CHAPTER 4.1

Fundamentals of Planetary Theory

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1 Introduction

The five planets visible to the naked eye can be divided into two groups which exhibit quite distinct observational features:

(1) the inner or inferior planets—Venus and Mercury; and

(2) the outer or superior planets—Mars, Jupiter, and Saturn.

(From a heliocentric perspective, the inner planets revolve between the Earth's orbit and the Sun and never reach opposition, while the outer planets revolve outside the Earth's orbit and reach opposition [see §3, p. 66].) To the untrained eye, the planets look much like stars and, in the ancient period, they were often considered to be a type of star.[1] It is said that stars twinkle more than planets; but the extent to which this can be perceived by the unaided eye depends on a number of factors, including the acuity of the eye in question and local atmospheric conditions. Nevertheless, the eye readily learns to differentiate the planets from the stars and to identify them by their motions, brilliance, and distinctive colors.

In this chapter, I discuss what one may observe of the planets first from the perspective of the local horizon over the course of some years and then as considered against a presumed background of the fixed stars, irrespective of the observer's location on Earth. The question of if, or when, ancient practitioners actually became aware of the phenomena discussed here are significant historical issues not covered in this chapter.

2 Observing the Planets

The more conspicuous phenomena associated with the planets are not necessarily the ones most useful to theoretical considerations. In the course of a single night, one cannot discern much about the planets to distinguish them from other stars. When the Sun sets, the planets can sometimes be found in

1 In Greco-Roman astronomy, the distinction was made between fixed and wandering stars.

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the night sky either, in the case of the inner planets, in the direction of the sunset or, in the case of the outer planets, anywhere in the general vicinity of the Sun's day-circle (that is, the circle on which the Sun appears to travel from east to west each daytime). The planets all move throughout the course of the night on day-circles parallel to those of the nearby stars and set at the western horizon, to all appearances as though they were themselves fixed stars. An outer planet can also sometimes be seen to rise into the night sky from the eastern horizon either just after sunset or at any time throughout the course of the night. When it does this, it crosses the sky with the nearby stars until the Sun rises and they all disappear into the dawning light. An inner planet, if it was not seen in the evening in the west, can also sometimes be seen to rise in the east the following morning. It rises in the general neighborhood of the coming sunrise as a bright point of light against the growing dawn and it quickly disappears into the light of the morning sky.

As one observes the planets over a number of nights, one may identify a range of phenomena. The most obvious of these is that the planets, like the fixed stars in the vicinity of the Sun's day-circles, repeatedly exhibit certain phenomena which we call appearances or phases. For example, like the stars, planets are sometimes visible and sometimes invisible, depending on their angular separation or elongation from the Sun; hence, like the stars they exhibit first and last appearances in a synodic cycle. (The synodic phenomena of the planets are, however, different from those of the stars.) The term "synodic", which derives from the Greek word «cύνοδοc», originally referred to a meeting or conjunction with the Sun but came by abstraction to indicate a more general relation to the Sun and its position. A full understanding of planetary synodic cycles takes a large number of observations made against a background of fixed stars. Hence, I will return to a discussion of the full synodic cycle after a brief discussion of the planetary phenomena as seen from the local horizon.

The wandering of the planets becomes clear after a relatively short time when they are observed in reference to the local horizon. If we look at the same planet over a sequence of nights, we find that, whether in relation to the horizon or the fixed stars, it moves around in the night sky. Consider Venus. If one had observed it just after sunset at the beginning and middle of every month in relation to a fixed object on one's western horizon at a location near Babylon in the year 199 BCE, one would have seen Venus appear at the sequence of positions given in Figure 1, p. 65. Then, on any given night, after sunset, Venus would descend from one of the dots in the diagram and set at the western horizon as it follows a path parallel to that taken by the Sun in its descent. Figure 1 was made by plotting the positions of Venus in local coordi-

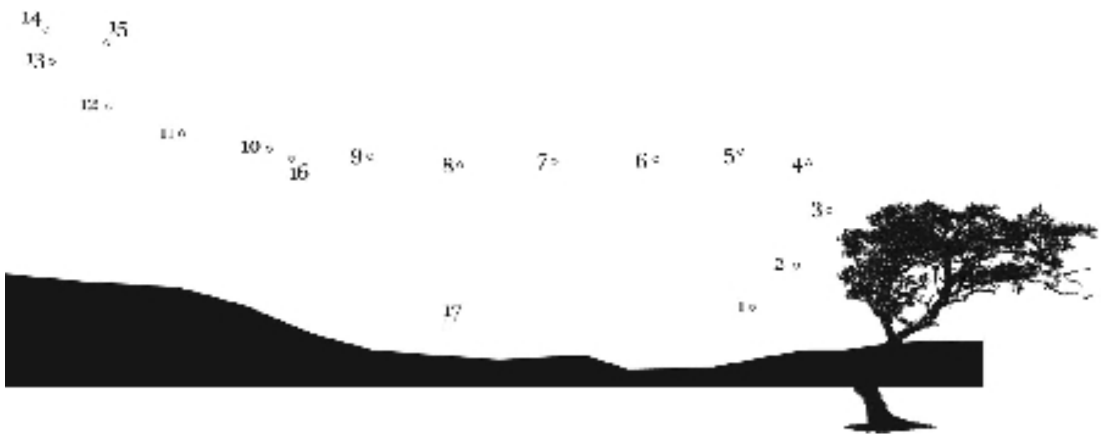

FIGURE 1 Positions of Venus as an evening star, viewed in relation to a fixed object on the western horizon at a location near Babylon just after sunset at 15-day intervals in 199/198 BCE

nates [see ch. 1 §4.3, p. 18] at 15-day intervals when the Sun was just below the observer's horizon. This means that everything other than the horizon is in a different location on each sighting—the local time and the location of sunset both vary and the positions of the fixed stars are displaced by roughly 15° west at each interval. Moreover, it is difficult to know whether Venus actually would have been seen at positions 1 or 17 because the timing of first and last evening appearances depends on many factors that we cannot now control, such as the physiology of the observer and the accidents of local weather. So let us assume a first evening appearance between 1 and 2, and a last evening appearance between 16 and 17.

Of course, there are no images of this sort from Antiquity; nor is there evidence that ancient scholars plotted successive positions of a planet in this way. Nevertheless, by considering this diagram, we can describe a number of planetary phenomena that ancient observers could have noticed even without precise, or carefully recorded, observations. Namely, as Venus moves away from the Sun to the east, it appears as an evening star in the western horizon, setting sometimes to the north and sometimes to the south of where the Sun sets. It moves farther away from the Sun for a while but then returns toward the Sun and disappears again. After some time, Venus will reappear as a morning star just before sunrise in the eastern sky and then exhibit a similar range of phenomena. Mercury, if it is seen at all, appears faintly for a short while, low in the sky, as either an evening or a morning star. This observational situation means that it was difficult for ancient observers to form a detailed understanding of the phenomena associated with Mercury.

The related phenomena for the outer planets, when considered with the horizon as a reference, is simpler than that of the inner planets—which fact, even without any developed theory, is sufficient to differentiate clearly between the two types of planets. Because the outer planets move through their eastward courses more slowly than the Sun, their synodic phenomena are similar to those of the fixed stars, with only one period of invisibility throughout the entire synodic cycle. Like the fixed stars, an outer planet has its first visibility in the eastern sky as it rises above the horizon for a few minutes before disappearing into the light of the coming dawn. In the following days, the outer planet will rise earlier and earlier each night and, during any given night, it will move with the nearby stars and eventually vanish with the sunrise. As time passes, it will become so far removed from the Sun that it will set at the western horizon just around sunrise and then rise at the eastern horizon when the Sun sets. In the following evenings, the planet will already be in the night sky after sunset, progressively approaching the western horizon, at which it sets. As this process continues, the planet is found farther and farther to the west at sunset until it is so close to the Sun that it is again invisible for some time.

3 The Synodic Planetary Phenomena

The forgoing account has been a qualitative description of the phenomena that are most obvious with respect to the local conditions of observation. Even without carefully recorded observations, it is clear that the most significant planetary phenomena are those based on the planets' angular separation from the Sun. Hence, in that phenomena such as these depend on some relation to the Sun, they are called synodic phenomena.

In order to form a clearer understanding of these phenomena, however, it is better to consider the position of the planets against a reference of the fixed stars. Such an understanding can only be developed on the basis of accurately made, and carefully recorded, observations of the relative positions of the planets and the stars in the vicinity of the zodiacal circle (that is, the ecliptic). In Figure 2, p. 67, we see the hypothetical path of an outer planet plotted against a background of fixed stars. The frame of this diagram is different from that of Figure 1, p. 65, which depicts positions of Venus relative to local coordinates. Figure 2 disregards the observer's horizon so that we have a theoretically constructed view of the motion of a planet as projected from the center of the Earth onto a point of the celestial sphere, in zodiacal coordinates. No observer on Earth will actually see such a series of planetary positions as is plotted here, even given perfect weather conditions and continuous observations. At best one could construct a sort of dotted line against the fixed stars. Moreover, Babylonian and Greek observers did not make continuous observations of planetary

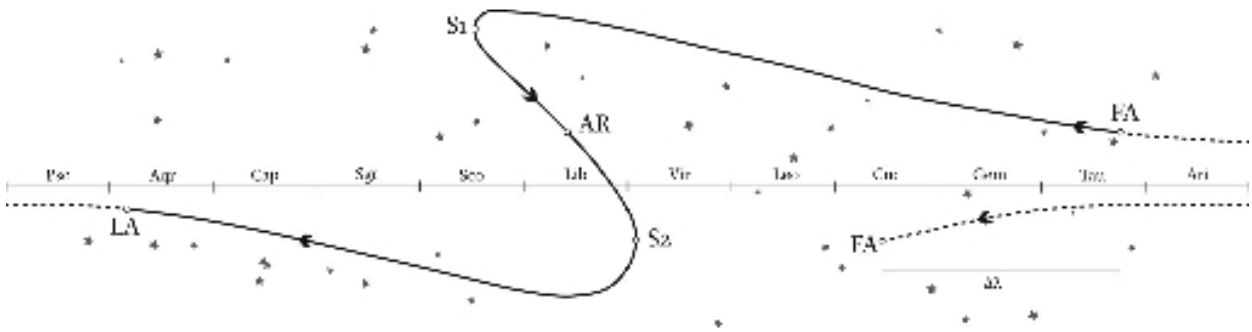

FIGURE 2 A hypothetical synodic cycle of an outer planet, considered against a background of fixed stars
The dotted paths indicate when the planet is too close to the Sun to be visible in the night sky. Note that the dotted line below the zodiacal circle on the right-hand side is a continuation of that from the left-hand side.

TABLE 1 The synodic phenomena or phases of the outer planets

FA	Γ	First appearance or phase	
S_1	Φ	First station	Retrogradation
AR	Θ	Acronychal rising (*Opposition*)	Retrogradation
S_2	Ψ	Second station	Retrogradation
LA	Ω	Last appearance or phase (*Conjunction*)	

positions at fixed temporal intervals as would be necessary to generate this sort of curve. Rather, they noted observationally significant events when they happened, such as first and last appearances, a planet's rising or setting opposite the Sun, the passage of a planet by a significant star, and so forth. Finally, there is no evidence in ancient sources for the sort of visual presentation of planetary phenomena presented in Figure 2.

Nevertheless, in order for us to speak clearly about the sorts of phenomena that formed the basis of ancient planetary theory, it is useful to consider diagrams of this sort. Since the synodic cycle of the inner planets is different from that of the outer planets, we will consider them separately. The synodic cycle of the outer planets is simpler; so let us start with it.

3.1 *The Outer Planets*

A synodic cycle for an outer planet may be said to begin on the morning when we first see the planet in the eastern sky just before sunrise. In Figure 2 [also

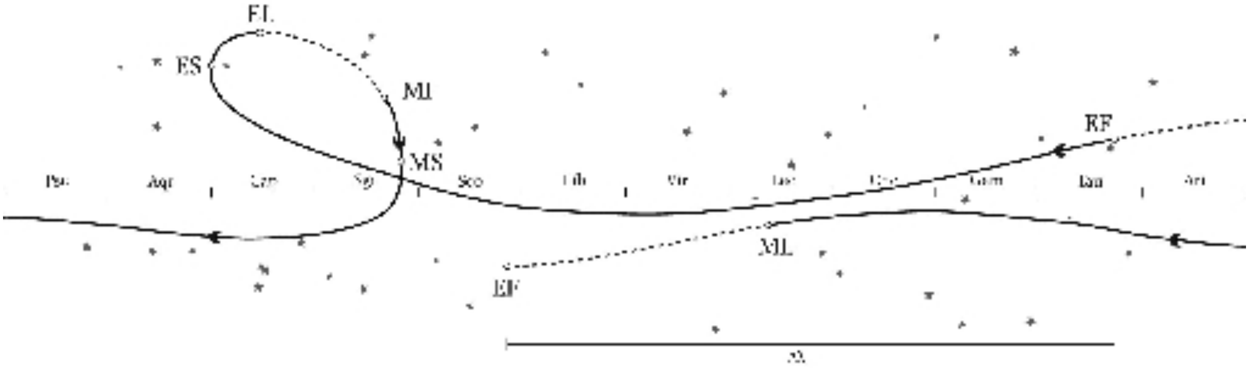

FIGURE 3 A hypothetical synodic cycle of an inner planet, considered against a background of fixed stars
The dotted paths indicate when the planet is too close to the Sun to be visible in the night sky.

Table 1, p. 67], we see a hypothetical outer planet plotted against the fixed stars. The diagram is orientated such that the daily motion of the stars westward is from left to right, so that the Sun and the planets all make their proper motion to the east from right to left in the order of the zodiacal signs. Our hypothetical outer planet begins a synodic cycle with its first appearance (FA or Γ)[2] and then has a direct (or prograde) motion to the east as its elongation from the Sun increases. When the planet begins to approach an elongation of 180°, its eastward motion appears to slow and stop against the background of the fixed stars. The short interval when the planet has no longitudinal motion along the zodiacal circle is called first station (S_1 or Φ). After first station, the planet moves backward through the zodiacal signs in a retrograde motion. During its course along this retrograde arc, when the planet reaches true opposition with an elongation of 180° from the Sun, the planet can be seen to rise at the same time as the Sun sets, which is called its acronychal rising (AR or Θ). The planet then continues in retrograde motion until second station (S_2 or Ψ), after which it returns to direct motion, moving forward in the order of the signs until it rises just before the Sun and finally disappears again in its last appearance (LA or Ω). While the planet is near the Sun in conjunction, it remains invisible until its next first appearance, which is displaced along the zodiacal circle and marks the start of a new cycle in which the same phenomena are repeated in the same order.

2 We use the abbreviations introduced in Ossendrijver 2012 for the planetary synodic phenomena but have also included the Greek letter abbreviations used in a number of classic studies, such as in Neugebauer 1975. Due to the use of the latter abbreviations, synodic phenomena are sometimes called Greek letter phenomena.

TABLE 2 The synodic phenomena or phases of the inner planets

<table>
<tr><td>EF</td><td>Ξ</td><td>First evening appearance or phase</td><td></td></tr>
<tr><td>ES</td><td>Ψ</td><td>Evening station</td><td></td></tr>
<tr><td>LE</td><td>Ω</td><td>Last evening appearance or phase
(Inferior conjunction)</td><td rowspan="3">Retrogradation</td></tr>
<tr><td>MF</td><td>Γ</td><td>First morning appearance or phase</td></tr>
<tr><td>MS</td><td>Φ</td><td>Morning station</td></tr>
<tr><td>ML</td><td>Σ</td><td>Last morning appearance or phase
(Superior conjunction)</td><td></td></tr>
</table>

3.2 *The Inner Planets*

A synodic cycle for an inner planet may be said to begin on the night when we first see the planet in the western sky just after sunset. In Figure 3, p. 68 [also Table 2], we see a hypothetical inner planet plotted against the fixed stars. The synodic cycle of the inner planets is slightly more involved than that of the outer planets because the inner planets have two periods of invisibility. An inner planet begins its cycle with its first evening appearance (FE or Ξ) as it moves in direct motion to the east in the order of the signs. It continues in direct motion until its first station, which we call evening station (ES or Ψ), after which it goes into retrograde motion and its elongation from the Sun begins to decrease. When the planet gets sufficiently close to the Sun it appears one last time in the western sky in its last evening appearance (LE or Ω). It then continues in retrograde motion, although it is not visible to the observer, during which period it has its inferior conjunction with the Sun. After some time, the planet reappears, now before sunrise in the western sky in its first morning appearance (MF or Γ). Fairly shortly after its reappearance, the planet will undergo its second station, known as morning station (MS or Φ), and return to direct motion. It will then continue in direct motion for most of its time as a morning star until it catches up with the Sun again and has its last morning appearance (ML or Σ). After its second period of invisibility, its superior conjunction, the star will again have a first evening appearance, EF, and the cycle will begin again, exhibiting the same phenomena in the same order.

4 Conclusion

It becomes clear, when we consider the phenomena of the planets with respect to a background of the fixed stars, that the planets not only wander but display a striking pattern of periodicity. After a sufficient length of time, we see that the same phenomena always occur in the same pattern. This gives rise to the idea that any given phenomenon occurs again and again after some period of time known as the synodic period Δt, and that succeeding phenomena are displaced from the preceding ones along the zodiacal circle by some whole number of circuits plus a difference in longitude known as the synodic arc $\Delta\lambda$. The core of ancient planetary theories—such as those found in the tablets of Babylonian mathematical astronomy or in Ptolemy's *Almagest*—build upon these period-relations by developing mathematical accounts that incorporate the time-intervals of these periods (Δt) and the longitudinal displacement of the synodic phenomena ($\Delta\lambda$), that is, their synodic arc. This may be done by considering the whole synodic cycle or by considering the interval between occurrences of an individual synodic phenomenon, such as one of the appearances, and its displacement along the zodiacal circle. Babylonian astronomers calculated the locations of the synodic phenomenon directly without regard for the motion of the planet between the synodic phenomena and, in rarer cases, where they computed an intermediate position of a planet, they used techniques of interpolation. Greek astronomers, however, tended to work through the intermediary of geometric hypotheses meant to depict continuous motion [see chs 4.2, p. 71; 4.3, p. 95; 4.4, p. 112], on the basis of which they hoped to exhibit some set of synodic phenomena.