Robust Network Design for Roadway Networks: Unifying Framework and Application

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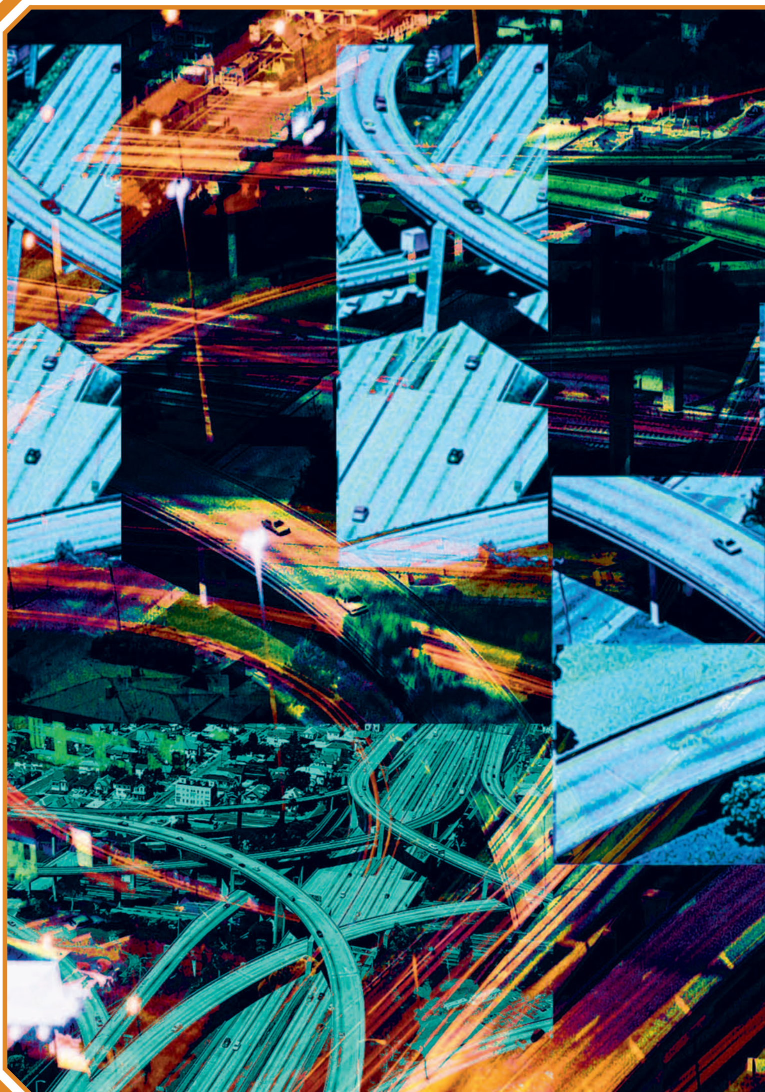

Digital Object Identifier 10.1109/MITS.2014.2386654
Date of publication: 20 April 2015

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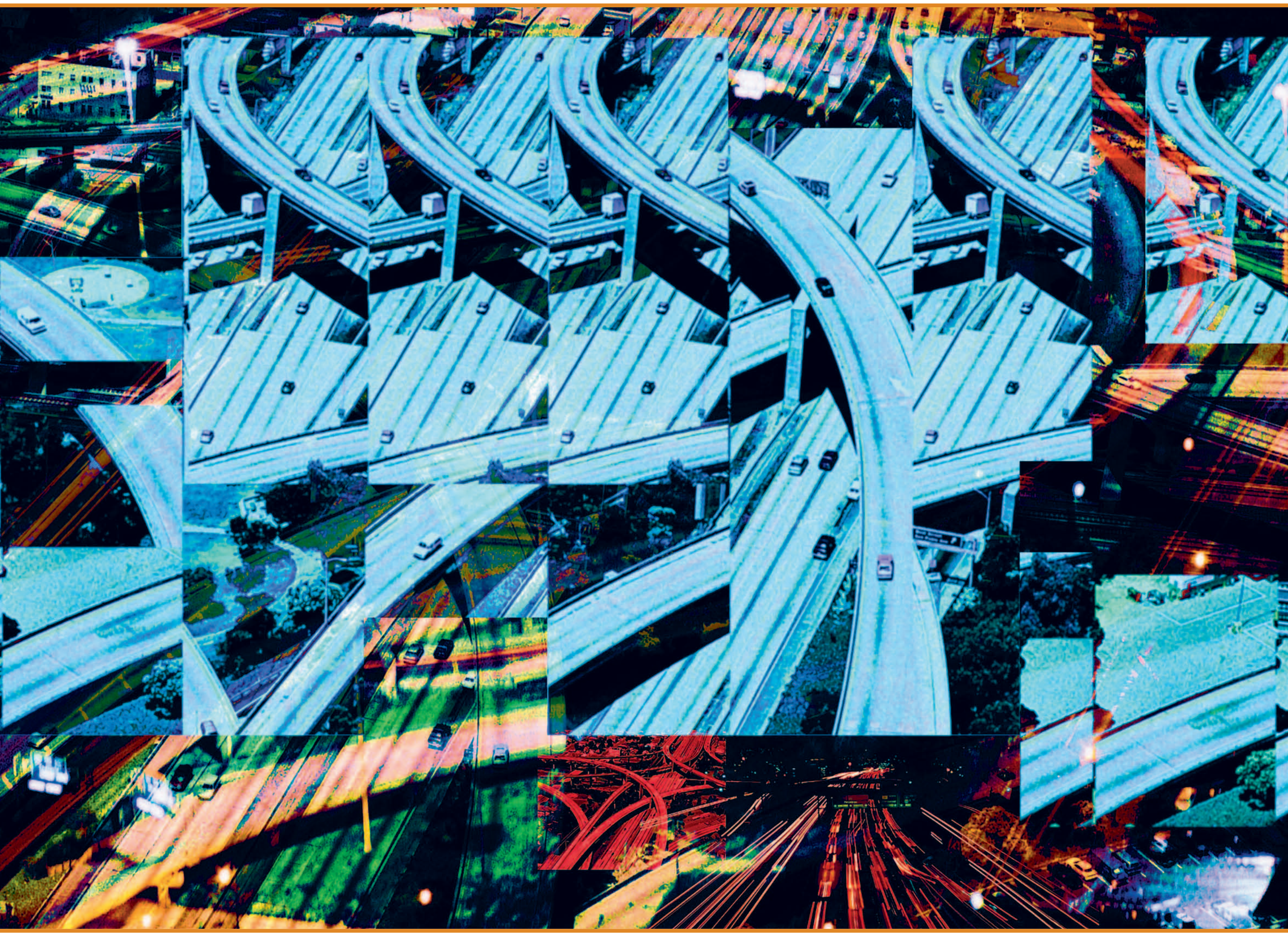
Abstract—Disturbances in roadway networks due to increases in demand or drops in network capacity can severely degrade the performance of the system. The robustness of a roadway network to such disturbances has been investigated using a variety of methods leading to disparate robust network designs. This paper introduces a unifying framework for understanding and applying different robust network designs based on the context of traffic disturbances and design goals. It presents the objectives, requirements and examples of robust network design with long-term (planning) and short-term (operation) goals. A sample case study is presented to assess a short-term robust network design using traffic assignment. The preliminary testing results compared to conventional User Equilibrium and System Optimal traffic assignment, demonstrate 20% and 10% travel time savings with demand increase and supply reduction, respectively.

I. Introduction

Recently, significant attention has been devoted to designing robust real-world networks that can handle the possible impacts caused by disturbances, such as system malfunctions, weather conditions, cyber attacks, terrorist activities, unexpected demands, incidents, etc. [1]–[3]. Examples of networks that can be affected by these disturbances include electric grids, computer networks, communication networks, financial networks, safe water networks, and transportation networks. In order to assess the robustness of a network, it is crucial to quantify the importance of its nodes and links to the performance of the overall network under such disturbances [4]. For example, the New York JFK airport is a very important node in the US air traffic network, because it acts as a hub between hundreds of other smaller airports. Therefore, any disturbance to the operations of the JFK airport could have considerable impact on the overall US air network performance.



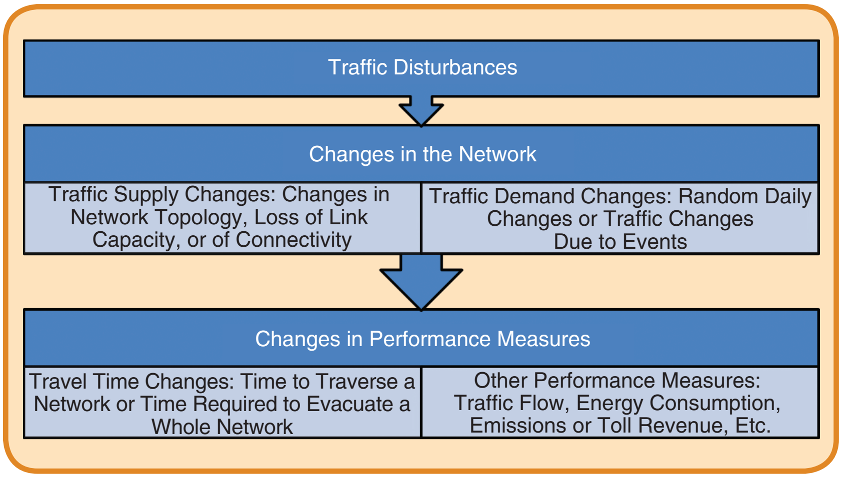


FIG 1 Impact of traffic disturbances [4].

In the context of road networks, several studies have looked at the general problem of designing road networks [5], [6]. Such studies relied on optimization models to design a network or make an optimal investment decision in order to satisfy the route choice specifications of network users, while minimizing total travel cost [5], [6]. The past few years in particular have seen a substantial effort in developing methods for measuring the robustness of road networks [7]–[11]. However, there has been a variety of approaches to network robustness addressing different types of traffic disturbances, resulting in disparate methods for robust network designs and their associated optimization problems. For example, Ip et al. [12] solved a resource allocation problem for choosing long-term road building projects by maximizing the robustness of a road network to severe traffic conditions. Koulakezian et al. [11] solved a traffic assignment problem for choosing short-term traffic routing by maximizing the robustness of a road network to frequent traffic demand/supply variations.

Contributions: This paper introduces a unifying framework enabling the research community and transportation planners/operators to understand and apply different robust network designs based on the context of traffic disturbances (severity and frequency) and design goals (short-term and long-term). Moreover, it systematically presents the objectives, requirements and examples of robust network design with long-term (planning) and short-term (operation) goals, which have been lacking in the literature. In addition, it illustrates the potential benefits of robust network design through a simple case study, while introducing a systematic evaluation methodology for robustness metrics; featuring additional simulations and novel formulations compared to the previous short paper [11].

In this paper, we first introduce this framework, starting with the various contexts of robustness based on the types of traffic disturbances in Section II. We then investigate in Section III the goals and requirements for developing a robust network design (short-term and long-term), discussing recent examples from the literature. In Section IV, we present a sample study using traffic assignment to achieve short-term robust network design with frequent disturbances. Finally, we conclude in Section V and discuss future work in Section VI. This research was funded by the Ontario Research Fund grant on Connected Vehicles and Smart Transportation [13].

II. Context of Robustness

In this section, we define the sensitivity of road networks to traffic disturbances and the possible contexts for robustness according to the severity and frequency of disturbances. We then define robustness and discuss how it can be measured.

A. Sensitivity of Road Networks

The impact of traffic disturbances on networks is illustrated in Fig. 1. Traffic disturbances cause various changes in the network, which may subsequently cause changes in some performance measures of the network.

Based on these definitions, the sensitivity of a road network to a disturbance [14] is defined as the change in a performance measure, divided by the magnitude of the change in the network due to the traffic disturbance, as shown in Equation 1.

$$\text{Sensitivity} = \frac{\text{Change in performance measure}}{\text{Change in network}} \tag{1}$$

For instance, an incident on a highway might cause changes in the network, including taking away 30% of the highway capacity (and possibly altering the demand on the highway and on alternate roads, etc.) Therefore, the highway could be considered insensitive to incidents of this nature if this 30% capacity drop only causes a 10% increase in travel times (a chosen performance measure of interest), and considered sensitive if it leads to a 90% increase in travel times.

B. Severity of Traffic Disturbances

The severity of traffic disturbances defines the extend of network changes. For example, disturbances can lead to:

- Normal traffic conditions, with minor variations in demand, minor incidents and changing weather conditions.
- Severe traffic conditions, including major incidents, natural disasters, and severe weather conditions.

In normal traffic conditions, the road topology is usually left intact, with certain performance degradation. However, severe traffic conditions include cases when demand tremendously exceeds capacity in parts of the network or when connectivity of the road network is greatly impacted, such as the case of evacuation scenarios. Severe traffic conditions typically lead to severe changes in performance measures also; thus certain special performance measures, such as the time required to evacuate a network, may be of interest in such cases.

C. Frequency of Traffic Disturbances

The frequency of traffic disturbances affecting the network is another factor to consider. For example, traffic disturbances can be classified as frequent and infrequent [4]. Traffic operators consider another associated notion of disturbances, namely their predictability of occurrence, defining them as predictable, if it is relatively easy to predict them, and unpredictable otherwise. Given these classifications of disturbances, a possible set of examples for traffic disturbances includes:

- Frequent predictable: A pedestrian at a crossing near a stadium before a sports game
- Frequent unpredictable: A pedestrian at the same crossing next to the stadium on a cold Sunday morning
- Infrequent predictable: Incidents during a storm, if the storm was predicted
- Infrequent unpredictable: An earthquake damaging roads.

Therefore, considering the severity, frequency and predictability of traffic disturbances helps better understand the impacts of disturbances on network performance.

D. Robustness of Road Networks

The robustness of a road network can thus be defined as the ability to maintain acceptable performance under a set of traffic disturbances [15]. Acceptable performance refers to reasonable travel times, delays, energy consumption, etc. compared to disturbance-free or average typical traffic conditions [14]. A formal definition of road network robustness should consider the context of robustness based on the severity, frequency and predictability of traffic disturbances. As illustrated in Fig. 1, each disturbance within this context leads to a corresponding set of network changes $\mathbf{n}_i$. Therefore, based on Equation 2, a road network is **robust** if the maximum change in a key performance measure of interest, divided by the magnitude of the change in the network $\mathbf{n}_i$, over all possible network changes due to disturbances within its context, is small:

$$\max_{\mathbf{n}_i}\left|\frac{\text{Change in performance measure}}{\text{Change in network}\,(\mathrm{n}_i)}\right| \text{is small} \quad (2)$$

This definition is independent of the context of disturbances. However, the magnitude of network and performance changes varies with this context. Therefore, to study the network robustness under emergency evacuation cases for example, the context of robustness is the set of severe, infrequent, and unpredictable traffic disturbances [12]. On the other hand, the context of a minor incident is the set of recurrent traffic disturbances in normal traffic conditions [1].

Note that even after considering the context of disturbances that could affect a road network and choosing a performance measure such as travel time in the network, calculating Equation 2 for all possible network changes, caused by all possible disturbances within this context, is usually not computationally feasible. Therefore, **robustness metrics** have been proposed to varying degrees of success in capturing the essence of Equation 2. Metrics have achieved this by modeling key changes in the network and/or performance measures (shown in Fig. 1) due to a set of disturbances within the context assumed for a network, rather than modelling all possible network and performance changes caused by disturbances.

For example, assume that average node degree (number of neighbors a node is connected to) is a robustness metric for a network under the context of severe conditions. Based on this metric, a network is robust if the change of its average node connectivity under this possible set of disturbances is small (see Equation 2), i.e. below a certain threshold chosen by the road planner/operator. However, since this metric only captures a change in the network (without taking into account any changes in performance measures), it offers an incomplete or partial view of network robustness. Similarly, other metrics capture network robustness with varying degrees of accuracy.

In addition, robustness metrics can be further classified as:

- Static, if they incorporate network changes and/or static performance measures that do not change with time, e.g. free-flow travel times, in the modelling process.
- Dynamic, if they incorporate both network changes and explicitly model temporal variations, such as travel times, queuing, spread of congestion and dynamic network and performance changes in general.

Static metrics have low computational complexity compared to their dynamic counterparts, but lack in the ability to capture short-term performance changes due to traffic disturbances. The next section will discuss the various objectives of robust network design, shedding light on the roles of static and dynamic metrics in various network design problems.

III. Robust Network Design

Road network designs can have both long-term and short-term objectives. While long-term objectives typically address regional planning needs, usually performed by transportation planners, short-term design objectives address managing the day-to-day operations of a road network, usually performed by traffic engineers and system operators. Recently, analyzing the robustness of road networks to traffic disturbances has played an influential role in road network design, including the decision of where to invest, aiming to make road networks more

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robust. This section discusses both long-term and short-term robust network designs, highlighting the benefits of analyzing network robustness in each case and providing insights on the robustness metrics proposed for these designs.

A. Long-Term Robust Network Design

Long-term network design implies designing a region-wide road network or updating an existing one to accommodate normal traffic disturbances, due to varying traffic demand, occurrence of incidents, and changing living and employment locations. This requires forecasting population growth, land use and traffic demand [16]. In addition, it could include planning for disaster situations with severe traffic disturbances, which could require emergency evacuation due to an earthquake, nuclear spill, etc. [17]. Defining robustness metrics is key to guide these long-term planning goals with explicit optimization of network robustness. Examples of robust long-term design goals include:

- Making policy decisions to prioritize building new roads to alleviate the performance degradation of critical roads
- Optimizing emergency evacuation plans to manage the usage of critical parts of the network
- Changing directions of existing roads or particular lanes of roads to limit the effect of seasonal traffic demand variations on critical roads.

To help planners achieve long-term robust network design goals, a robustness metric should satisfy the following criteria:

- Network-wide measure: The metric should quantify the impact of traffic disturbances on the network as a whole.
- Component-specific measure: should identify the critical links/nodes for a robust system performance [4].
- Directed: should be capable of modeling the asymmetric demand inherent in road networks, as they are represented by directed graphs (some existing metrics for instance assume symmetrical links between nodes, which is not the case in transportation networks).

In Table 1 we provide a set of proposed robustness metrics for long-term robust network design, and summarize these metrics against the requirements defined above. In this section, we summarize the robust metrics proposed in the literature (with their original metric names) by identifying the context of disturbances considered and the associated design goals, in light of the framework setup in Section II and Section III.

Resilience and **Friability** [12] are static robustness metrics in the context of severe, infrequent and unpredictable disturbances, with the goal of guiding policy decisions on where to invest in expanding the road network. In a network that models cities as nodes and passageways between cities as paths/routes, the resilience of a node is defined as the weighted average of the number of passageways it has to all other nodes using city population as a node weight. Consequently, network resilience is calculated as the weighted sum of node resiliences. Moreover, friability is defined as the reduction in network resilience upon removing a node or a link from the network. Based on these robustness metrics, a robust network design problem is formulated to prioritize the investment on road expansion project(s)—among several possible projects - to maximize the resilience and minimize the friability of the network subject to budget constraints [12]. The definitions of resilience and friability metrics do not fully capture network robustness as defined in Equation 2. For instance, they do not consider the capacities of links, their importance to the whole network, or their impact on performance measures such as travel time or throughput. In addition, friability needs to be

Table 1. Robustness metrics for long-term road network design.

Metric	Source	Solution	Network-Wide Measure	Component-Specific Measure	Directed
Resilience	[12]	Average of node resiliences	✓	✓	✓
Friability	[12]	Decrease in network resilience	✓	✓	✓
Diameter change	[18]	Min. net. diameter increase	✓		
Alternative paths	[7]	Based on no. of alternate paths	✓		
Fastest-path betweenness	[8]	Weight: no. of restaurants	✓	✓	✓
Estimating travel time	[9]	Shortest path assignment		✓	✓
Vehicle loss hours	[4]	Estimating partial link blocking	✓	✓	✓

A network is considered to be robust if the increase in the network diameter (the largest number of links that need to be traversed between any pair of nodes) upon the removal of nodes, due to disturbances, is small.

computed for every network node after removing it and recomputing the resiliences of its neighboring nodes, which is computationally inefficient. While the aim of prioritizing road expansion investment to improve resilience and friability aligns with transportation planner goals, an additional requirement to convince policy makers to secure funding for these roads requires clarifying the potential savings this additional achieved robustness provides relative to a base case of not building new roads for example. For instance, this could possibly be in terms of time required to evacuate a network.

Diameter change [18] is a static metric in the context of severe, infrequent unpredictable disturbances for bus traffic networks. A network is considered to be robust if the increase in the network diameter (the largest number of links that need to be traversed between any pair of nodes) upon the removal of nodes, due to disturbances, is small. This metric is dependent on few low-degree but high-load nodes in the network, a property that defines the topology of bus networks [18]. Therefore, it could be used to promote bus network designs with highly-connected stations. However, since roadway networks have more complex routing options and design parameters compared to a fixed pre-defined set of routes for bus networks, this metric does not well represent road network robustness.

Derrible [7] presents a similar static metric called **Alternative Paths** for metro networks. This metric is calculated as $\mathbf{r}^{\mathrm{T}} = (\mu - |\mathbf{L}^{\mathrm{m}}|)/|\mathbf{N}|$, where μ is the number of cycles in a network with $|N|$ nodes and $|L^{m}|$ multiple links (between nodes, used to provide redundancy). This metric works well for the topology of metro networks, but seems implausible for urban road networks, which have different network topologies.

Leung et al. [8] propose the static **Node-Weighted Fastest-Path Betweenness** metric in the context of frequent disturbances within normal traffic conditions. The betweenness of a node k with respect to flows from source node s to destination node d is defined as the proportion of the shortest paths from s to d that traverse node k. The overall betweenness of node k is the sum of the betweenness values over all source-destination (s-d) pairs [19]. Node-weighted fastest-path betweenness uses the free-flow travel times of each link based on road class and length to determine the shortest paths in the betweenness calculation and also considers the importance of a node by using the number of restaurants in its vicinity as a node weight. To show the potential benefits of this metric, correlations with traffic conditions were performed in [8]. At first glance, this metric looks promising for promoting long-term designs to handle frequent disturbances; however, it uses an odd node weight and assumes free-flow travel times in its calculation. These assumptions are questionable for common traffic flow patterns in real roadway networks, as shown by the average correlation results with traffic conditions provided in [8].

Another metric proposed in the context of frequent disturbances within normal conditions is the **estimating travel time heuristic** [9]. The objective of this metric is to estimate the impact of a link failure on the total system travel time and design the network to reduce this impact. For this purpose, it does not require removing each link and re-running a traffic assignment algorithm. Instead, it requires running an assignment algorithm once to find the initial assignment, then fixing link costs based on this assignment and calculating link failure effects on total travel time by only rerouting the affected traffic using shortest path routes. However, given the dynamics of road networks and the considerable effect of traffic flowing from shortest-path routes into alternate routes, this metric cannot sufficiently model the dynamic movement of vehicles in the presence of frequent disturbances in the network.

Vehicle loss hours [4] is a similar static metric for frequent incidents within normal traffic conditions. It is evaluated with travel times from a macroscopic traffic assignment tool and a marginal incident computational model based on probabilities and properties of incidents on links. It estimates the effect of drivers using alternate routes using percentages of rerouting upstream of an incident. Performance results with this metric [4] show that it is unable to model spillbacks caused by incidents. Such a robustness metric can possibly provide a modeling capability to design road networks with reduced impact from incidents. However, this capability is limited by its macroscopic model that simplifies driver behavior and by its inability to model spill-back effects to accurately represent the propagation of shockwaves upstream of an incident.

B. Short-Term Robust Network Design

Traffic engineers are responsible for the day-to-day operations of the road network and the associated short-term network designs. In most cases, traffic engineers consider the context of frequent and infrequent disturbances under normal traffic conditions. However, severe disturbances might be of interest in the case of preparing a real-time dynamic emergency evacuation plan that can cope with short-term traffic disturbances. Short-term network design requires managing both the traffic demand and the network supply in a road network. Supply-side management strategies include, for instance, ramp metering [20], Variable Message Signs (VMS), Variable Speed Limits (VSL) [21], and changing road directions dynamically. Demand-side management includes Advanced Travel Information Systems (ATIS) [22] and congestion pricing [23].

Analyzing network robustness metrics helps in directing these operation goals to achieve a robust road network in day-day operations in the presence of disturbances. It helps to pro-actively control traffic demand and supply, considering the possible effect of disturbances that might occur, by optimizing for robustness rather than minimizing travel times during daily operations [24]. In other words, the impact of disturbances can be mitigated, leading to more stable road networks, by incorporating robustness metrics into the objective functions of real-time demand management and supply optimization, such as ramp metering, congestion pricing, VSL, VMS and dynamic road direction change. To guide such short-term robust network design objectives, a robustness metric should satisfy the following requirements (in addition to the requirements of long-term design discussed earlier):

- Scalable: should have low computational complexity relative to network size.
- Dynamic: Should capture both dynamic changes in traffic performance measures and effects of network changes as shown in Fig. 1.
- Measurable in Real-time: Should be easily measured using real-life traffic monitors, such as road sensors and sensors in connected vehicles.

A set of robustness metrics proposed for short-term robust network design is shown in Table 2. Here, we describe and compare them based on the established requirements.

Vulnerability [1] is a dynamic robustness metric proposed in the context of frequent incidents within normal traffic conditions. Defined as the ratio of actual flow (demand) to available capacity of links, it is calculated for network links using an initial traffic assignment and its variation is studied with a reduction of the number of lanes for links in the network. The results in [1] show that link-based metrics, including this metric, are insufficient to capture the effects of blocking a link (for example due to an incident or construction) on the whole network [1]. Using loop detectors on the roads, the flow on the links can be estimated and therefore this metric is scalable and can guide a response towards achieving robustness by possibly maximizing the weighted average for flow to available capacity ratios in network links. It is important to note that this metric assumes that when demand exceeds capacity, the network (or link) will operate at capacity. Although this is the case in many networks types such as computer networks, it is not the case in road networks. In road networks if demand exceeds capacity, throughput will degrade due to excessive turbulence in the traffic stream [25]. This could lead to measuring low flow values not because there is unused capacity but because the traffic stream is congested and not moving well. As a result, this metric is not suitable for congested networks.

The **Unified Network Performance Measure** (UNPM) [10] is a dynamic metric proposed in the context of frequent disturbances in normal traffic conditions. Given a network topology $\mathbf{G}$ and the equilibrium demand vector $\mathbf{D}$ between its source-destination (s-d) pairs, UNPM is defined as:

$$\epsilon = \epsilon(\mathbf{G}, \mathbf{D}) = \frac{\sum_{\mathbf{s,d}} \frac{\mathbf{D_{sd}}}{\lambda_{\mathbf{sd}}}}{\mathbf{n_{sd}}} \tag{3}$$

where $\mathbf{n_{sd}}$ is the number of s-d pairs in the network, and $\mathbf{D_{sd}}$ and $\lambda_{\mathbf{sd}}$ are the equilibrium demand (numbers of cars) and the equilibrium disutility for s-d pair **sd,** respectively [10]. This is called a measure of efficiency as it computes the average demand to cost ratio for all s-d pairs, where cost is the total travel time for vehicles on an s-d pair. Therefore, this ratio is the service rate of vehicles (and the inverse of average vehicle travel time) for each s-d pair and ϵ is an average of this ratio over all s-d pairs in the network [10]. This metric is suitable for directed networks and can guide a response maximizing the UNPM measure by rerouting traffic for s-d pairs with high $\lambda_{\mathbf{sd}}$ to other routes. However, the demand vector it needs cannot be calculated from real-time road and vehicle sensors, which provide traffic data in terms of flows for example, without identifying which s-d pairs they belong to. Consequently, this measure provides an offline analysis

Table 2. Robustness metrics for short-term road network design.

Metric	Source	Solution	Network-Wide Measure	Component-Specific Measure	Directed	Scalable	Dynamic	Measurable in Real-Time
Vulnerability	[1]	Link flow over available capacity		✓	✓	✓	✓	✓
UNPM	[10]	Mean of s-d pair throughputs	✓	✓	✓	✓	✓	
UNPM robustness	[10]	Variation with capacity decrease	✓		✓	✓	✓	
Network criticality	[11]	Weight over betweenness	✓	✓		✓	✓	✓

tool of robustness through a simulator, assuming full knowledge of traffic demands between s-d pairs.

Robustness metrics for short-term network design are dynamic and primarily deal with frequent disturbances in normal traffic conditions.

Nagurney et al. [10] present another metric called **UNPM Robustness** based on Equation 3. Given a vector of link capacities $\mathbf{C}$, this metric ($\mathbf{R}^{\alpha}$) is defined as the relative performance retained when the vector of link capacities is reduced to $\alpha \cdot \mathbf{C}$ with the scalar factor $\alpha \in (\mathbf{0}, \mathbf{1}]: R^{\alpha} = ((\epsilon^{\alpha}/\epsilon) \times \mathbf{100}\%)$, where ϵ^{α} and ϵ are the network performance measures calculated by Equation 3 with the original capacities and remaining capacities, respectively. However, this scalar reduction in link capacities should not be generalized as incidents only affect specific links in the network and reduce various link capacities differently depending on the severity of the incident. Therefore, this metric is not suitable for robust short-term network design.

Lastly, **network criticality** [11] is a dynamic metric proposed in the context of frequent disturbances within normal traffic conditions. Assume that a random-walker starts from a source node s in the network, then chooses a neighbor at random with equal probability and goes there. It continues wandering around until it reaches a specified destination d, where it stops. Thus, similar to the notion of fastest-path betweenness explained earlier, the random-walk betweenness of a node k with respect to flows from s to d is the proportion of the random walks from s to d that traverse node k. The overall betweenness of node k is the sum of this quantity over all s-d pairs. Therefore, the point-to-point network criticality of node $\mathbf{k}$ for trajectories from $\mathbf{s}$ to $\mathbf{d}$ is [19]:

$$\tau_{\mathbf{sd}}^{\mathbf{k}} = \frac{\mathbf{b_{sk}(d)} + \mathbf{b_{dk}(s)}}{\mathbf{W_k}} \tag{4}$$

where $\mathbf{b_{sk}(d)}$ is the random-walk betweenness of a node $\mathbf{k}$ for pair s-d, the inverse of travel time is used as the link weight $\mathbf{w_l}$ for link $\mathbf{l}$ (taking into account that road links with long travel times are undesirable), node weight is defined as $\mathbf{W_k} = \sum_{\mathbf{l} \in \mathbf{A^o(k)}} \mathbf{w_l}$, and $\mathbf{A^o(k)}$ denotes the set of outgoing links attached to node $\mathbf{k}$.

In generic random-walks, in which the probability of transitioning along a link is proportional to the weight of the link, $\tau_{\mathbf{sd}}^{\mathbf{k}}$ is independent of $\mathbf{k}$ [19]. Consequently, the average network criticality τ (of the whole network) is defined as the mean of all point-to-point network criticalities and it can be shown to be proportional to the trace of $\mathbf{L}^{+} = [\mathbf{l}_{\mathbf{ij}}^{+}]$, the Laplacian matrix of the graph [19]:

$$\tau = \frac{\mathbf{1}}{\mathbf{n(n-1)}} \sum_{\mathbf{s,d}} \tau_{\mathbf{sd}} = \frac{\mathbf{2}}{\mathbf{n-1}} \mathbf{Tr(L^{+})} \tag{5}$$

This metric supports short-term robust design as it captures variations in traffic demand or supply using real-time measurements with graph weights, where optimizing for network criticality minimizes steep increases in average link travel times by guiding short-term responses for rerouting away from critical links (with low weights). Note that network criticality is inherently for undirected graphs. Therefore, to be used for directed road networks, an undirected symmetric matrix of the graph defined as $\mathbf{W_{sym}} = (\mathbf{W} + \mathbf{W^T})/\mathbf{2}$ is used, where $\mathbf{W}$ denotes the link weight matrix and $\mathbf{W^T}$ denotes the transpose of $\mathbf{W}$ [19]. Thus, network criticality has a limitation as it approximates directed road networks by undirected graphs. However, this can be resolved by developing a similar robustness metric for road networks by adapting the directed version of network criticality defined in [26] for usage in road networks.

C. Robust Network Design Discussion

After analyzing robustness metrics proposed for long-term and short-term network design, we can see that they capture the essence of Equation 2 with varying degrees of success.

Robustness metrics proposed for long-term network design have mostly been static, with some metrics using estimation techniques by modeling some aspects of traffic flow variation. For this reason, they mostly fail in modeling changes in performance measures as needed by Equation 2. Metrics such as resilience or friability can help transportation planners make policy decisions to build new roads while reducing the impact of unpredictable disturbances. However, for developing long-term robust network designs in the presence of regular disturbances, further analysis is required to clearly model the impacts of such disturbances and determine the critical components of a network. This requires using dynamic metrics showing the impact of disturbances on various traffic flow measures and conditions. In addition to long term planning, the network can benefit from short-term robust design techniques, such as, for instance, robustness-maximizing dynamic traffic assignment, presented in the next section. Operating the network with robust traffic assignment will not only enhance the daily operation but may also reduce the need for more expensive long term infrastructure expansion.

Robustness metrics for short-term network design are dynamic and primarily deal with frequent disturbances in normal traffic conditions. The most promising metrics are the UNPM and network criticality metrics as they can help detect the onset of congestion due to disturbances and guide

For real-time use, novel methods should use distributed and high-performance computing strategies to reduce the computation times for these metrics.

a short-term response to re-optimize network robustness. Nevertheless, both of these metrics have a computational complexity of $\mathbf{O}(|\mathbf{N}| \cdot |\mathbf{L}|)$, for a network with $|\mathbf{N}|$ nodes and $|\mathbf{L}|$ links. Therefore, for real-time use, novel methods should use distributed and high-performance computing strategies to reduce the computation times for these metrics, yet still achieve a high degree of calculation accuracy. Moreover, the practical usage of the UNPM measure requires the estimation of the demand vector between s-d pairs in the network based on real-time traffic measurements captured in the field.

IV. Sample Study

This section presents a sample study for a short-term robust network design for a traffic assignment problem using robustness metrics. The goal is to show benefits of robust network design and introduce a systematic evaluation methodology for robustness metrics. We first formulate the traffic assignment problem and then present the performance results.

A. Traffic Assignment Problem Formulation

This section provides the problem formulation for a static traffic assignment [27], including with the system model used and the traffic optimization problem formulation.

1) System Model: Suppose that the road network topology is given by a directed graph $\mathbf{G}(\mathbf{N}, \mathbf{E}, \mathbf{W})$, where $\mathbf{N}$, $\mathbf{E}$, and $\mathbf{W}$ denote the node set, link set, and link weight matrix, respectively. While a link represents a road segment between nodes $\mathbf{i}$ and $\mathbf{j}$ and denoted by $\mathbf{l} = (\mathbf{i}, \mathbf{j})$ with weight $\mathbf{w}_\mathbf{l}$, a node represents a trip origin/destination/junction of road segments. The sets of outgoing links and incoming links of a node $\mathbf{k}$ are denoted by $\mathbf{A}^\mathbf{o}(\mathbf{k})$ and $\mathbf{A}^\mathbf{i}(\mathbf{k})$, respectively. We assume that the analysis period of interest is taken as a peak period with relatively high demand and vehicles request, receive and follow guidance information from the network operator. From the operator's perspective, these requests make up the triple $(\mathbf{s}, \mathbf{d}, \gamma_\mathbf{s}(\mathbf{d}))$, where $\mathbf{s}, \mathbf{d},$ and $\gamma_\mathbf{s}(\mathbf{d})$ denote the traffic source, destination and the demand from $\mathbf{s}$ to $\mathbf{d},$ respectively, for each $\mathbf{s}$-d pair. Travel times are calculated using the volume-delay function (VDF) referred to as the BPR (Bureau of Public Roads) Formula, shown in Equation 6 [25]:

$$\mathbf{t}(\mathbf{V}_\mathbf{l}) = \mathbf{t}_{\mathbf{fl}}\left(\mathbf{1} + \mathbf{0.15}\left(\frac{\mathbf{V}_\mathbf{l}}{\mathbf{C}_\mathbf{l}}\right)^4\right) \tag{6}$$

where $\mathbf{t}(\mathbf{V}_\mathbf{l})$ is the average vehicle travel time as a function of the flow $\mathbf{V}_\mathbf{l}$ on link $\mathbf{l}, \mathbf{t}_{\mathbf{fl}}$ t is the free-flow travel time on link $\mathbf{l}$, and $\mathbf{C}_\mathbf{l}$ is the theoretical capacity of a highway link, around 2000 veh/hr/lane × number of lanes for highways, as defined by the Highway Capacity Manual 2000 [28]. Note that lower capacity values are to be used for arterials and local roads.

2) Traffic Problem Formulation: The objective is to find the assignment of vehicles on each path and link during the period of interest. We can formulate the optimization problem for a network minimizing total travel time, called System Optimal (SO), subject to traffic flow conservation constraints [27].

$$\begin{aligned} &\textit{minimize} \quad \sum\nolimits_{l \in E} t(V_l) \times V_l \\ &\textit{subject to} \\ &\forall s, d \in N, \quad \forall l, e \in E, \quad \forall k \in N \\ &\sum\nolimits_{l \in A^o(k)} V_l^{sd} - \sum\nolimits_{e \in A^i(k)} V_e^{sd} = \gamma_s(d)\,\delta(k-s) - \gamma_s(d)\,\delta(k-d) \\ &V_l = \sum\nolimits_{s,d} V_l^{sd} \\ &V_l \geq 0 \end{aligned} \tag{7}$$

The objective function in problem (7) is the total travel time for all vehicles in the network and is based on both the flows $\mathbf{V}_\mathbf{l}$ and the individual travel times $\mathbf{t}(\mathbf{V}_\mathbf{l})$ on each link in the network. The flow conservation constraint needs to be satisfied for every node k and entry $\gamma_s(d)$ of the traffic matrix, where $V_l^{(sd)}$ is the flow of link l for traffic from source s to destination d and $\delta(x)$ is the Kronecker delta function. Under very light traffic conditions, SO would assign all traffic to the shortest paths (also called an all-or-nothing assignment). As demand levels increase relative to capacities, SO assigns traffic to routes with lower marginal travel times, minimizing the total travel time in the road network [25].

We also define the traffic optimization problem for the User Equilibrium (UE) Assignment, which seeks to maximize user welfare by minimizing individual travel times of all users in the system [25]. This is done through an assignment where all alternate routes between an s-d pair have the same travel time. As no user can reduce their travel time by choosing another route, UE is an equilibrium assignment. Therefore, the objective function in (7) is replaced with $\mathbf{minimize}\ \sum_\mathbf{l} \int_0^{\mathbf{V}_\mathbf{l}} \mathbf{t}_\mathbf{l}(\mathbf{x})\,\mathbf{dx}$, where $\int_0^{\mathbf{V}_\mathbf{l}} \mathbf{t}_\mathbf{l}(\mathbf{x})\,\mathbf{dx},$ is the area under the volume-delay curve defined by Equation 6, given volume $\mathbf{V}_\mathbf{l}$ on link $\mathbf{l}$ and the minimization is done over all network links [25].

The traffic optimization problem minimizing network criticality (Tau) replaces the objective function in (7) with: $\mathbf{minimize}\ \tau$ as defined in Equation 5, where link weight $\mathbf{w}_\mathbf{l} = \mathbf{1}/\mathbf{t}(\mathbf{V}_\mathbf{l})$. To calculate this metric, an undirected symmetric matrix of the graph defined as $\mathbf{W}_{\mathbf{sym}} = (\mathbf{W} + \mathbf{W}^\mathbf{T})/\mathbf{2}$

is used, where $\mathbf{W}$ denotes the link weight matrix and $\mathbf{W}^T$ denotes the transpose of $\mathbf{W}$ [19]. This optimization distributes traffic flows such that it minimizes the use of critical links, making the network only incur minimal increases in link travel times subject to traffic disturbances (as needed by Equation 2). In addition, this optimization problem remains convex (since the weight defined satisfies the condition that an increase in the weight also increases the desirability of using the link), and thus can be solved efficiently using a similar solution procedure to that of SO, reaching a unique solution [19].

Lastly, we define the traffic optimization problem maximizing the UNPM measure by replacing the objective function in (7) with **maximize UNPM**, as defined in Equation 3. This problem maximizes the network robustness by maximizing the traffic demand-to-cost ratio for all s-d pairs in the network, in order to get the highest network efficiency possible. The convexity of this optimization problem and the uniqueness of its solution are also guaranteed given the monotone link weight function used, using a similar solution procedure to that of SO [10]. The robustness of the assignment algorithms defined here are discussed next.

B. Performance Results

The simulation test network includes the major highways in the metropolitan Toronto area, as shown in Fig. 2. Note that there are 2 links in opposite directions between every pair of nodes in the network (shown with bidirectional links for simplicity) and the link weights shown represent the number of lanes in each direction. The traffic demand includes equal traffic from 6 s-d pairs, producing various levels of congestion throughout the network. We are interested in analyzing the robustness of the solutions of the 4 static traffic assignment algorithms defined in subsection IV-A (we are implementing these algorithms with a dynamic traffic assignment simulator). Therefore, we first run these traffic optimization problems and find the assignments for SO, UE, Tau and UNPM based on the initial traffic demand and supply. Next, we model frequent traffic disturbances in terms of increases in demand and decreases in number of lanes due to incidents. After applying these disturbances and **without rerunning the traffic optimization again**, we measure the increases in travel time given the original assignment solutions for each of SO, UE, Tau and UNPM to assess their robustness to traffic disturbances. Based on Equation 2, the set of changes considered in the network include increases of up to 30% in demand and removing up to 2 lanes from a link, and the key performance measures of interest are the average travel time in the network and its increase. The assignment algorithms have no a priori knowledge of disturbances or their probability distribution. This is an important distinction from algorithms based on stochastic optimization that explicitly consider within the optimization problem: the traffic disturbance uncertainty using probability distributions [3], [29], or the probability of network links to operate below their capacities when serving different traffic patterns deviating from the average condition [30].

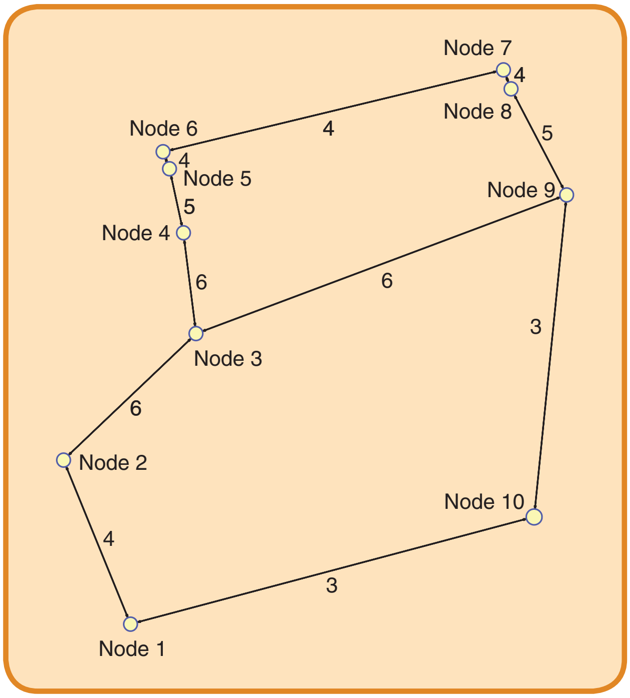

FIG 2 Greater Toronto highway network.

1) Effect of Increasing Demand: We study the robustness of SO, UE, Tau and UNPM by analyzing the effect of increasing demand on their solutions. This increase is the difference between the ideal case of expected demand and the actual demand occurring the road network. Examples for the demand increase include: special events, unexpected surge in demand in parts of the network in response to road closures nearby the network, and additional traffic demand to vehicles that do not have any wireless or vehicular communication capabilities; thus cannot receive en-route navigation updates, resorting to a shortest path route based on their prior experience. This traffic, which is randomly generated on the 6 s-d pairs with a normalized Gaussian distribution, is equivalent to 10% to 30% of the original traffic and results are averaged over 10 runs.

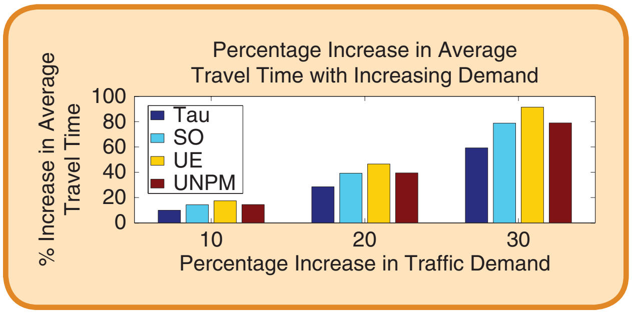

FIG 3 Percentage increase in average travel time with increasing demand.

Note that we only increase demand by up to 30% as the network is already congested at this point and further increases will lead to the breakdown of several highways in the network.

Fig. 3 shows the percentage increase in average travel times for the assignment methods while increasing demand by 10% to 30%. The % increase of the travel times is the smallest for Tau, followed by SO and UNPM with UE having the largest increase in travel times. The performance degradation is far worse in the case of a 30% increase in demand, where travel times go up by more than 90% for UE while they increase by around 80% for SO and UNPM, and by less than 60% for Tau. This indicates that meeting the equilibrium conditions for UE results in a *less forgiving* network and does not lead to robustness in the presence of disturbances. Note that we only increase demand by up to 30% as the network is already congested at this point and further increases will lead to the breakdown of several highways in the network.

We can also use the raw average travel times to evaluate the robustness of these methods, instead of the % increase of average travel times. These are shown in Fig. 4, where the first 4 columns indicate the travel times of the original assignments and the subsequent columns indicate the resulting/new calculated average travel times using the original assignments with increases in traffic demand of 10%, 20% and 30%. The travel times for Tau are the highest before any increases in demand, with the difference being around 5% compared to the SO travel times. This is the price to pay or the trade-off for gaining robustness. However, with increasing demand, Tau provides the lowest travel times starting from a 10% demand increase and performs much better than SO, UE, and UNPM for the case of a 30% increase. This explains the significant deterioration in terms of travel time increases for UE, SO and UNPM also observed in Fig. 3. In addition, the performance results of UNPM are almost comparable to that of SO, deteriorating slightly more than SO with increases in traffic demand.

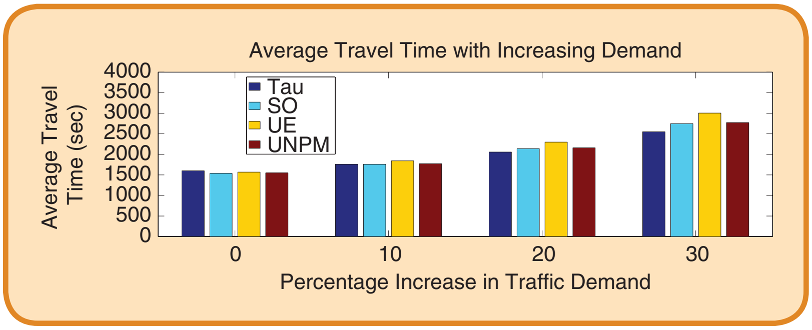

FIG 4 Average travel time with increasing demand.

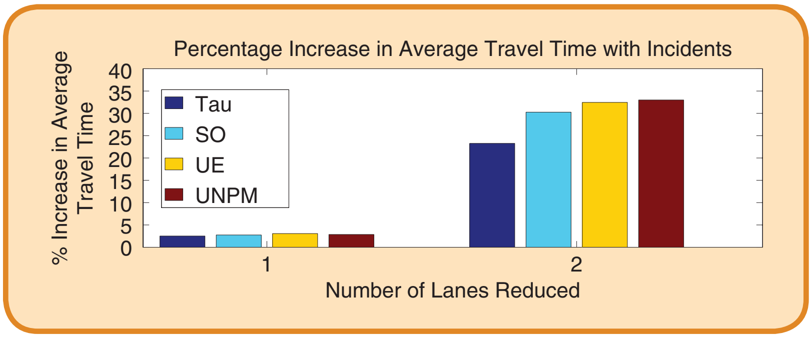

FIG 5 Percentage increase in average travel time with incidents.

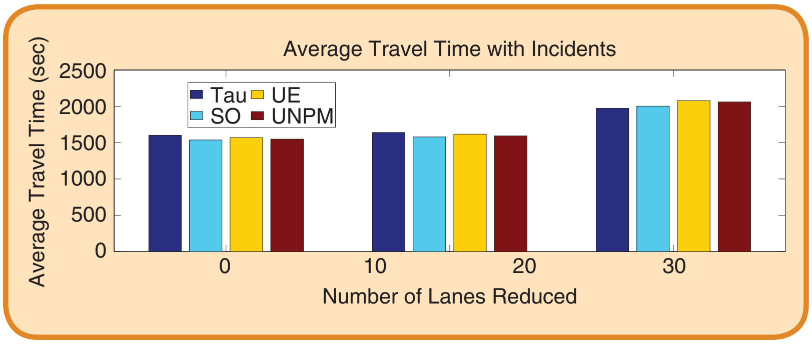

FIG 6 Increase in average travel time with incidents.

2) Effect of Decreasing Supply: Here, we study the effect of decreasing traffic supply due to an incident or weather conditions by removing 1 lane from 1 link, and measuring the new travel time experienced by the original traffic assignment. The lane removal is repeated for all links 1-by-1 and the results are averaged. The same procedure is repeated for removing 2 lanes. Fig. 5 shows the % increase in average travel times for the assignment methods due to removing one lane and 2 lanes. It is the smallest for Tau, especially in the 2-lane case, followed by SO, UE and UNPM, which leads to the largest travel time increase. The performance of UNPM is especially deteriorating in this case due to the high traffic it originally assigns to the link from node 1 to node 10 in Fig. 2, which only has 3 lanes and its capacity has been reduced by 33% and 66% in the cases of removing 1 lane and 2 lanes, respectively. In the latter case, the performance degradation is around 33% for UE and UNPM, while it is only around 23% for Tau.

The raw average travel times are shown in Fig. 6, with the original travel assignment times and those with reductions in the number of lanes. The original travel times for Tau are the highest as before. They are also high for the case of removing 1 lane, since a lane is removed from only one of the links at a time, making it a lighter disturbance compared to adding 10% additional demand. However, Tau provides the best performance when 2 lanes are removed, with UNPM providing the worst performance, as explained earlier.

V. Conclusion

In this paper, we provided a unifying framework for developing a robust design for road networks. After establishing the various contexts of robustness based on the types of traffic disturbances, we presented a definition of robustness in road networks. Moreover, we discussed the objectives, requirements and examples of long-term and short-term robust network design, including planning and operations. Finally, a sample study was presented for a short-term robust network design with a design using traffic assignment. This showed that robust assignment algorithms, especially with network criticality as a robustness metric, lead to solutions that yield suboptimal travel times under expected traffic conditions, but that deliver high performance over a range of unexpected traffic disturbances, even achieving better travel times than the SO algorithm under certain traffic disturbances.

VI. Future Work

Although encouraging results were obtained in this research and a number of research questions were answered for developing a robust design for road networks, a number of questions still exist to further extend and enhance the system. The following are our plans for future research:

1) Integration with a dynamic traffic assignment (DTA) model: This step would include formulating a robust DTA algorithm to capture the time-dependent demand and the dynamic interaction between demand and supply while capturing congestion within transportation networks using a large-scale network model for the Greater Toronto Area.
2) Computational efficiency and system scalability: This step would include a modified mathematical representation of robust metric values to minimize the number of function evaluations and have a reasonable run-time for the assignment algorithm.

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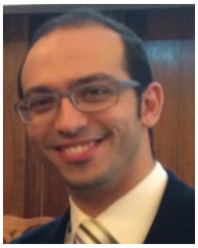

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