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Reviews

Edited by Adrian Rice and Antoni Malet

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Classics in the History of Greek Mathematics

Edited by Jean Christianidis. Dordrecht (Kluwer Academic). 2004. ISBN 14-0200-081-2. x + 461 pp. $187

This book gathers together a number of important papers on the history of Greek mathematics and organizes them according to the main currents of late 20th-century historiography in the field. In fact, only 10% of the articles are from the first half of the century, while over 60% are from the last quarter. The articles are grouped thematically into six sections, each of which is edited and introduced by a major historian of Greek mathematics. A few of these introductions are useful discussions of the historiographic issues that go beyond a simple explanation of the editor's selection of articles. The story this book tells is coherent, but not exhaustive. The view of Greek mathematics presented is quite traditional: only pure mathematics is considered and only from the perspective of internal, intellectual history.

The first section, edited by Hans-Joachim Waschkeis, treats the little-known but much-discussed origins of Greek mathematics. Waschkeis's introduction relates the discussions about the beginnings of Greek mathematics to some of the evidence for the mathematical practices of nearby, more ancient cultures. Three papers follow this. Mittlestraß (1962) argues for the view that Greek mathematics begins with the practice of stating general propositions, and is trusting enough of the late ancient sources to attribute this development to the Ionian sages, such as Thales. Szabó (1956) takes mathematics to be proof, and seeks to show that it arose much latter in response to, and by adopting the methods of, the Eleatic philosophy founded by Parmenides. Knorr (1981), on the other hand, argues that the practices of Greek mathematics were produced for purely internal reasons, conditioned by the needs of working mathematicians. This leads him, however, to argue that the systematization of mathematical knowledge took place only as late as the 3rd century. The fact that three competent scholars can arrive at such different interpretations of the context and the timeline of the development of early Greek mathematics tells us a good deal about the state of our sources for this period. There are good reasons that this topic is no longer the favorite subject of historians of Greek mathematics.

Reviel Netz edits a section on Greek geometry. In the first paper, Knorr (1983) convincingly argues against Zeuthen's claim that we should take constructions in Greek mathematics as proofs of existence. This paper discusses a number of ways in which existence actually is dealt with in Greek mathematics and suggests some of the many functions of construction; nevertheless, we are still a long way from a complete understanding of the role of construction in Greek geometry. Saito (1985) gives a detailed explanation of how the application-of-area theorems of *Elements* II and VI can be understood as motivated by the needs of geometric research in conic theory, without the need to appeal to an algebraic tradition for which there is little or no evidence. Finally, Lloyd (1993) shows that an obscure passage in Plato's *Meno* can be read as intentionally obscure in order to make a philosophical point; that it does not have to have a definite mathematical interpretation. This would explain why there have been so many conflicting interpretations put forward for the mathematics. Along with giving a good sense for Greek geometry, this section touches on some of the core issues of late-20th-century historiography of Greek mathematics: the role of construction, application of area methods, and the use of philosophical sources as historical documents.

A section on proportion theory and incommensurables is edited by Ken Saito. Due to the subject matter, the papers share some overlap with the first section in terms of the methodology and style of scholarship. It begins with a good example of the old style of reconstructive history by Becker (1933). He uses the *Elements* and comments in Aristotle to develop a proportion theory that he attributes to Eudoxus. Von Fritz (1954) provides a mathematically appealing, although purely speculative, approach to the discovery of incommensurables using constructive methods. Freudenthal (1966), however, calls into question the very idea that there was a "crisis" of incommensurability in Greek mathematics. The final paper, by Knorr (2001), is a posthumous publication of a talk given in 1975. Here, Knorr points out a number of ways in which the concerns of the 20th-century mathematical research community have shaped the preconceptions of historians of mathematics, often distorting our views of past mathematical concerns and practices.

The section on algebra is edited by Jacques Sesiano, whose introduction is primarily a summary of our current understanding of Diophantus's *Arithmetic*. This is followed by a paper by Vogel (1933) discussing methods for solving quadratic equations found in the Babylonian sources. An important paper by Toomer (1984) points out the wealth of work to be done on the history of Greek mathematics by examining Arabic sources. Finally, a chapter from Heath's book (1964) on Diophantus summarizes the mathematical methods found in the *Arithmetic*.

As well as editing the entire book, Jean Christianidis contributed a short section on fractions in Greek mathematics. His introduction provides a discussion of some passages in Diophantus and generally establishes that the *Arithmetic* is relevant to the issues, although typically idiosyncratic. Knorr (1982) gives a good overview of the subject and shows the continuity of Greek methods with the Egyptian tradition. Fowler (1992) presents a brief version of his argument that the Greek system of fractions was quite different from ours and, in particular, did not admit the arithmetic operations on common fractions.

The section edited by Sabetai Unguru is titled "Methodological Issues in the Historiography of Greek Mathematics," but it is only concerned with the debate occasioned by his 1975 paper "On the Need to Rewrite the History of Greek Mathematics." It is quite misleading to present the controversy about the so-called geometric algebra as the only historiographically significant event to have affected the field in the 20th century. Some examples of methods not addressed by Unguru are investigations of Arabic sources to shed light on the text histories of the Greek texts and to supply translations of lost texts, studies of Eastern sources to develop our still largely unsatisfactory picture of the transmission of Eastern mathematical sciences into the Greek world, examinations of applied mathematics texts to flesh out our understanding of mathematical methods and practices, and use of nonmathematical texts to help contextualize mathematics and its practitioners. Indeed, every other section of this book presents interesting methodological issues, many of which are not addressed by Unguru's 1975 paper.

Finally, the absence of any section on applied mathematics is perplexing, given the number and importance of applied mathematical texts and the fact that they were, on the whole, written by the same small group of individuals who wrote the pure mathematical texts. There are entire episodes of Greek mathematics and innovative ways of conceptualizing mathematical objects and relations that are only found in the applied texts. Moreover, 20th-century scholarship in this area has been particularly rich. Hence, this absence is disappointing. On the whole, however, this book brings together a wealth of important papers on Greek mathematics and gives a good overview of the progress in a number of major areas of research during the 20th century.

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Histoire des nombres complexes : Entre algèbre et géométrie
By Dominique Flament. Paris (CNRS Editions). 2003. ISBN 2-271-06128-8. 501 pp. EUR 39

A capsule outline of the history of complex numbers might go something like this: In Europe in the early 16th century, the appearance of the square root of a negative number in a calculation signified merely that the desired problem was impossible. As Cardano said "quaestio ipsa est falsa, nec esse potest quod proponitur." It was Bombelli in his Algebra of 1572 [see Bombelli, 1579] who first made a systematic use of these strange quantities to arrive at real solutions of certain cubic equations. Descartes introduced the word imaginary when an equation of degree n had fewer than n roots, because he says we can imagine the missing roots. Euler a century later called these impossible quantities imaginary numbers, but pointed out that even though they are impossible, yet nothing stops us from making calculations with them and deriving many useful results. And indeed, the 18th century saw a great flourishing of research involving trigonometric functions, exponentials, logarithms, and infinite series, in all of which the use of imaginary numbers played an essential role, even though no one had explained what they really were, nor had rigorously justified their use. It was only the geometrical representations of Wessel and Argand around the beginning of the 19th century, backed by the authority of Gauss, and the later developments of Cauchy and Hamilton that put the theory on a solid logical foundation. Apparently we owe the terminology complex number to Gauss, who introduced "numeros integros complexos" in his theory of biquadratic residues.

The present text is a thorough study of these developments from the 16th to the 19th century, describing the work of all the major and some minor figures in this story, and documenting the nonlinear progression of concepts that began in total mystery and gradually developed into a tool that we take for granted today. The first chapter (100 pp.) introduces imaginary numbers and shows their multiple uses in the 16th to 18th centuries. Chapter Two (150 pp.) analyzes in detail the work of Wessel, Argand, and Warren and Mourey, which gave a logical basis to the theory via geometry. Chapter Three (50 pp.) describes the solidifying work of Gauss and Cauchy, while the long last chapter (120 pp.) is devoted to Hamilton and the English school of algebra. As the author's goal is to describe the mathematical thinking as it was at the time, he says he will stay as close as possible to the original texts. There are numerous citations throughout the book and an extensive bibliography. However, the many misprints, errors, and inconsistencies make for difficult reading, as I will illustrate with a few episodes from my encounter with the book.

I know of no other book-length study of this subject, but the excellent article of Loria [1917]—cited incorrectly in footnote 5, and with a spelling error in the bibliography—could be taken as a template for this study. Subsequent footnotes refer to pages 47, 48, 51, and 54 of this article, which appears on pp. 101–121 of the journal! It turns out that *Scientia* printed the original article in Italian, together with a French translation in a supplement, with separate pagination, and Flament is referring to the corresponding pages of the French translation.

On p. 20, Flament cites Cardano, *Ars magna*, Ch. XI. However, the text given (in French) is not an exact translation. It is an abbreviated paraphrase. A footnote points to [Libri, 1840, p. 254]. There is nothing relevant on that page, or nearby, in Libri, though I could find the Latin text of Cardano starting on p. 437. The correct reference would be [Cardano, 1570, 58], or the excellent English translation by Witmer [Cardano, 1968, 96], which unfortunately Flament does not list in his bibliography.