a selection of intermediate quantities. This confirms what had been observed already for the planets, namely that the synodic tables are representations of the algorithms for the final quantities, rather than tables containing only final results of these algorithms. Second, while some intermediate quantities are regularly included, there are others that are always omitted; this is especially true for the algorithms for the Lunar Six intervals in system A, to a lesser extent in system B. Hence as representations of algorithms, the lunar tables are remarkably deficient with regard to the (highly complex) subalgorithms for the Lunar Six intervals. Also here the content of the tables may be guided by conventions that do not have any formal or practical justification. Third, since most synodic tables (especially those of system A) contain numerous intermediate quantities it is interesting that there are also template tables containing only a selection of intermediate quantities, but likewise associated with unique dates. As was remarked for the planetary tables, this suggests that lunar template tables are stages in the production of single synodic tables, rather than templates to be used for producing different synodic tables for different years. Some intermediate quantities were first written on template tables, after which a synodic table was compiled by combining data from the template tables, and adding columns for the final quantities.

Mathematical tables in Ptolemy's *Almagest*

Nathan Sidoli

In the *Almagest*, mathematical tables fit into Ptolemy's overall goals of presenting a mathematical structure of the cosmos, presented in an essentially single, structured argument. Indeed, Ptolemy makes a number of explicit assertions that the structure of the tables in the *Almagest* should agree with the overall project of mathematical astronomy and that they should exhibit both the true nature of the phenomena in question and have a suitable correspondence with the mathematical models [1, ex. vol. 1, 208 and 251]. Hence, in order to understand the role of tables in the *Almagest*, we must consider their function in the deductive framework of Ptolemy's mathematical presentation.

It is well known that Greek mathematical texts have quite specific forms that are so conspicuously marked that a knowledgeable reader would be able determine what part of a mathematical argument is being developed after reading just a sentence or two [2]. For example, in Euclid's *Elements* we encounter *theorems* and *problems*, and in the writings of Apollonius and Archimedes we find also *analyses* and *calculations*. Furthermore, many of these types of texts also have subdivisions that have been recognized and discussed by scholars at least as far back as Proclus [3]. In this context, we can understand tables as a *kind of mathematical* text in Ptolemy's argument.

One of Ptolemy's principle strategies, which he probably adopted from his predecessors, is to apply the types of mathematical texts found in the purely theoretical treatises, along with new kinds that seem to have arisen within the applied mathematical tradition, to the investigation of objects that he regarded as mathematical, such as harmonic intervals or heavenly bodies ([4], 93). We find the following types of *mathematical text* in Ptolemy's works:

Description: General application of a mathematical model to a physical situation.

Theorem: Straightforward mathematical proof (which must be interpreted by means of the model).

Problem: Demonstration that a certain construction is possible. (Rare, but ex. in *Planisphere*.)

Analysis: Argument by means of 'givens' that a certain calculation can be carried out. (Metrical resolution.)

Calculation: Use of the model to produce numerical values.

Table: List of numerical values based on the model and generally of use in further calculations.

Algorithm: General description of how we use the values in the table.

Of these, the last four are directly related to the construction and use of tables. Indeed, in the *Almagest*, tables are not found alone but are always in a group of related texts, which I call a 'table nexus.' The table nexus has a distinctive logical structure.

The tables in the *Almagest* are sets of numerical values that correspond to lengths and arcs in the geometric models from which they are derived. At least in principle, they are produced by direct *derivation* from geometric objects with assumed numeric values, or from a given geometric model with specific, astronomically determined, parameters. We can understand the tables themselves as a numerical *representation* of the underlying model, which is geometric. The tables are then used, either by Ptolemy or by his supposed reader, to provide an *evaluation* of specific values that relate both to the underlying model and to the heavenly bodies themselves.

In this way, we can outline the structure of the **table nexus** and relate its parts to the types of mathematical text that Ptolemy employs:

Derivation: A *calculation* or *analysis* that shows that given the model and its parameters, the numbers in the table are determined. (The table is, in fact, not always derived by the method Ptolemy provides ([5], [6]).)

Representation: A *table*, or series of *tables*, that gives a numerical representation of all of the key components of the geometric model. (Each moving part of the model has a separate entry.)

Evaluation: An *algorithm* that describes how the various entries in the tables can be used to calculate phenomena that we actually see.

Ptolemy himself does not explicitly address his methods and he gives no general statements of why he thinks they are valid or effective. Nevertheless, it is possible to construct an argument for the validity and purpose of the tables by considering the overall table nexus. For example, an understanding of how the components of the geometric model move can be derived from studying the table, the argument that the table represents a certain kind of function can be seen from the fact that the different columns refer to specific components of the model, an understanding of what each of the components of the table means can be based on an analysis of the figure, and arguments that the algorithms using the table actually produce apparent motion can be based on an assessment of how the terms of the table relate to the diagram.

We can take the solar theory, set out in book III of the *Almagest* as a general example of how the table nexus should function. We will consider the table nexus for the table of solar anomaly, *Alm.* III 6, which is composed of *Alm.* III 5, 6 and 8.

Alm. III 5 gives a *calculation* for the equation of anomaly (α), given the parameters of the model and a mean normed longitude ($\bar{\kappa}$) of 30°. *Analyses* are then used to show that given any other values for one of $\bar{\kappa}$, κ or α, the other two are also given. This can be taken as a demonstration that the numeric values corresponding to these angles in the model are all determined.

This is then followed by the *table* of solar anomaly, *Alm.* III 6, which sets out corresponding values of $\bar{\kappa}$ and α. Each of the entries in this table are direct representations of an angle in a possible diagram of the model. Hence, the nature of the function depicted by the table can be understood as predicated on the basis of an intuitive understanding of the geometric properties of the model.

After the values of $\bar{\kappa}$, κ and α at epoch are established in *Alm.* III 7, the final section of the table nexus, *Alm.* III 8, gives an *algorithm* for using the values in the table of solar anomaly, along with the table of mean solar motion, *Alm.* III 2, to compute the apparent position of the sun for any time since epoch. There is no attempt to justify the operations of the algorithm; however, since all of the values in the computation are direct representations of objects in the model, which in turn directly represent the celestial bodies, we may take everything that has lead up to the the algorithm as its justification.

As usual in the Almagest, we can read the solar theory as a sort of model; however, because of its simplicity, none of the other theories can be patterned on it exactly. In the solar theory, each of the entries in the table can be related to a specific object in the model. Hence, one could argue that the justification for the algorithm, *Alm.* III 8, can be read off the model itself.

In the more involved theories, where Ptolemy tabulates functions of more than one variable (*Alm.* V 8 and XI 11), individual entries in the tables will correspond to multiple positions of the model, and interpolation will be used for the intervening positions. Nevertheless, the justification for the final algorithm can still be made by referring to the geometric features of the model, although Ptolemy does not, himself, do this. Perhaps he thought the derivation was sufficient.

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Astronomical tables in second millennium Sanskrit sources

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One important strand in the study of numerical tables in Sanskrit sources comprises of the so-called *koṣṭhakas* or *sāraṇīs*, astronomical tables that rose in popularity from the tenth century onwards. It has been argued that the prominence of these types of tables in Sanskrit astronomy was linked to Islamic inspiration particularly through the influence of the *zīj* compositions ([1], 41; [4]). Unlike the standard Sanskrit astronomical formats that contained enumerations of important parameters and fundamental algorithms composed in verse, these works used spatial arrangement, ruled rows and columns, alignment, and accompanying explanatory prose to present precomputed data intended for practical astronomical application.

The earliest text of this type that we know of is a work authored by the Indian astronomer Durlabha who was working at Multān in Sind. We know of this work not directly, but through descriptions of it that are given by al-Bīrūnī in his work *India*. The epoch of Durlabha's tables are given as Śaka 854 (= 932 CE) and in subsequent centuries dozens of tables were produced in India. The tables that have been catalogued so far by modern efforts come predominantly from North Western India, with a handful from Kāśī and Bengal. This geographical concentration further evidences the impact that Islamic sources had on the Indian tradition. Tables do exist from other regions of India, however, their content reveals them to be largely independent of Islamic inspiration. This is particularly pertinent to the tables found in South India.

In addition to new works, many authors found their inspiration in key astronomical texts from previous times. Extracting the base parameters which had been expressed in prose, they cast and developed the relevant astronomical data in a tabular format. For example, Bhāskara II's (b. 1114) work, the *Karaṇakutūhala* was recast as tables by Nāgadatta and called the *Brahmatulyasāraṇī* (or often the *Karaṇakutūhalasāraṇī*) with an epoch of 23 February 1183. Other similar instances abound: a tabular version of Brahmagupta's *Khaṇḍakhādyaka*, entitled the *Khaṇḍakhādyakasāraṇī* was written (and exists in incomplete manuscript form), the *Sūryasiddhāntasāriṇī* based on the *Sūryasiddhānta* was prepared, and the *Grahalāghavasāraṇī* was composed by Nīlakaṇṭha in 1630 based on Gaṇeśa's *Grahalāghava* (1520), as well as another work by the same name by Porema in 1656.

To rationalise the corpus, astronomical tables have been classified into several distinct types [2]). These include Planetary tables, Pañcāṅgas or Calendrical tables, Eclipse tables, and Geographical tables. The parameters of the tables can generally be further classified, both with respect to their base parameters and their