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Book Review

Domninus of Larissa, *Encheiridon and Spurious Works. Introduction, Critical Text, English Translation, and Commentary by Peter Riedlberger.*

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This book, which is based on Riedlberger's Ph.D. thesis at Ludwig-Maximilians-Universität München, is a welcome contribution to our understanding of the mathematical scholarship of late antiquity, especially that of the Platonic school of Athens.

The book is organized as follows: an introduction to Domninus of Larissa's intellectual context, focusing on the mathematical scholarship of the imperial and late-ancient Platonists; three chapters on Domninus, reappraising the evidence and making a sound case that Domninus practiced mathematical scholarship in much the same vein as the other Athenian Platonists; prolegomena, critical editions and English translations of three short texts usually associated with Domninus; commentaries on these texts; indexes and other apparatus.

The introduction includes a useful summary of what we know about the mathematical interests of the late-ancient Athenian Platonists and a reassessment of Domninus' place in this context. Riedlberger convincingly argues that the previous assessment of Domninus as a sort of maverick among his peers who placed more emphasis on deductive mathematics than philosophy is unwarranted by the evidence. Instead, Domninus, an upperclass Syrian who studied in Athens and then returned to his homeland, appears as a typical late-ancient Platonist—a man who had an interest in mathematics as part of philosophy, who practiced mathematical scholarship in a fairly philosophical mode, but who had no special competence in the mathematical sciences.

The core of this book are critical editions and translations of the following three texts: (1) Domninus' *An Educational Handbook to the Arithmetical Introduction* (*Encheiridion*), and two shorter works that Riedlberger cogently argues should be regarded as anonymous, (2) *How to Separate a Ratio* (*How-to*) and (3) *Rough Notes, or Scholia, on Nicomachus.*

The *Encheiridion* is series of definitions related to numbers and number theory, which Riedlberger shows can be usefully compared to the work of Euclid, Theon of Smyrna and Nicomachus and was most likely composed for educational purposes. In this regard it is similar to the Heronian *Definitions*. From the perspective of the history of mathematics this is a meager work. The goal of the text is to give names and descriptions to various types of numbers and their relations, and to try to organize these names according to some philosophical, as opposed to mathematical, scheme. Riedlberger aptly characterizes the treatise as motivated by an "overwhelming yearning for classification and regularity, while sacrificing logic" (p. 182). A noteworthy indication of this approach is the use of the word "theory" (*theōria*). Domninus appears to use "theory" to mean a way of naming and arranging (sec. 14, 19, 31–35; pp. 153–154), which is his goal in this work. While this text may have been helpful in an educational context, it is not really useful as mathematics. Rather, it is a representative example of a form of mathematical scholarship much in vogue with philosophers, which essentially ends where mathematicians usually begin—that is, with definitions.

The *How-to*, although shorter, is more interesting from the perspective of the history of mathematics. This text—along with a passage from Theon of Alexandria's *Commentary on the Almagest*, passages from Eutocius' commentaries on Archimedes and Apollonius, and a passage from the anonymous introduction to the *Almagest*—belongs to a tradition of late-ancient texts that sought to produce a numerical understanding of ratio and the operations of ratio *composition* and *separation*—mathematically, but not always conceptually, equivalent to $(a:b)\times(c:d)$ and $(a:b)\div(c:d)$. The goal of the text is to show how working with ratios as having, or being, a certain "value" (*pēlikotēs*) enables one to treat ratio separation computationally. The use of values to handle ratios works in two ways: (1) a ratio can be treated as a value, so that multiplication of two values "makes" the value of a composed ratio (sec. 3), and (2) the value of a ratio can be used to "make" the ratio, expressed as two numbers, given any number (sec. 16). The text ends with a general description of the procedure for separation that mixes the language of givens with the language of operations in a way that was made common by Ptolemy (cf. 217; for example, *Alm.* H.I.240, H.II.426–427).

All of this late-ancient work on a numerical theory of ratios was clearly influenced by reading Ptolemy, and probably Heron. Although historians of mathematics have a tendency to overlook the tradition of mixing computational methods (*logistikē*) with geometric methods (geometry, ratio manipulation, givens terminology) that developed in the Hellenistic and imperial periods as being exact science and not "pure" mathematics, it is important for us to remember that this was not a distinction that would have been meaningful to ancient mathematicians. Curriculums of Greek mathematics could encompass elementary geometry, number theory, harmonics, spherics (a discipline that itself covers aspects of what we separate into mathematics and astronomy), geometric analysis, mechanics, computation, astronomy, and so on. It is clear from Theon of Alexandria's *Commentary on the Almagest* that reading Ptolemy's work was taken as a good opportunity to teach students some fundamentals of computation. The *How-to* is a text that was produced in an era in which the strict separation of mathematical practices into those which treat discrete magnitudes and those which treat continuous magnitudes was no longer carefully maintained—if, indeed, it ever had been maintained to the extent that historians of Greek mathematics seem sometimes to believe.

Rough Notes, or Scholia, on Nicomachus appear to be scholia to Nicomachus's *Introduction to Arithmetic* that have become separated from the text. They are perhaps notes to a lecture based on a reading of the text accompanied with exposition. Although this work is not of great intrinsic interest, it is curious that it survived in the manuscript tradition and it may help us understand philosophical education in the late-ancient or medieval periods. As Riedlberger's discussion makes clear, it is not really possible to date this text with any certainty, and it may be much later than the other two.

The level of scholarship in this book is very high. I have not checked the text against the manuscripts, but the translation is sound, and although I, naturally, do not agree with all of Riedlberger's translation choices, he usually gives his reasoning for a particular choice in the commentaries. The commentaries cover philological, philosophical and mathematical issues. In some cases, I wished these different layers of commentary had been separated more clearly, so that I could skip directly to the one in which I was interested. The mathematical commentary often takes its point of departure from modern conceptions of mathematics, an issue to which I will return below. The philological commentaries are thorough, carefully evaluating all previous suggestions for a particular passage.

I have two fairly modest criticisms along historiographic lines. The first is that the discussion of mathematical methods is often based on how the Greek approach differs from modern conceptions, so that instead of characterizing the text positively, in terms of what is actually done, Riedlberger describes the text negatively, in terms of what he thinks should have been done. A few examples may help make my point. In discussing the operation of ratio separation, Riedlberger tells us that we would characterize this as a division of fractions. He says, "if we were to divide $a : b$ by $c : d$, we would multiply by the reciprocal of $c : d$ and immediately write down the result: $\frac{a \times d}{b \times c}$." He then goes on to observe that the text does not state "this in a straightforward procedure" (p. 207). This description of the text is rather misleading. Indeed, the fact that the text does not state the procedure that he expects—although it does, in fact, state straightforward procedures—should give him pause to reconsider his expectations. Although ratio separation is mathematically equivalent to division with common fractions, this is not how it functions for Greek mathematicians, because ratio manipulation is not "a substitute for fraction handling"—despite Riedlberger's claim to the contrary (p. 202). As the papyrological evidence makes clear, the Greeks handled fractions using the Egyptian system of unit fractions, or in later periods the Babylonian sexagesimal system for the astral sciences. Both of these methods of computing with fractions were suitable for the needs of ancient practitioners. It is not the case that there is, on the one hand, what we call common fractions, and then, on the other hand, these "other systems," with which the Greeks "replaced" them (p. 199). All three of these are different ways of handling and computing with fractional parts; unit fractions and sexagesimal fractions *were the fractions* that Greek mathematicians used. The question that the *How-to* addresses is how to use ratio operations directly to do calculations, where these calculations may themselves involve operations with fractions, carried out in the usual ancient way—as, indeed, occurs in the text.

Another example comes from Riedlberger's treatment of the "rule of three." Here, we are told that the author of the *How-to* must transform (1) $c : d :: x : b$ to (2) $b : x :: d : c$ in order to see that x can be calculated by (3) $x = \frac{b \times c}{d}$, because he "mechanically uses the rule of three with the searched-for term as the consequent" (p. 208). The implication is that it should have been obvious to late-ancient students that (3) follows directly from (1), as it is for us. In pre-symbolic mathematics, however, algorithms are statements such as "I multiply the second number by the third, and divide by the first" (p. 125), and in order to work they must be applied *as they are stated*. Of course, one eventually develops an intuitive sense of how the ratios can be inverted, and so on, but the *How-to* is an educational text, so all of the steps must be spelled out. In general, it would be more useful to use such educational texts to illustrate how mathematics was actually practiced in antiquity, with less focus on how ancient practitioners failed to do mathematics as we do.

My second criticism, which is related to the first, is that Riedlberger tends to follow a historiography of Greek mathematics that is heavily influenced by the philosophical tradition. For example, he adheres to the notions that there is a strict sense in which "number" (*arithmos*) must be understood in Greek mathematics, and that ratios could not be directly subjected to arithmetical operations (p. 211), despite the fact that the text under discussion, the *How-to*, calls both of these claims into question. The persistence of the ideas that the unit was regarded as inviolable and that *arithmos* denoted only natural numbers is a result of historians of mathematics focusing their attention on philosophical authors and the *Elements* while neglecting the papyrological evidence and the exact sciences. In authors such as Heron, Ptolemy and Diophantus, the word *arithmos* has various technical meanings that are unrelated to its meaning in the numerical books of the *Elements*. This is presumably because these authors held that a common word like "number" might have rather different meanings in different contexts. In computation (*logistikē*), Greek mathematicians showed no "self-restraint in reference to fractions" nor any "reluctance to use fractions" (p. 199, 200). These claims about how the Greeks thought about the unit and *arithmos* come from an over reliance on philosophical writings and it is not clear that they should be adopted by historians of mathematics to characterize actual mathematical activity. A possible reconstruction of what may have happened is the following: Number theory developed in conjunction with harmonics, in which the indivisibly of the unit had technical, empirical and philosophical meanings. Philosophers latched onto this indivisibility and raised it to a sort of principle. Mathematicians, including Euclid, developed number theory, in which the indivisibility of the unit played a technical role, because the object of study was the natural numbers, while, at the same time, they both divided the unit and used a wider conception of *arithmos* in other works. Whether or not these mathematicians agreed with the philosophical discussions is unknown. The exact sciences were developed by combining *logistikē* with geometry, in which the divisibility of the unit and a broader conception of *arithmos* were assumed. In the work of Ptolemy and Diophantus, *arithmos* came to have other technical meanings that were different from, even incompatible with, that in the *Elements*; but this situation was apparently unproblematic to mathematicians, since they took the term as relative to its context. Imperial and late-ancient philosophers, however, influenced by the philosophical tradition, and focusing on elementary mathematical texts, continued to try to argue that there was only one "proper" mathematical understanding for the concepts of unit and *arithmos*, and they sought to dismiss the obvious diversity found in mathematical practice as due to the contamination of inferior, practical traditions. Whatever the case, it is clear that the diversity of practices in the evidence does not support the idea of a single notion of number or ratio in Greek mathematics.

In summary, Riedleberger's book provides a well-argued reevaluation of Domninus as a mathematician and Platonic philosopher, provides critical editions, English translations and detailed discussions of three late-ancient mathematical texts, and is a valuable source for understanding the mathematical scholarship of the late-ancient period, particularly in Athens.

This is the second book—following a treatment of Diophantus' *Polygonal Numbers* by F. Acerbi (2011)—in a new series of classical texts put out by Fabrizio Serra of Pisa, entitled Mathematica Graeca Antiqua. The series is edited by F. Acerbi and B. Vitrac, two of the most prolific and careful scholars working on Greek mathematics today. The stated goal of this series is to produce "*editiones principes* as well as revised editions of already published texts, especially of so-called 'minor' works, of late texts, and of treatises in which the tensions within the literary code of Greek mathematics are particularly evident" (http://www.libraweb.net). Riedleberger's book fits these criteria quite well. It is clear that this series will be indispensable for libraries of classical texts and to scholars of the Greco-Roman exact sciences.