Apollonius's *Conics*

The Greek and Arabic Traditions

By Nathan Sidoli*

Apollonius de Perge. *Coniques.* Volume 1, *Part 1: Livre I: Commentaire historique et mathématique, edition et tradition du texte arabe.* Volume 1, Part 2: *Livre I: Édition et traduction du text grec.* Edited by **Roshdi Rashed**, **Micheline Decorps-Foulquier**, and **Michel Federspiel**. xiv + 666 pp., lxxxiv + 275 pp. Berlin/New York: Walter de Gruyter, 2008. $207 (cloth).

Apollonius de Perge. *Coniques.* Volume 2, Part 1: *Livres II et III: Commentaire historique et mathématique, edition et tradition du texte arabe.* Edited by **Roshdi Rashed**. xiv + 682 pp. Berlin/New York: Walter de Gruyter, 2010. $140 (cloth).

Apollonius de Perge. *Coniques.* Volume 2, Part 2: *Livre IV: Commentaire historique et mathématique, edition et tradition du texte arabe.* Edited by **Roshdi Rashed**. x + 319 pp. Berlin/New York: Walter de Gruyter, 2009. $140 (cloth).

Apollonius de Perge. *Coniques.* Volume 2, Part 3: *Livres II–IV: Édition et traduction du text grec*. Edited by **Micheline Decorps-Foulquier** and **Michel Federspiel**. xxx + 506 pp. Berlin/New York: Walter de Gruyter, 2010. $140 (cloth).

Apollonius de Perge. *Coniques.* Volume 3: *Livre V: Commentaire historique et mathématique, edition et tradition du texte arabe*. Edited by **Roshdi Rashed**. xi + 550 pp. Berlin/New York: Walter de Gruyter, 2008. $140 (cloth).

Apollonius de Perge. *Coniques.* Volume 4: *Livres VI et VII*: *Commentaire historique et mathématique, edition et tradition du texte arabe*. Edited by **Roshdi Rashed**. x + 572 pp. Berlin/New York: Walter de Gruyter, 2009. $137 (cloth).

This massive project furnishes us with a new Greek text of the first four books of Apollonius's *Conics* (ca. third century B.C.E.), in the edition made by Eutocius (ca. fifth

* School for International Liberal Studies, Waseda University, 1-6-1 Nishiwaseda, Shinjuku, Tokyo, Japan.

Isis, 2011, 102:537–542

0021-1753/2011/10203-0012$10.00

century C.E.), the first complete edition of an Arabic version, produced by a group of scholars centered around Baghdad, including the Banū Mūsā and Thābit ibn Qurra (ca. ninth century C.E.), editions of a number of related Arabic texts, and French translations of all of the texts, with mathematical and philological commentaries. The Arabic text and translation, as well as the mathematical commentaries, have been handled by Roshdi Rashed, while the Greek text was edited by Micheline Decorps-Foulquier and translated by Michel Federspiel. All of the diagrams were produced by Françoise Rashed.

The mathematical work of Apollonius has recently been the subject of considerable scholarship, and this project is a welcome addition to these endeavors. Having a complete text of the Arabic version, and an up-to-date Greek version, will be useful for those of us who are interested in studying Apollonius's mathematical methods. In terms of inspiring original mathematical work, in both the medieval Islamic world and the early modern Christian world, Apollonius ranks with Archimedes as one of the masters whose methods supplied a foundation for creative work, with Apollonius receiving somewhat more attention from medieval mathematicians and Archimedes somewhat more from early modern mathematicians. For all of these mathematicians, however—even down to the time of Newton and Halley—the *Conics* remained a paradigmatic example of pure mathematics, containing useful methods, significant results, and intricately developed theories.

For all of this, our entire knowledge of the *Conics*—indeed, the very fact that the text survived into the medieval and early modern periods—is the result of just a few Greek manuscripts that we can infer were in existence in the ninth century, all of which are now lost.

Our oldest Greek manuscript, Vat. gr. 206—on which all of the other, mostly early modern, Greek manuscripts depend (Vol. 1, Pt. 2, p. xxxvi)—is a Byzantine copy made around the end of the twelfth century C.E. Although it is uncertain, it is reasonable to suppose that this manuscript was made on the basis of a text produced in Constantinople sometime during the long ninth century, a time of revival of classical scholarship in the eastern empire. The tradition preserved in this manuscript is the edition of the first four books of Apollonius's treatise made by Eutocius of Ascalon. It has long been clear that Eutocius introduced changes to the texts, but until fairly recently there has been little direct study of his editorial methods.[1] One of the greatest hopes for an Arabic edition of these first four books was that it would shed more light on Eutocius's editorial decisions and practices.

Unfortunately, the situation is not so simple. The "official" history of the Arabic translation (Vol. 1, Pt. 1, pp. 25–28), as told in the preface to the Arabic edition (Vol. 1, Pt. 1, pp. 501–507), can be recounted briefly. Around the middle of the ninth century, the Banū Mūsā came into possession of a manuscript of the *Conics* that contained Books 1–7 in a version apparently descending from Apollonius's original work, but initially they could not understand it owing to textual corruption and the difficulty of the subject matter. Since the Banū Mūsā were not scholars of Greek, however, it remains unclear who studied

[1] E.g., Wilbur Knorr used reconstructive methods to argue that Eutocius added *Conics* 2.4 to the original text, and Micheline Decorps-Foulquier used text-critical methods to analyze Eutocius's scholarship. See Wilbur Knorr, "The Hyperbola-Construction in the *Conics*, Book II: Ancient Variations on a Theorem of Apollonius," *Centaurus*, 1982, *25*:253–291; and Micheline Decorps-Foulquier, "Eutocius d'Ascalon éditeur du traité des *Coniques* d'Apollonius de Pergé et l'exigence de clarté: Un exemple des pratiques exégétiques et critiques des héritiers de la science alexandrine," in *Sciences exactes et appliquées à Alexandrie*, ed. Gilbert Argoud and Jean-Yves Guillaumin (Saint-Étienne: Publications Univ. Saint-Étienne, 1998), pp. 87–101.

the manuscript itself and to what extent and in what manner this work was carried out. Nevertheless, an initial breakthrough in the project occurred when al-Ḥasan ibn Mūsā established the geometrical properties of the ellipse by studying the section of a cylinder and relating this to the section of a cone. Another major advance came when Aḥmad ibn Mūsā took up a post in Damascus. During his stay in Syria, he found a copy of the Eutocius edition of the first four books and was able to study this carefully, working through all the details and producing some kind of commentary. When he returned to Baghdad, he finished his mathematical study of the remaining three books. We are, then, told that Hilāl ibn Abī Hilāl translated the first four books and Thābit ibn Qurra translated the last three.

So much for the official narrative. As Rashed shows, however, there is evidence that at least one other version was in circulation in the tenth century (Vol. 1, Pt. 1, pp. 29–44). According to al-Khāzin, there was a complete version by Thābit ibn Qurra and a second by Hilāl ibn Abī Hilāl and Isḥāq ibn Ḥunayn, the son of the famous medical translator. Moreover, there are discussions of conic sections in a number of authors that contain technical expressions not used in the extant version and that may be evidence for earlier stages in the transmission. Whether or not we agree with Rashed that there were two different translations from the Greek, it seems clear that there were different Arabic versions of the text in use. Nevertheless, the extant manuscripts of the Arabic text all contain the same text—that attributed to the research project under the direction, and funding, of the Banū Mūsā.

We can regard the text that has come down to us as the product of what Dmitri Gutas has called a translation complex—a group of scholars working together, combining their respective scientific, mathematical, and philological strengths to address the problem of reading and understanding difficult texts, preserved in one or two, often paltry, copies, and rendering them into a language whose technical vocabulary was being established by the very efforts of these complexes.[2] A by-product of this mode of scholarship was that productive, original work was often done using texts that are different from the versions that have come down to us, preserving technical idioms that appear to have been developed in transitional phases of the complex's activity. The Arabic text of the *Conics* that survives in our manuscript sources is the product of one of the most active of these translation complexes, that centered around the Banū Mūsā. When we consider the difficulties involved in reading the manuscript sources, in understanding their mathematical content, and in incorporating this knowledge into an ever-growing body of active research, there can be little doubt that the scholars involved in the production of the Arabic *Conics* extended their considerable talents in producing a mathematically coherent text, probably often at the expense of preserving exactly what they found in the Greek source.

I have provided this background to lend support to my claim that we are no longer in possession of any text of the *Conics* that Apollonius wrote. What we have now are a Greek edition of this text, made by Eutocius, and an Arabic version, made by a group of mathematical scholars; both of these recensions have been altered in various, now often obscure, ways by the scholars who produced and transmitted them. This should be weighed against Rashed's general claim that we can use the Arabic version of the text as a source for determining the text that Apollonius himself wrote. His procedure for this is quite incredible. He supposes that we can simply use the current Arabic version to

[2] Dmitri Gutas, *Greek Thought, Arabic Culture* (London: Routledge, 1999), pp. 141–150.

establish the characteristics of the Greek manuscript on the basis of which one (or both?) of the original translations was made; he calls this text M.[3] He then proceeds to compare the text of *Conics* 4 in the Eutocius edition with that in the Arabic version and shows, for example, that 4.1–43 is considerably more developed, from a mathematical perspective, in the Arabic version, whereas the two versions become much closer from 4.44 to the end of the book. On this basis, Rashed argues that the Arabic text can be regarded, unproblematically, as evidence for the text that Apollonius wrote and, furthermore, that the deficiencies of the Greek can be attributed to a now lost Greek interpolation that served as Eutocius's source (Vol. 1, Pt. 1, pp. 12–21).[4] Given the current state of the evidence, however, we cannot be certain that these improvements were not due to the scholars of the Banū Mūsā circle, many of whom had the mathematical competence to make such changes.[5] Hence, while the Arabic text of *Conics* 4 may make more mathematical sense, it is hardly possible to be certain that this is due to any superiority in the readings of the Greek manuscript that was in Baghdad in the ninth century, and it is certainly not possible to say anything definitive about how closely this manuscript, now lost, was related to whatever Apollonius wrote.

We can consider these questions again from the perspective of some concerns about Eutocius's text raised by historians studying the Greek tradition. For example, Heike Sefrin-Weis has pointed out that the version of the construction of a hyperbola given a diameter and latus rectum, *Conics* 1.54–55, that Pappus read in the pre-Eutocian version of the *Conics* available to him was apparently presented in analytical form (see *Collection* 4, prop. 33), whereas Eutocius's text includes only a synthetic problem.[6] The extant Arabic text, however, agrees with the Eutocius edition in preserving no analytical treatment (Vol. 1, Pt. 1, pp. 451–461). Nevertheless, as Rashed has shown (Vol. 1, Pt. 1, p. 32), both al-Khayyām and Abū al-Jūd read a version of this part of the text that had four extra theorems between our 1.54–55 and theirs (1.58–59). Two or three of these extra propositions could well have been analyses of the problems that conclude *Conics* 1, on analogy with the analyses of the problems that conclude *Conics* 2 in both the extant versions. If this was, indeed, the case, however, these propositions are lost to us now, and the current Arabic version has been brought into structural conformity with the Eutocius edition.

As another example, we may consider Wilbur Knorr's claim that *Conics* 2.4—which gives the construction of a hyperbola through a given point between two given asymp-

[3] Rashed acknowledges that the use of M to denote both a lost Greek manuscript and the whole Arabic tradition is strange, but this does not prevent him from using this misleading construct.

[4] Rashed displays a similar sort of philological audacity in his decision to give a privileged place to the manuscript transcribed by Ibn al-Haytham, Aya Sofia 2762. He does this by simply claiming, with no philological argument, that Aya Sofia 2762 was copied from a lost archetype prior to the lost archetype from which the other manuscripts were copied, both of which are simply called x (Vol. 1, Pt. 1, p. 232). In both cases, what we require is some sort of argument for why we should believe that the superior readings—of the Arabic tradition over the Greek, and of Aya Sofia 2762 over the other Arabic manuscripts—are due to the fact that they are *closer to the source*, as opposed to improvements introduced by the mathematical scholars who produced them.

[5] E.g., when we compare the Greek versions of Theodosius's *Spherics* or Aristarchus's *On the Sizes and Distances of the Sun and Moon* with Arabic versions attributed to Thābit, we find that the majority of the differences are improvements in the mathematical and logical details that can be attributed to Thābit's editorial work. See, e.g., J. L. Berggren and Nathan Sidoli, "Aristarchus's *On the Sizes and Distances of the Sun and the Moon*: Greek and Arabic Texts," *Archive for History of Exact Sciences*, 2007, *61*:213–254; and Sidoli and Takanori Kusuba,"Naṣīr al-Dīn al-Ṭūsī's revision of Theodosius's *Spherics*," *Suhayl*, 2008, *8*:9–46.

[6] See Heike Sefrin-Weis, *Pappus of Alexandria: Book 4 of the* Collection (London: Springer, 2010), pp. 151 n 10, 283.

totes—was interpolated into the text by Eutocius.[7] On the basis of a direct statement by Eutocius in his *Commentary on Archimedes' Sphere and Cylinder*, some propositions in Pappus, and some mathematical deficiencies in the current Greek version of this proposition, Knorr argued that the original version of the text contained no such construction. The Arabic version, however, contains essentially the same approach as the Greek, but the mathematical presentation has been improved. As usual, Rashed claims that this improved version is simply the text that Apollonius wrote (Vol. 2, Pt. 1, pp. 11–16). His best argument that Apollonius's text contained some form of this construction is the fact that it is used five times in *Conics* 5 (5.51–52, 55, 58–59), which is probably decisive. Again, however, even once we acknowledge that some version of this construction was probably written by Apollonius, it is not possible to be sure that this original version is still found in our sources. The confusion of the Greek sources seems to indicate that, even prior to Eutocius's activity, there were multiple versions of the text; and, again, we have no independent way of determining that the mathematically superior features of the Arabic version were based on a Greek manuscript and not due to Thābit or someone else. Again, the overall impression is that the final Arabic version was prepared so as generally to agree with the Eutocius edition and then corrected from a mathematical and logical perspective.

We may take the *Conics* as an example to illustrate the general claim that an investigation of the text history is essential to writing the history of ancient and medieval sciences. For the majority of the major figures of these periods we have no material evidence, no archives, and almost no direct sources. Thus, the broad outline of our understanding of the sciences of these periods must be based on the canonical sources that were transmitted through the manuscript tradition. There is now undeniable evidence, however, that this transmission has not left these texts unchanged. The methods of classical philology, which were developed in the early modern period and, largely in the nineteenth and twentieth centuries, made possible the critical establishment of our primary sources; however, are based on a set of assumptions that derive from studies of religious and literary texts, which cannot simply be transferred to scientific texts. For religious and literary texts, the actual words of the original author were always considered inviolable, and no editor—ancient, medieval, or modern—could justifiably see the task at hand as anything other than an attempt to sort through the available evidence and present what was most likely to have been the expression of the author. Before the development of the techniques of critical philology, however, the mathematical and scientific texts were transmitted as part of a living tradition, often re-edited and reworked by scholars who were themselves practitioners of the fields the texts treated. Hence, the core object of transmission was often seen to be primarily the ideas themselves—and only secondarily the specific words in which they were presented. For this reason, the transmission of scientific texts was beset by two conflicting tendencies: on the one hand, the operative belief that the canonical texts were the products of genius and that it was the role of scholarship to preserve that genius; and, on the other hand, the realization that the texts had become corrupt over time but that the principles of the science in question could be used to recover the ideas and restore the argument. For these reasons, the texts of ancient and medieval science must be regarded as historically contingent objects, in some ways created by the process of transmission itself. Hence, it is not possible to study the history of the ancient sciences seriously without also investigating the various medieval transmissions.

[7] See note 2, above.

The major, and lasting, contribution of the project under review is the texts and translations themselves, which will serve scholars for many years to come. The work on the Greek text has been carried out to the highest standard of scholarship and is the culmination of years of research on this work.[8] The edition of the Arabic text is also very good, although Rashed has a preference for Ibn al-Haytham's copy for reasons that are not fully argued.[9]

There are a couple of missed opportunities in this project. There is no Greek-Arabic index, which would have been very useful for scholars working on the transmission of the Greco-Arabic sciences in the medieval period. Furthermore, all of the diagrams have been redrawn according to modern standards of linear perspective and the analytical definitions of the curves, and the same figures have been used for the texts and the translations. Both the Greek and the Arabic manuscripts of the *Conics*, however, all agree on the general practice of depicting the conic sections with arcs of circles. This means that the diagrams for the text bear only a loose relation to the manuscript tradition; they are not a visual representation of what we find in the manuscripts, nor do they contain any critical notes of the manuscript variants.[10] Hence, if we wish to study the diagrams that medieval mathematicians used to understand conic theory, we must still consult the manuscripts themselves. Since there are repeated diagrams for both text and translation, better practice would have been to produce a medieval style diagram for the text and a modern style diagram for the translation.

On the whole, this project is a highly valuable contribution to scholarship on the ancient and medieval exact sciences, and the volumes will be a necessary acquisition for any reasonably complete library.

[8] See esp. Micheline Decorps-Foulquier, *Recherches sur les* Coniques *d'Apollonios de Pergé et leurs commentateurs grecs: Histoire de la transmission des Livres I–IV* (Paris: Klincksieck,1999); and a series of papers by Michel Federspiel that appeared in *Revue des Etudes Grecques*, 1994–2002 (see Vol. 1, Pt. 2, of the work under review for full references).

[9] Rashed never refutes G. J. Toomer's argument that we should not attribute the superior readings of the al-Haytham (Toomer's H) autograph to the archetype. See G. J. Toomer, *Apollonius,* Conics, *Book V to VII: The Arabic Translation of the Lost Greek Original in the Version of the Banū Mūsa* (New York: Springer, 1990), note 1, pp. lxxxix–xc.

[10] This oversight is especially strange, since Decorps-Foulquier has written an interesting paper discussing what we can learn about the Eutocius edition from a study of diagrams in the manuscripts. See Micheline Decorps-Foulquier, "Sur les figures du traité des *Coniques* d'Apollonius de Pergé edité par Eutocius d'Ascalon," *Revue d'Histoire des Mathématiques*, 1999, *5*:61–82.