (ed.), The Philosophy of Mathematical Practice. Oxford University Press, Oxford, pp. 65–79.
——— 2008b. The Euclidean diagram (1995). In: Mancosu, P. (ed.), The Philosophy of Mathematical Practice. Oxford University Press, Oxford, pp. 80–133.

Mansfeld, J., 1998. Prolegomena Mathematica: From Apollonius of Perga to Late Neoplatonism. Brill, Leiden.

Masià-Fornos, R., 2010. A "lacuna" in Proposition 9 of Archimedes' On the Sphere and the Cylinder, Book I. Historia Mathematica 37, 568–578.

Mendell, H., 2007. Two traces of two-step Eudoxan proportion theory in Aristotle: A tale of definitions in Aristotle, with a moral. Archive for History of Exact Sciences 61, 3–37.

Mercier, R., 2011. Ptolemy's Handy Tables, Volume 1b, Tables A1-A2. Institute Orientaliste, Louvain.

Meskens, A., 2009. Reading Diophantos. In: Van Kerkhove, B. (ed.), New Perspectives on Mathematical Practices. World Scientific, Hackensack, pp. 28–46.

——— 2010. Travelling Mathematics: The Fate of Diophantos' Arithmetic. Berkhäuser, Basel.

Miller, N., 2007. Euclid and His Twentieth Century Rivals: Diagrams in the Logic of Euclidean Geometry. CSLI Publications, Stanford, CA.

Mogenet, J., 1950. Autolycus de Pitane : Histoire du texte suivie de l'édition critique des traités de La sphére en mouvement et Des levers et couchers. Publications universitaires de Louvain, Louvain.

Morelli, G., 2009. Lo Stomachion di Archimede nelle testimonianze antiche. Bollettino di Storia delle Scienze Matematiche 29, 181–206.

Mueller, I., 2003. Remarks on Euclid's Elements I,32 and the parallel postulate. Science in Context 16, 287–297.

Mugler, C., 1959. Dictonnaire historique de la terminologie géomeétrique des grecs. C. Klinckseick, Paris.

Netz, R., 1998. Deuteronomic texts: Late antiquity and the history of mathematics. Revue d'Histoire des Mathématiques 42, 261–288.

——— 1999a. The Shaping of Deduction in Greek Mathematics. Cambridge University Press, Cambridge.

——— 1999b. Proclus' division of the mathematical proposition into parts: How and why was it formulated? Classical Quarterly (N. S.) 49, 282–303.

——— 1999c. Archimedes transformed: The case of a result stating a maximum for a cubic equation. Archive for History of Exact Sciences 54, 1–47.

——— 2000. Why did Greek mathematicians publish their analyses? In: Suppes, P., Moravcsik, J., Mendell, H. (eds.), Ancient & Medieval Traditions in the Exact Sciences. CSLI Publications, Stanford, pp. 139–157.

——— 2002a. Greek mathematicians: A group picture. In: Wolpert, L., Tuplin, C.J., Rihll, T. E. (eds.), Science and Mathematics in Ancient Greek Culture. Oxford University Press, Oxford, pp. 196–216.

——— 2002b. Counter culture: Towards a history of Greek numeracy. History of Science 40, 312–352.

——— 2004a. The works of Archimedes, Vol. I: The Two Books on the Sphere and the Cylinder, Translated into English, Together with Eutocius' Commentaries, with Commentary, and Critical Edition of the Diagrams. Cambridge University Press, Cambridge.

——— 2004b. Eudemus of Rhodes, Hippocrates of Chios and the earliest form of a Greek mathematical text. Centaurus 46, 243–286.

——— 2004c. The Transformation of Mathematics in the Early Mediterranean World: From Problems to Equations. Cambridge University Press, Cambridge.

——— 2009. Ludic Proof: Greek Mathematics and the Alexandrian Aesthetic. Cambridge University Press, Cambridge.

——— 2010. Imagination and Layered Ontology in Greek Mathematics. Configurations 17, 19–50.

Netz, R., Acerbi, F., Wilson, N., 2005. Towards a Reconstruction of Archimedes' Stomachion. SCIAMVS 5, 67–99.

Netz, R., Noel, W., Wilson, N., Tchernetska, N. (eds.), 2011. The Archimedes Palimpsest. 2 vols. Cambridge University Press, Cambridge.

Netz, R., Saito, K., Tchernetska, N., 2001–2002. A new reading of Method Proposition 14: Preliminary evidence from the Archimedes palimpsest. (Parts I and II.) SCIAMVS 2, 9–29; 3, 109–125.

Netz, R., Wilson, N., 2010. The Archimedes Codex: Revealing the Secrets of the World's Greatest Palimpsest. Phoenix, London.

Noack, B., 1992. Aristarch von Samos: Untersuchungen zur Überlieferungsgeschichte der Schrift περὶ μεγεθῶν καὶ ἀποστημάτων ἡλίου καὶ σελήνης. Ludwig Reichert Verlag, Wiesbaden.
Pambuccian, V., 2008. Axiomatizing geometric constructions. Journal of Applied Logic 6, 24–46.
Panza, M., 2012. The twofold role of diagrams in Euclid's plane geometry. Synthese 186, 55–102.
Rashed, R., Bellosta, H., 2010. Apollonius de Perge, La section des droites selon des rapports. Walter de Gruyter, Berlin.
Rashed, R., Decorps-Foulquier, M., Federspiel, M., 2008–2010. Apollonius de Perge, Coniques : Text grec et arabe établi, traduit et commenté. Walter de Gruyter, Berlin.
Robson, E., 2005. Influence, ignorance, or indifference? Rethinking the relationship between Babylonian and Greek mathematics. BSHM Bulletin 4, 1–17.
Rommevaux, S., Djebbar, A., Vitrac, B., 2001. Remarques sur l'histoire du texte des Éléments d'Euclide. Archive for History of Exact Sciences 55, 221–295.
Saito, K., 2003. Phantom theories of pre-Eudoxean proportion. Science in Context 16, 331–347.
——— 2006a. Between magnitude and quantity: Another look at Archimedes' quadrature (trans. of Sūgaku 55, 166–179). Sugaku Expositions 19, 35–52.
——— 2006b. A preliminary study in the critical assessment of diagrams in Greek mathematical works. SCIAMVS 7, 81–144.
——— 2008. Diagrams in Greek Mathematical Texts: Report of Research Grant 17300287 of the Japan Society for the Promotion of Science. Sakai, Osaka. (See http://www.hs.osakafu-u.ac.jp/~ken.saito/diagram/ for further updates.)
——— 2009. Reading ancient Greek mathematics. In: Robson, E., Stedall, J. (eds.), The Oxford Handbook of the History of Mathematics. Oxford University Press, Oxford, pp. 801–826.
——— 2012. Traditions of the diagram, traditions of the text: A case study. Synthese 186, 7–20.
Saito, K., Sidoli, N., 2010. The function of diorism in ancient Greek analysis. Historia Mathematica 37, 579–614.
——— 2012. Diagrams and arguments in ancient Greek mathematics: Lessons drawn from comparisons of the manuscript diagrams with those in modern critical editions. In: Chemla, K. (ed.), History of Mathematical Proof in Ancient Traditions. Cambridge University Press, Cambridge, pp. 135–162.
Sefrin-Weis, H., 2010. Pappus of Alexandria: Book 4 of the Collection. Springer, London.
Sesiano, J., 1998. An early form of Greek algebra. Centaurus 40, 276–302.
——— 1999. Sur le Papyrus graecus genevensis 259. Museum Helveticum 56, 26–32.
Sidoli, N., 2005. Heron's Dioptra 35 and analemma methods: An astronomical determination of the distance between two cities. Centaurus 47, 236–258.
——— 2006. The sector theorem attributed to Menelaus. SCIAMVS 7, 43–79.
——— 2007. What we can learn from a diagram: The case of Aristarchus's On the Sizes and Distances of the Sun and the Moon. Annals of Science 64, 525–547.
Sidoli, N., Berggren, J.L., 2007. The Arabic version of Ptolemy's Planisphere or Flattening the Surface of the Sphere: Text, translation, commentary. SCIAMVS 8, 37–139.
Sidoli, N., Saito, K., 2009. The role of geometrical construction in Theodosius's Spherics. Archive for History of Exact Sciences 63, 581–609.
——— 2012. Comparative analysis in Greek geometry. Historia Mathematica 39, 1–33.
Smith, A.M., 1999. Ptolemy and the Foundations of Ancient Mathematical Optics. A Source Based Guided Study. Transactions of the American Philosophical Society, 89, Pt. 3. American Philosophical Society, Philadelphia, PA.
Solomon, J., 2000. Ptolemy's Harmonics. Brill, Leiden.
Stanley, R.P., 1997. Hipparchus, Plutarch, Schröder, and Hough. The American Mathematical Monthly 104, 344–350.
Stenius, E., 1978. Foundations of Mathematics: Ancient Greek and Modern. Dialectica 32, 255–290.
Stückelberger, A., Grasshoff, G., 2006. Ptolemaios: Handbuch der Geographie (2 Bände). Schwabe, Basel.
Taisbak, C.M., 2003a. Euclid's Data. Or the Importance of Being Given. Museum Tusculanum Press, Copenhagen.

Takahashi, K., 2001. A manuscript of Euclid's De speculis: a Latin text of MS 98.22 of the Archivo y Biblioteca Capitulares de la Catedral, Toledo. SCIAMVS 2, 75–143.

Thiel, C., 2005. Becker und die Zeuthensche These zum Existenzbegriff in der antiken Mathematik. In: Peckhaus, V. (ed.), Oskar Becker und die Philosophie der Mathematik. Wilhelm Fink Verlag, Paderborn, pp. 32–45.

Thomaidis, Y., 2005. A framework for defining the generality of Diophantos' methods in Arithmetica. Archive for History of Exact Sciences 59, 591–640.

——— 2011. Some remarks on the meaning of equality in Diophantos' Arithmetica. Historia Mathematica 38, 28–41.

Tihon, A., 1999. Le Grand commentaire de Théon d'Alexandrie aux Tables faciles de Ptolémée. Livre IV. Biblioteca Apostolica Vaticana, Vatican City.

——— 2011. Les Tables faciles de Ptolémée, Volume 1a, Tables A1-A2. Institute Orientaliste, Louvain.

Tybjerg, K., 2004. Hero of Alexandria's mechanical geometry. Apeiron 37, 29–56.

Unguru, S., Fried, M., 2007. Apollonius, Davidoff, Rorty, and Zeuthen: From A to Z, or, What else is there? Sudhoffs Archiv 91, 1–19.

Van Brummelen, G., 2009. The Mathematics of the Heavens and the Earth: The Early History of Trigonometry. Princeton University Press, Princeton.

Vitrac, B., 1990–2001. Euclide : Les Élements. Presses Universitaires de France, Paris.

——— 2002. Note textuelle sur un (problème de) lieu géométrique dans les Météorologiques d'Aristote (III.5, 375 b 16–276 b 22). Archive for History of Exact Sciences 56, 239–283.

——— 2003. Les scholies grecques aux Éléments d'Euclide. Revue d'histoire des sciences 56, 27–292.

——— 2004. À propos des démonstrations alternatives et autres substitutions de preuves dans les Éléments d'Euclide. Archive for History of Exact Sciences 59, 1–44.

——— 2012. The Euclidean ideal of proof in the Elements and philological uncertainties of Heiberg's edition of the text. In: Chemla, K. (ed.), History of Mathematical Proof in Ancient Traditions. Cambridge University Press, Cambridge, pp. 69–134.

Vitrac, B., Djebbar, A., 2011–2012. Le Livre XIV des Élements d'Euclid : versions greques et arabe (première partie, seconde partie). SCIAMVS 12, 29–158; 13, 3–158.

Wagner, R., 2009. For some histories of Greek mathematics. Science in Context 22, 535–565.

Wilson, N.G., 1999. Archimedes: The palimpsest and the tradition. Byzantinische Zeitschrift 92, 89–101.

Winter, T.N., 2007. The Mechanical Problems in the corpus of Aristotle. Faculty Publications, Classics and Religious Studies, University of Nebraska.

Wright, M.T., 2007. The Antikythera Mechanism reconsidered. Interdisciplinary Science Reviews 32, 27–43.

Zhmud, L., 1998. Plato as "architect of science." Phronesis 43, 211–244.

——— 2006. The Origin of the History of Science in Classical Antiquity. Chernoglazov, A. (trans.), Walter de Gruyter, Berlin.

——— 2012. Pythagoras and the Early Pythagoreans. Windle, K., Ireland, R. (trans.), Oxford University Press, Oxford.