Robust Traffic Assignment in Transportation Networks Using Network Criticality

Agop Koulakezian, Hazem M. Soliman, Tang Tang and Alberto Leon-Garcia
Department of Electrical and Computer Engineering
University of Toronto, Canada,
Email: agop.koulakezian@utoronto.ca

Abstract—**Congestion in a transportation network is usually the result of either an increase in traffic demand, i.e. the desire of drivers to use the transportation network, or a decrease in traffic supply, i.e. the traffic capacity, which is affected by weather conditions, incidents, etc. In either case, congestion reduces the efficiency of the transportation network and increases the travel time of vehicles in the network. In this paper, we leverage the benefits that Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications provide in Intelligent Transportation Systems (ITS), to optimize traffic assignment in transportation networks. In particular, we formulate a convex optimization problem for a transportation network and minimize network criticality, a new graph metric that measures centrality. The robustness of this solution is studied and compared to that of the System Optimal Equilibrium (SOE) Optimization. The results show that using network criticality provides robustness (lack of sensitivity) to both increases in traffic demand and decreases in traffic supply, thus reducing traffic congestion.**

Index Terms—**Network Criticality, Robustness, Transportation, ITS, Flow Assignment.**

I. Introduction

Urban transportation optimization has long been an active research field. Over the past decades, the development of mathematical optimization theories as well as faster and more capable computers has enabled solutions to problems that were previously intractable in this field [1]. A widely adapted objective function for traffic optimization and a major factor affecting customer satisfaction is travel time, defined as the time for a vehicle to travel from its source to its destination. The problem of minimizing travel time is often solved by finding the best timetable for the network, because scheduling is realized by determining the departure time for each vehicle. The problem is generally difficult to solve as even testing the feasibility of a timetable is NP-hard [1]. Some approaches to solving this problem include: integer programming [2], extension of integer programming using cycle bases [3], quadratic semi-assignment models [4], graph-theoretical approaches [5], etc.

The robustness of solutions has also been studied extensively. Yim et al [6] measure robustness by estimating the probability of overloading the links and they solve the problem of minimizing such probability using a genetic algorithm. Yao et al [7] take uncertainty into account when building the linear programming model for a surface transportation network. They show how the solution outperforms the nominal deterministic solution when the system parameters change due to uncertainty. Ceylan et al [8] take a different approach by performing sensitivity analysis on travel time in a transportation network under equilibrium. Based on the analysis, a genetic algorithm-based flow estimator is designed to solve the Area Traffic Control (ATC) problem and Stochastic User Equilibrium (SUE) problem [8], [9] simultaneously.

Contributions: In this paper, instead of incorporating estimation of the uncertainties into the traffic model, we design a convex optimization problem that is agnostic to possible system changes. The proposed solution is inherently more robust to system changes than the System Optimal Equilibrium (SOE) Optimization, which minimizes the total travel time of all users in a transportation network [9]. Our novel approach solves the traffic assignment problem using network criticality, a graph metric measuring robustness of a network. We show that this method achieves robustness (lack of sensitivity) to unforeseen events, such as more traffic or accidents on the road. This will be especially useful during the deployment stages of Vehicular Ad-hoc NETworks (VANETs), where not all vehicles have Vehicle-to-Vehicle (V2V) or Vehicle-to-Infrastructure (V2I) communication devices installed, thus traffic estimation on the transportation network could be inaccurate. In addition, our optimization problem remains convex, and thus can be solved efficiently.

The rest of this paper is structured as follows. Section II provides a brief review of network criticality in graphs. Section III introduces the system model and details the proposed optimization problem, while achieving robustness or minimizing travel time. It also poses the sensitivity analysis and discusses how it is measured in this paper. Evaluation results are discussed in section IV. The paper is concluded in section V.

II. Review of Network Criticality

Network criticality in network science is a graph metric measuring the robustness of a network [10]. It reflects the effect of environmental changes such as traffic variation and capacity changes. A network is modeled as an undirected weighted graph, where the weight of a link denotes the desirability of the link. On this network, a random-walk is defined with transition probability matrix $[pr(l) = pr(i \to j)]$ where the elements are functions of link weights and denote the probability of transitioning from node i to neighbor node j along link $l = (i, j)$.

Suppose a random-walker starts at s and stops when it arrives at node d for the first time. The random-walk betweenness $b_{sk}(d)$ of a node k for source-destination pair $s - d$ is defined as the average number of visits to node k by the random-walker. The betweenness reflects the centrality of that node, which measures the importance of the node relative to the entire network. Based on betweenness, the point-to-point network criticality of node k for trajectories from s to d is defined as [11]:

$$\tau_{sd}^k = \frac{b_{sk}(d) + b_{dk}(s)}{W_k} \tag{1}$$

where $W_k = \sum_{l \in A^o(k)} w_l$, and $A^o(k)$ denotes the set of outgoing links attached to node k.

In the generic random-walks considered in this paper, the transition probability from node i to its neighboring node j is proportional to the weight of link $l = (i, j)$ (i.e. high weight due to low travel time):

$$pr(l) = \frac{w_l}{\sum_{e \in A^o(i)} w_e} \tag{2}$$

In addition, for generic random-walks, τ_{sd}^k is independent of k [10]. Consequently, the average network criticality τ (of the whole network) is defined as the mean of all point-to-point network criticalities and it can be shown to be proportional to the trace of $L^+ = [l_{ij}^+]$, the Laplacian matrix of the graph [10]:

$$\tau = \frac{1}{n(n-1)} \sum_{s,d} \tau_{sd} = \frac{2}{n-1} Tr(L^+) \tag{3}$$

Network criticality has many useful interpretations in communication networks such as congestion and average travel cost [12],[13]. Specifically, as it captures the average centrality of the network through betweenness, a higher criticality value implies that part of the network is more critical, thus more sensitive to environmental changes such as variation in capacity or load distribution. As a result, minimizing criticality reduces the centrality of links in the network, and therefore leads to a more robust solution. In this paper, network criticality is used as an alternative objective function to the total travel time to achieve robust traffic assignment in a transportation network.

III. Problem Formulation

In this section we define the system model and derive the convex optimization problem with two different objectives. We also describe our approach for analyzing the sensitivity of the solutions.

A. System Model

Suppose that the network topology is given by a directed graph $G(N, E, W)$, where N, E, and W denote the node set, link set, and link weight matrix, respectively. A link between nodes i and j is denoted by $l = (i, j)$ with link weight w_l. The sets of outgoing links and incoming links of a node k are denoted by $A^o(k)$ and $A^i(k)$, respectively. The weight matrix is in general asymmetric; however, in calculating the network criticality in this paper, we use an undirected symmetric matrix of the graph defined as $W_{sym} = \frac{W+W^T}{2}$, where W^T denotes the transpose of W [10].

We take the viewpoint of an Intelligent Transportation System (ITS) Service Provider (Public or Private) that receives traffic requests from vehicles to use the transportation network, through V2V or V2I communications. Each request is a triple $(s, d, \gamma_s(d))$, where s, d, and $\gamma_s(d)$ denote the traffic source node, traffic destination node, and the number of vehicles requesting to go from source s to destination d, respectively. This is summarized in a traffic matrix $\Gamma = [\gamma_s(d)]$ showing the traffic demand between each pair of nodes. In transportation, unlike what is common in wireless networks, metrics such as capacity, flow and density are all defined as rates (at a specific point of the road network and at a specific instant) [9]. Thus, we consider the traffic matrix to be a set of traffic rates, i.e. traffic requests per unit time.

In order to quantify the travel time of each vehicle on a link, we use the link performance function referred to as the BPR (Bureau of Public Roads) Formula [9]:

$$t(V_l) = t_{fl}\left(1 + 0.15\left(\frac{V_l}{C_l}\right)^4\right) \tag{4}$$

where $t(V_l)$ is average vehicle travel time as a function of demand volume (flow) V_l on link l, t_{fl} is free-flow travel time (no congestion) on link l, and C_l is the practical capacity of the link (around 1600veh/hr/lane $\times$ number of lanes). Therefore, if demand V_l exceeds C_l by 60% on a link due to rush hour congestion for example, travel time will be double that of free-flow travel time on that link. Finally, t_{fl} is calculated as the ratio of the length of the link to the average vehicle free-flow speed.

We can now formulate an optimization problem in which a desired convex objective function is minimized subject to traffic flow conservation constraints.

B. Optimization Problem Formulation

We aim to find a traffic flow assignment strategy such that a traffic matrix Γ is satisfied and a convex objective is minimized. For a specific node k and entry $\gamma_s(d)$ of the traffic matrix, the conservation of flow can be stated as:

$$\sum_{l \in A^o(k)} V_l^{sd} - \sum_{e \in A^i(k)} V_e^{sd} = \gamma_s(d)\delta(k-s) - \gamma_s(d)\delta(k-d) \tag{5}$$

where $V_l^{(sd)}$ is the flow of link l for traffic from source s to destination d and $\delta(x)$ is the Kronecker delta function.

After adding link flow summation and non-negativity constraints, the optimization problem for a transportation network minimizing network criticality, called *MinNC*, can be summarized as follows:

$$
\begin{aligned}
& minimize \quad \tau \qquad (6)\\
& subject \quad to\\
& \forall s,d \in N, \quad \forall l,e \in E, \quad \forall k \in N\\
& \textstyle\sum_{l \in A^o(k)} V_l^{sd} - \sum_{e \in A^i(k)} V_e^{sd} = \gamma_s(d)\delta(k-s) - \gamma_s(d)\delta(k-d)\\
& V_l = \textstyle\sum_{s,d} V_l^{sd}\\
& V_l \geq 0
\end{aligned}
$$

We would like to have robustness in the traffic distribution; therefore, we choose the inverse of the average travel time $t(V_l)$ as the weight of each link l: $w_l = \frac{1}{t(V_l)}$, where $t(V_l)$ is calculated using Equation 4. After symmetrization, this weight can be used to evaluate τ in optimization problem (6). *MinNC* distributes traffic flows such that τ is minimized, which in turn enhances the robustness in the traffic distribution.

We also define the traffic optimization problem minimizing total travel time, called *MinTT*, as follows:

$$
\begin{aligned}
& minimize \quad \textstyle\sum_{l \in E} t(V_l) \times V_l \qquad (7)\\
& subject \quad to\\
& \forall s,d \in N, \quad \forall l,e \in E, \quad \forall k \in N\\
& \textstyle\sum_{l \in A^o(k)} V_l^{sd} - \sum_{e \in A^i(k)} V_e^{sd} = \gamma_s(d)\delta(k-s) - \gamma_s(d)\delta(k-d)\\
& V_l = \textstyle\sum_{s,d} V_l^{sd}\\
& V_l \geq 0
\end{aligned}
$$

The objective function in problem (7) is the total travel time for all vehicles in the network and is based on both the flows V_l and the individual travel times $t(V_l)$ (calculated using Equation 4) on each link in the network. At low congestion, *MinTT* maximizes the flow V_l assigned to links with short lengths (small t_{fl}) and large number of lanes (large C_l). At high congestion, *MinTT* intelligently adjusts the flow assignment to reduce the flow V_l assigned to links with high t_{fl} or low C_l. Thus, it provides the optimal static flow assignment for minimizing total travel time in transportation [9]. In Section IV, we will compare the performance results of *MinNC* and *MinTT*, along with the robustness of their solutions.

C. Sensitivity Analysis

Sensitivity analysis deals with studying how much the value of an optimal solution will change in case of a change in the input or in one or more of the problem parameters [14]. It is important here to distinguish between the notion of criticality and optimality. An optimal solution is the one with the lowest objective value. A critical (sensitive) solution is the one that changes significantly when there is a small change in the input or in the problem parameters [15]. In the context of transportation, it is very hard to get an exact estimation of the traffic traveling through the transportation network [9]. Therefore, it is almost as important to get a robust solution, i.e. one with low criticality, as it is to get an optimal one. The main argument in this paper is that although *MinNC* gives a slightly larger total travel time than *MinTT*, the *MinNC* solution is more robust, i.e. less sensitive.

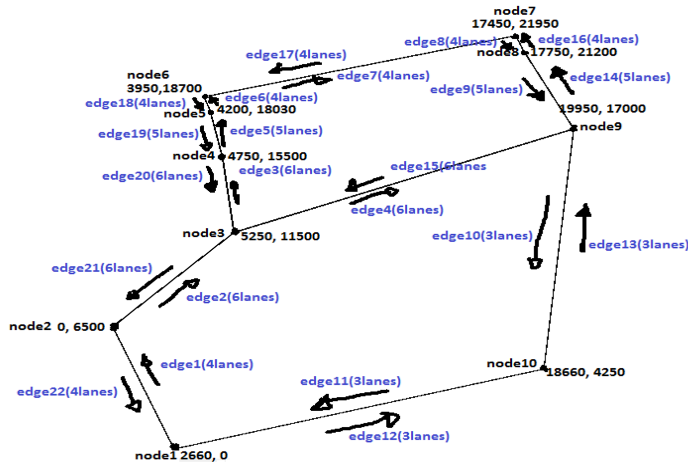

Fig. 1. Metropolitan Toronto highway map

We measure sensitivity with two approaches. The first approach is more theoretical and is based on the shadow price interpretation of the Lagrange multiplier [16]. The Lagrange multiplier is defined as the rate of change of the optimal value divided by the rate of change of a constraint; thus the smaller the value of the Lagrange multiplier, the less sensitive the solution. This means that you can relax the constraint, in our case the flow conservation constraint, and still have a small increase in the optimal value of the problem. The second approach (shown in Section IV) directly measures the change in travel time in case of uncertainty in demand and supply. In the first test, we change the demand by adding some extra traffic to the network. This additional traffic is assumed to be from vehicles that do not have any V2V or V2I communication capabilities; thus cannot send their traffic requests to the ITS Service Provider. These vehicles therefore resort to taking a shortest path solution. In the second test, we decrease the supply capacity by assuming the network is experiencing some kind of event, an accident for example. This is done by blocking one or two lanes in different links of the network. We average the effect over all links and study the increase in the average travel time of all vehicles in the network.

IV. Simulation Results

The simulation test network is shown in Fig. 1, representing the major highways in the metropolitan Toronto, Canada. The ten nodes define highway boundaries or key points on a highway where the number of lanes

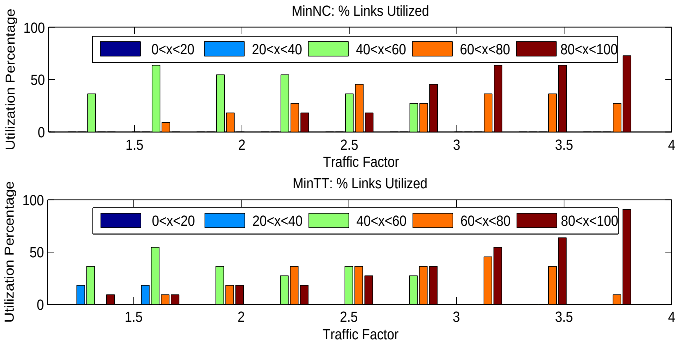

Fig. 2. Link Utilization for *MinNC* and *MinTT*

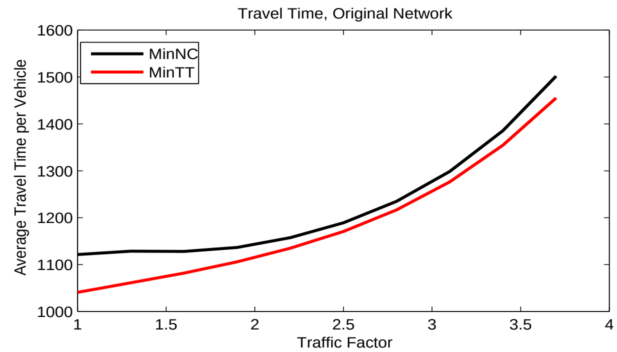

Fig. 3. Average travel time resulting from *MinNC* and *MinTT*

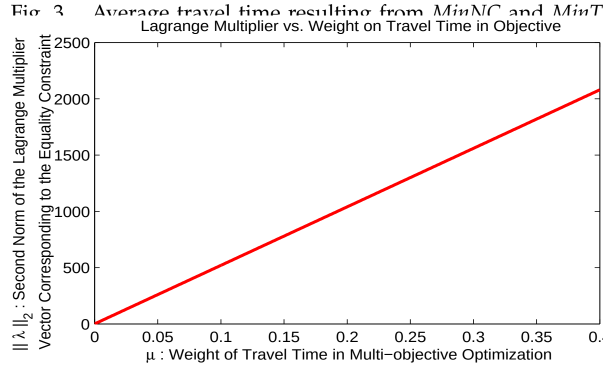

Fig. 4. Lagrange Multiplier versus weight of the *MinTT* objective

change. These nodes could be both inputs and outputs of traffic, e.g. A traffic source-destination pair can be represented by node 1 and node 9, whereby a certain number of vehicles enter the highway map from node 1 (generating node) and leave the map at node 9 (absorbing node). It is evident from the existence of loops in Fig. 1 that there are multiple possible routes for such a traffic to be satisfied. Moreover, the coordinates of these nodes correspond to accurate geographic locations, based on an origin defined in a 2-dimensional space. Therefore, the edges joining these nodes (shown in blue in Fig. 1) represent highway segments with accurate lengths, used to calculate t_{fl} using a 100 Km/hr average vehicle free-flow speed. The capacity of each link is calcuated as 1600veh/hr/lane × number of lanes.

Here, we provide the simulation results based on the network setup discussed above. The simulations include the solution of *MinNC* and *MinTT* when the traffic matrix has 6 source-destination pairs, enough to show different levels of congestion. Note that these pairs of nodes are not connected directly, thus, flows have to be sent through other nodes in the network. We start the simulations with this traffic matrix and multiply it with a constant traffic scaling factor in order to observe the response of the network with increasing congestion.

Fig. 2 shows the change in the distribution of link utilization as a function of the traffic factor. By distributing the flows better among links, *MinNC* manages to load balance the network, leading to less congestion in the operating links. For example, even in the highest traffic scenario, with a factor of 3.9, only 70% of the links are utilized more than 80% with *MinNC*, as opposed to 92% of the links being utilized more than 80% with *MinTT*. This is achieved by maintaining high weights based on the required traffic flows in the network. *MinNC* forces the distribution of flows among various links, in order to provide more robustness to unexpected events, which are not taken into account in the *MinTT* case.

Fig. 3 shows the average travel time resulting from *MinNC* and *MinTT*. Though *MinNC* results in a longer average travel time, the percentage difference is at most 8%. This is the small price paid to achieve robustness with the *MinNC* solution.

As a theoretical measure of robustness, Fig. 4 shows the second norm of the Lagrange multiplier vector corresponding to the flow conservation equality constraint, versus the weight of the *MinTT* objective in the multi-objective optimization problem (based on both *MinTT* and *MinNC*). As seen in Fig. 4, increasing this weight makes the problem more sensitive; resulting in a less robust solution. This means that any change in the state of the network (ex. higher traffic demand) will lead to a lower change in the objective if this weight is reduced. Therefore, *MinTT* is more sensitive to such changes.

A. Effect of Increased Demand

In this section, we study the robustness of *MinNC* and *MinTT* by analyzing the effect of increasing demand on their solutions (Fig. 5). We assume that there is additional traffic demand from vehicles that do not have any V2V

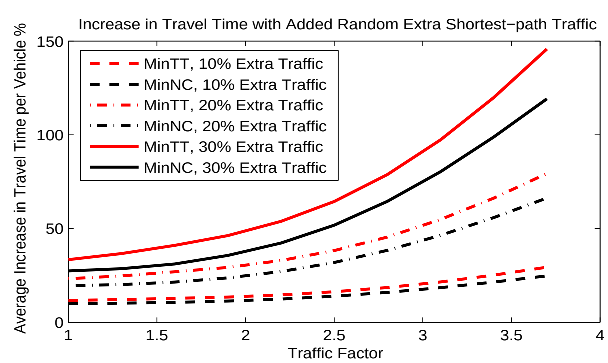

Fig. 5. Percentage increase in average travel time per vehicle

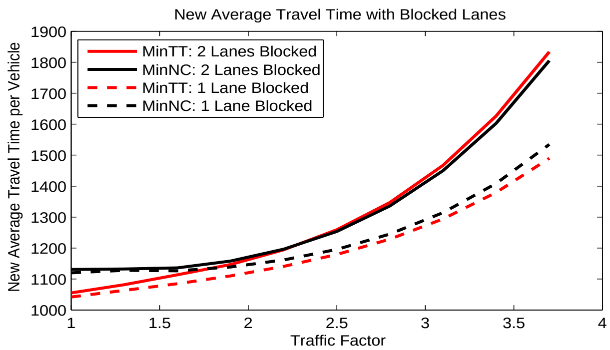

Fig. 6. New average travel time for 1 or 2 blocked lanes

or V2I communication capabilities; thus cannot send their requests to the ITS Service Provider. Therefore, they resort to a shortest path solution. The sum of this traffic (randomly generated on the 6 source-destination pairs with a normalized Gaussian distribution) is equivalent to 10% to 30% of that of the original traffic and the results are averaged over 10 different runs. As more traffic enters the network, the advantage of criticality becomes more apparent. The percentage increase in average travel time for all vehicles in case of *MinNC* is less than that of *MinTT*. The gain ranges from around 5% in the first case with 10% extra traffic, up to over 30% in the third case with 30% extra traffic.

B. Effect of Decreased Supply

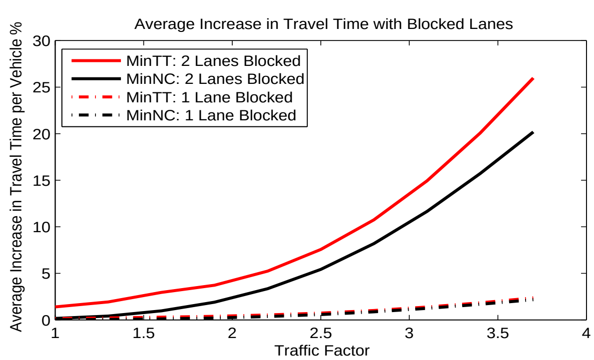

Fig. 7. Percentage increase in average travel time for 1 or 2 blocked lanes

In this section, we study the effect of decreasing supply, as an effect of an accident, malfunctioning vehicles or even some serious weather conditions that degrade the state of the road network. We simulate this effect by removing a lane from each link separately, and measure the new travel time experienced by the same original traffic. Then we average over all network links. The same is done for the case of 2 lanes removed. In terms of average travel time Fig. 6, *MinTT* performs better for lightly loaded network. The two curves approach each other as the network becomes more congested and eventually, *MinNC* gives a lower average travel time than *MinTT* in the case of 2 blocked lanes. In terms of the percentage increase in average travel time Fig. 7, *MinNC* always performs better with a gain of up to 5% in the two blocked lanes case.

V. Conclusion

In this paper, we formulated a convex optimization problem to assign the traffic flows to transportation links while achieving robustness. Our approach to formulate the optimization problem incorporates network criticality, a robustness metric from graph theory. This solution was tested under various changes in traffic conditions, such as increases in traffic demand or decreases in traffic supply. The results showed how our optimization solution is more robust than the System Optimal Equilibrium (SOE) Optimization, especially when approaching high levels of congestion.

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