Ahmad Osman

PART A

General Strategy

The problem is to solve for the required mass flow rate of mercury in a three duct system to cool air entering the middle duct at a mass flow rate of 12 kg/min with a temperature of 400 °C to a temperature of 150 °C. It is known that mercury enters the top and bottom ducts with a temperature of 20 °C. The required mass flow rate will be found at various thicknesses of steel (t = 0 mm, t = 10 mm, t = 20 mm) separating the ducts.

To solve this problem ANSYS workbench will be used to generate the geometry of the problem and mesh it. Geometry will only be created for the fluid zones, walls will not be created. Instead varying wall thicknesses will be taken into account later. This will simplify the geometry and meshing process. The geometry will be created as a single part composed of 3 bodies. The meshing strategy is discussed in Part A: Meshing.

After the geometry has been created and it has been meshed, the mesh will be exported to Fluent to solve the problem. The model will be chosen and both cell zone and boundary conditions will be set (described in **Part B**). Materials and material properties will be set (described in **Part D**).

At this point a guess for the velocity of the mercury will be input and the problem will be solved. A velocity of 1 mm/s will be used which represents a reasonable mass flow rate. If this velocity is close to generating the target air outlet temperature of 150 °C then the mesh will be refined to include more control volumes in order to capture more data. This will be done to determine if the result is mesh independent. If the result is shown to be independent of the mesh being used then that mesh will continue to be used to solve the rest of the problem. If it is not independent the mesh will be changed and tested in the same manner until it shows that it is. Details of this are described in **Part C**.

Once a mesh is developed that produces reliable results the velocity (representing mass flow rate) of mercury will be varied to determine the velocity at which air is cooled to exactly 150 °C. This will be done with a steel duct separation of 10 mm. Once the flow rate is determined at this steel thickness the process will be repeated at a thickness of 0 mm and a thickness of 20 mm. These results will be reported in **Part E**.

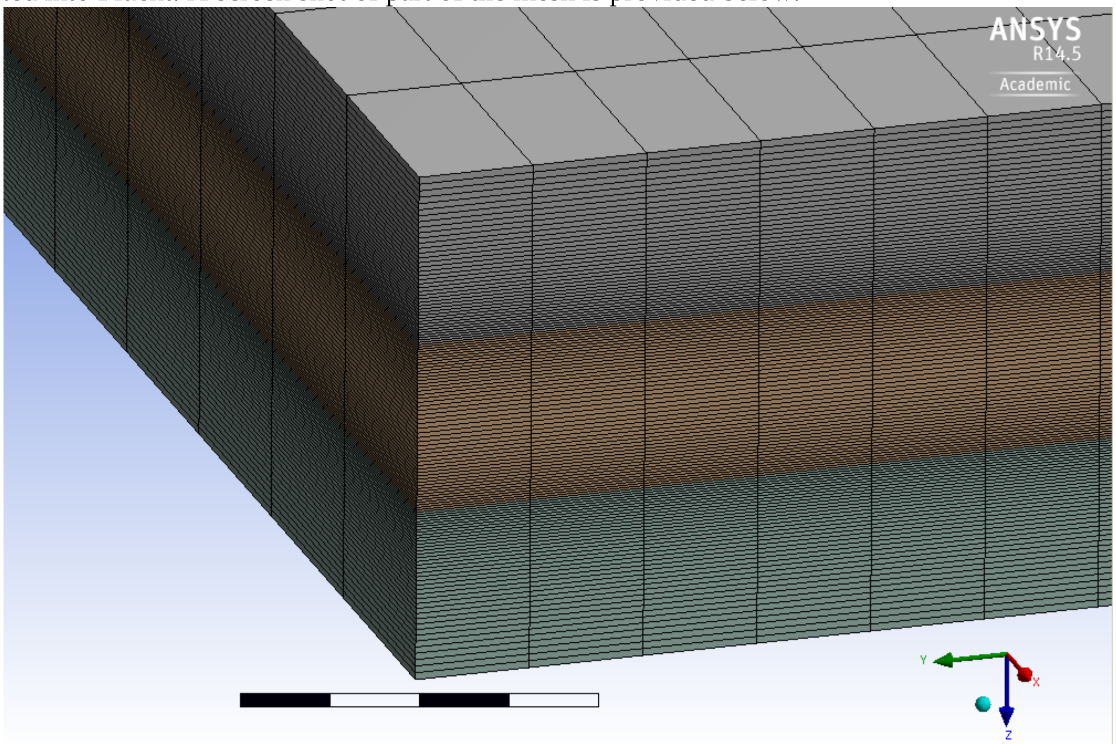
The optimum case will be chosen and explained in a discussion that begins in **Part F**. Further results will be provided in **Part G** and finally the results will be checked against the fundamental equations in **Part H** to show that they make physical sense.

Meshing

In the ANSYS workbench meshing tool, mapped face meshing was used to map the mesh to each of the faces, then edge sizing was used for all edges. For the top and bottom mercury domains the z-axis was edge sized to 40 divisions with a bias factor of 3. For the middle air domain the z-axis was edge sized to 50 divisions with a bias factor of 2. Along the y-axis (1 m dimension) each of the three domains were edge sized to 30 divisions without any bias. Along the x-axis (4 m dimension) each of the three domains were edge sized to 60 divisions without any bias. A total of 234,000 control volumes resulted. The reasoning for these choices is that data needs to be captured most where heat transfer occurs. Since this occurs at the wall boundaries between air and mercury, the mesh was refined most in those areas.

Note: The mesh should have had larger control volumes at the boundaries to have the required near-wall treatment applied so that the turbulent boundary layer could be skipped instead of being resolved incorrectly and affecting the rest of the results. If done this would result in y+ values would be in the proper range of 30 to 300-500. Time limitations prevented this from being done as the mesh forms the basis of the rest of the work and everything would have to be redone.

Named selections were created for each of the faces, creating boundaries for air and mercury inlets, outlets, boundary walls and interior domains. This allowed for easier identification when the mesh was exported into Fluent. A screen shot of part of the mesh is provided below.



PART B

In order to choose the type of model the type of flow present has to be determined (either turbulent or laminar), this is shown below. Once this is done materials need to be chosen and properties for them need to be defined, the procedure for this will be presented in **Part D**. Afterwards, cell zone conditions must be set and then boundary conditions must also be established, the method for this is shown later in Part B.

Flow Type Determination to Choose Model

In addition to the given values, the density of air at 400°C and its kinematic viscosity are [1]:

$$\rho = 0.524\ kg/m^3$$

$$\nu = 62.53 \times 10^{-6}\ m^2/s$$

Hydraulic diameter can be defined as [2]:

$$D_H = \frac{4lw}{2(l+w)} = \frac{4(0.05m \times 1m)}{2(0.05m + 1m)} = 0.0952\ m$$

Given mass flow rate, velocity can be found using the following equation:

$$\dot{m} = \rho \vec{\mathrm{v}} A$$

$$\vec{\mathrm{v}} = \frac{\dot{m}}{\rho A} = \frac{12kg/min}{(0.524kg/m^3)(0.05m^2)} = \frac{0.2kg/s}{(0.524kg/m^3)(0.05m^2)} = 7.63\ m/s$$

Reynolds number can be defined as [3]:

$$Re = \frac{\vec{v} \cdot D_H}{\nu} = \frac{(7.63m/s)(0.0952m)}{62.53 \times 10^{-6} m^2/s} = 11616$$

Where, $\vec{v}$ is the mean velocity of the fluid, D_H is the hydraulic diameter, and ν is the kinematic viscosity. It can be seen that the Reynolds number is 11616 which is greater than the turbulence threshold of 2300. This indicates that a turbulent model is required for this problem. The exported mesh from ANSYS Workbench was imported into Fluent and after checking the mesh the energy equation was turned on because this is a heat transfer problem and the k-ε model for turbulent flow was also chosen because the air flow is turbulent.

Cell Zone Conditions

Each of the domains were assigned their appropriate material. However there was uncertainty whether mercury would be experiencing turbulent or laminar flow depending on the chosen velocity. Calculations were performed to make this determination.

The kinematic viscosity of mercury at 293 °K is [5]:

$$\nu = 0.114 \times 10^{-6} \ m^2/s$$

The hydraulic diameter for mercury can be defined as [2]:

$$D_H = \frac{4lw}{2(l+w)} = \frac{4(0.05m \times 4m)}{2(0.05m + 4m)} = 0.09876 \ m$$

At a velocity of 2.7 mm/s the Reynolds number can be defined as [3]:

$$Re = \frac{\vec{v} \cdot D_H}{\nu} = \frac{(0.0027 \ m/s)(0.09876 \ m)}{0.114 \times 10^{-6} m^2/s} = 2339$$

This is higher than the turbulence threshold of a Reynolds number eq0ualing 2300. Therefore, if the velocity is greater than 2.7 mm/s at 293 °K then mercury will undergo turbulent flow. In cases when the velocity was set to be less than 2.7 mm/s a laminar zone was used for the mercury cell zones.

Boundary Conditions

Turbulence intensity needs to be calculated for both air and for mercury at all velocities above 2.7 mm/s. The turbulence intensity for air at the given conditions is given below. The turbulence intensity for mercury at a sample velocity of 1 m/s is also given below:

$$I_{air} = 0.16Re^{-\frac{1}{8}} = 0.16(11616)^{-\frac{1}{8}} = 0.0496$$

$$I_{mercury} = 0.16Re^{-\frac{1}{8}} = 0.16(866363)^{-\frac{1}{8}} = 0.0289$$

For the air inlet, a velocity inlet was chosen, and under the momentum tab a velocity of 7.63 m/s was set with a turbulence intensity of 4.96 % and a hydraulic diameter of 0.0952 m. Under the thermal tab a temperature of 673.15 °K was set.

For the both of the mercury inlets, a velocity inlet was chosen, and under the momentum tab various velocities were input and if the velocity was above 2.3 mm/s then a turbulence intensity was entered along with the hydraulic diameter of 0.0987 m, otherwise only the velocity was entered. Under the thermal tab a temperature of 293.15 °K was set.

Each of the outlets were set as pressure outlets. Each of the walls that did not interface between mercury and air were set as stationary walls with a no slip condition. The two walls that interfaced between mercury and air were also set as stationary walls with a no slip condition and for the thermal tab the walls were set as coupled.

PART C

Mesh Independence

In order to determine that the results were independent of the mesh, several meshes with different dimensions were created. Each mesh had more control volumes than the last and the dimensions were varied. All parameters were kept the same (1 mm/s velocity of mercury and 10 mm wall thickness) to determine how the temperature of the air would change, if at all, for the same mass flow rate of mercury. It was found that the larger meshes did not have a temperature change greater than ±2 °K from the median value. This is an acceptable deviation. The largest mesh is 2.1 times larger (495,000 control volumes) than the mesh being used (234,000 control volumes) and so this gives confidence that there is mesh independence.

Mesh Size (Control Volumes)	Dimensions (T + M + B) x L x W	Mercury Velocity (mm/s)	Average Air Outlet Temperature (°K)	Average Air Outlet Temperature (°C)
234,000	(40+50+40)x60x30	1	426.09	152.94
352,000	(30+50+30)x80x40	1	427.65	154.50
416,000	(40+50+40)x80x40	1	427.63	154.48
495,000	(30+50+30)x90x50	1	428.57	155.42

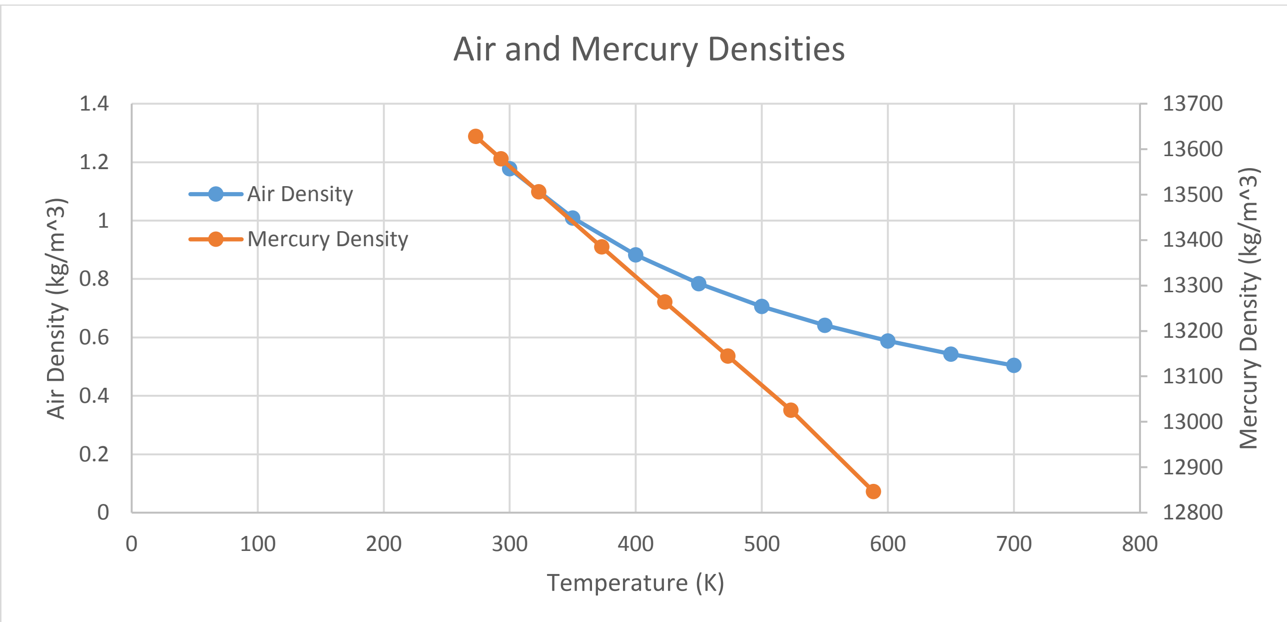
PART D

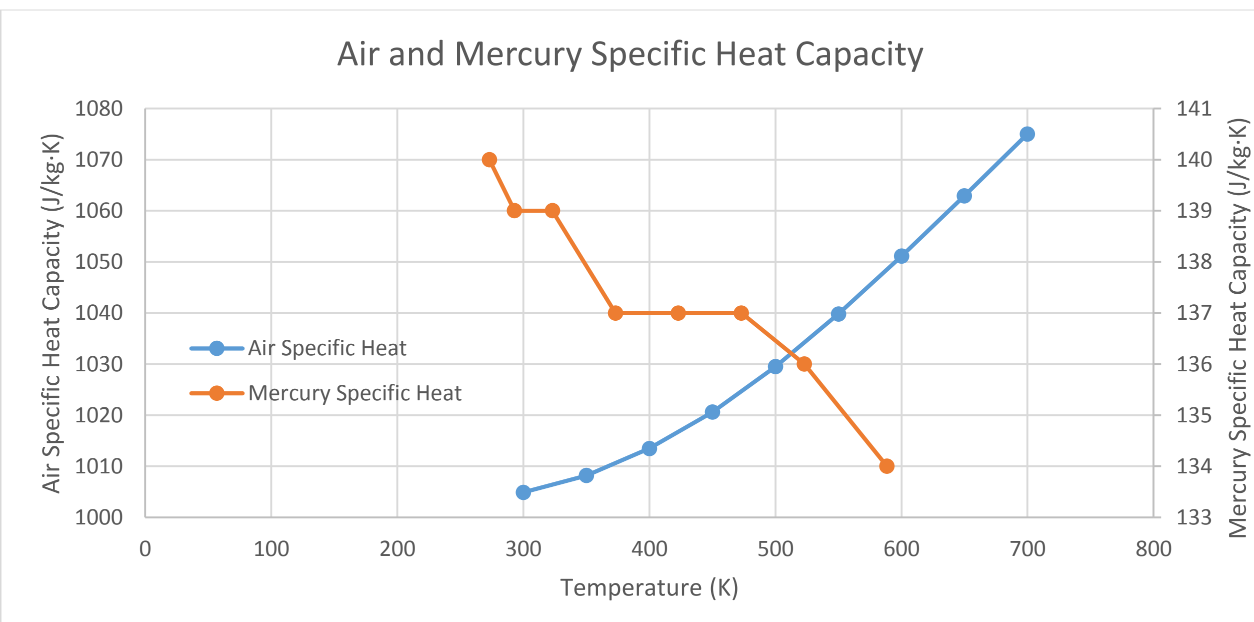
Materials and Material Properties

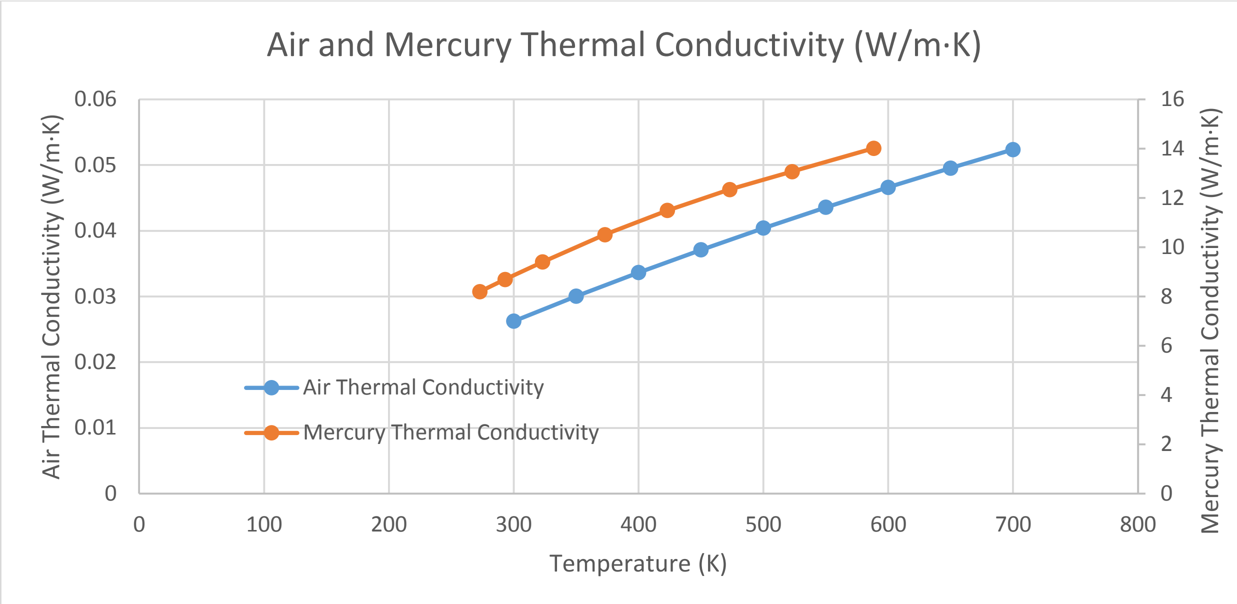
For this problem, air, mercury and steel were chosen as materials in Fluent. For air, dry air properties were obtained at various temperatures and values for density, specific heat capacity, thermal conductivity and dynamic viscosity were input to generate a piece-wise linear function for each property. This was done because the properties experience significant change over the range of temperatures that these fluids are expected to experience. This same process was repeated for mercury. For air 9 points were used, and for mercury 8 points were used. For steel the constant values in the Fluent database were used.

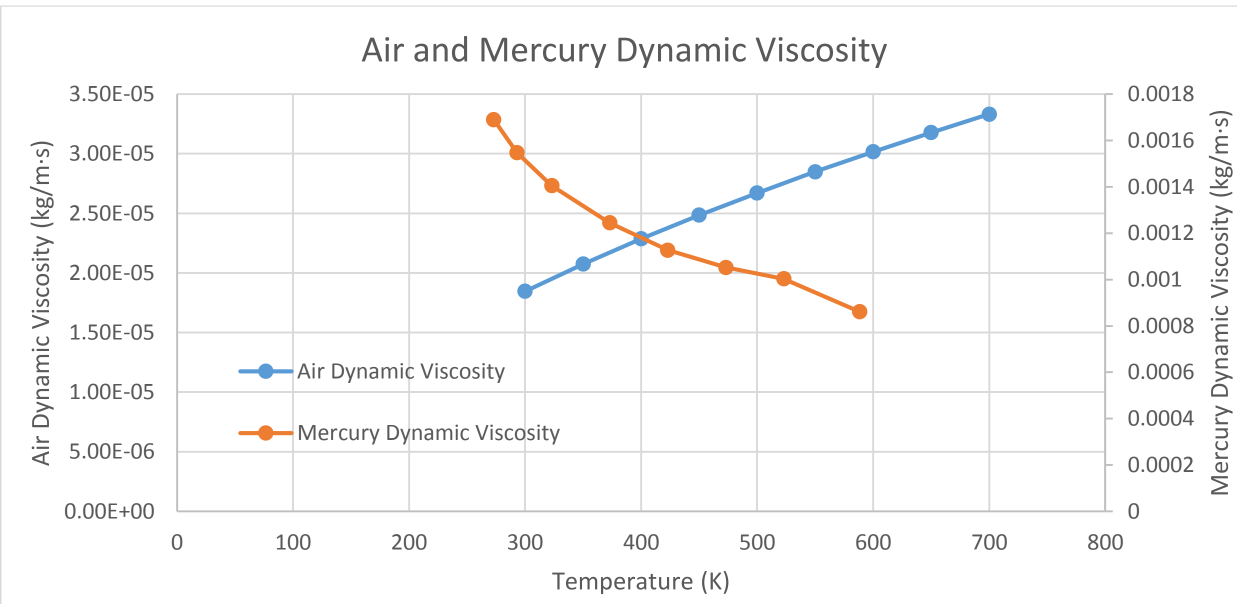
Dry Air Properties [4]				
Temperature (°K)	Density (kg/m^3)	Specific Heat Capacity (J/kg·K)	Thermal Conductivity (W/m·K)	Dynamic Viscosity (kg/m·s)
300	1.177	1004.9	0.02624	1.846E-05
350	1.009	1008.2	0.03003	2.075E-05
400	0.8824	1013.5	0.03365	2.286E-05
450	0.7844	1020.6	0.0371	2.485E-05
500	0.706	1029.5	0.04041	2.67E-05
550	0.6418	1039.8	0.04357	2.849E-05
600	0.5883	1051.1	0.04661	3.017E-05
650	0.543	1062.9	0.04954	3.178E-05
700	0.5043	1075	0.05236	3.332E-05

Mercury Properties [5]				
Temperature (°K)	Density (kg/m^3)	Specific Heat Capacity (J/kg·K)	Thermal Conductivity (W/m·K)	Dynamic Viscosity (kg/m·s)
273	13628	140	8.2	0.00169
293	13579	139	8.69	0.001548
323	13506	139	9.4	0.001405
373	13385	137	10.51	0.001245
423	13264	137	11.49	0.001127
473	13145	137	12.34	0.001052
523	13026	136	13.07	0.001003
588.5	12847	134	14.02	0.000861









PART E

Results

It was found that for a wall thickness of 0 mm the required mass flow rate for mercury is 5.97 kg/s total in order to achieve an average outlet temperature of 150 °C. For a wall thickness of 10 mm the required mass flow rate of mercury is 6.79 kg/s. For a wall thickness of 20 mm the required flow rate of mercury is 8.15 kg/s. In each of these cases the mercury does not reach the boiling temperature of 356 °C and so the requirement that the mercury remain a fluid is always satisfied. Since it is easier to pump mercury at a lower flow rate than a higher one, this is preferable. If it was physically possible to have a wall thickness of 0 mm then 5.97 kg/s would be the optimal flow rate for mercury. However, this is not possible, therefore a flow rate of 6.79 kg/s with a wall thickness of 10 mm is the best choice. A chart showing the results of all attempts at the chosen mesh is provided in **Part F**.

PART F

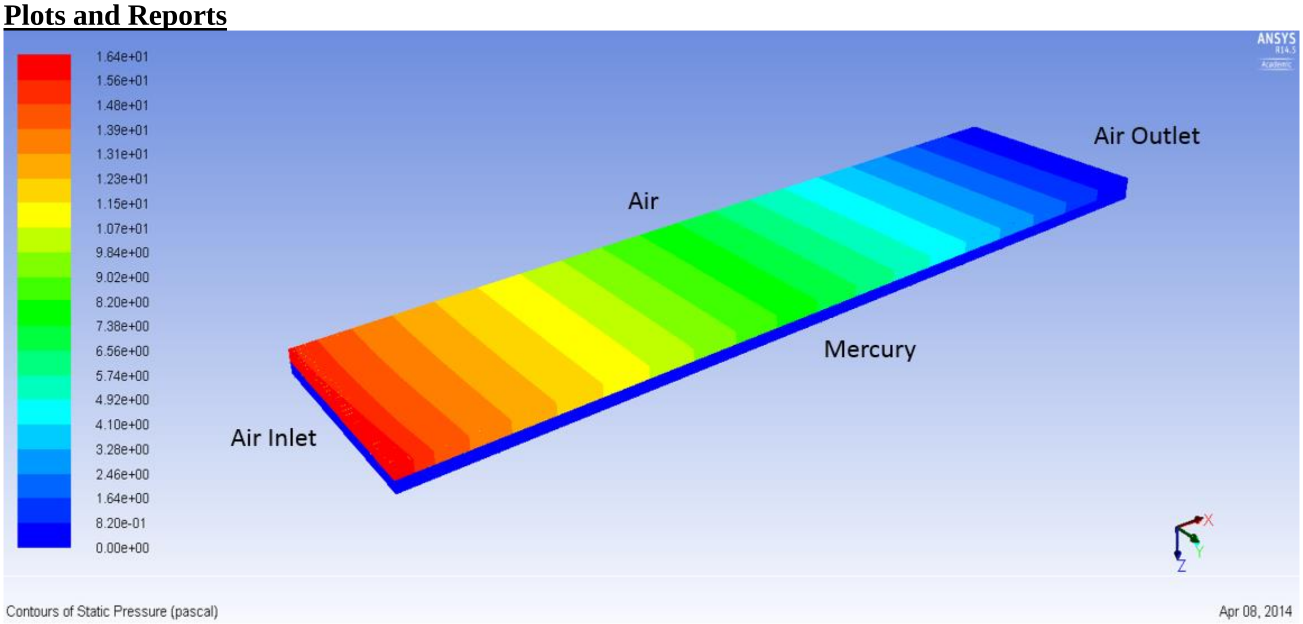
Discussion

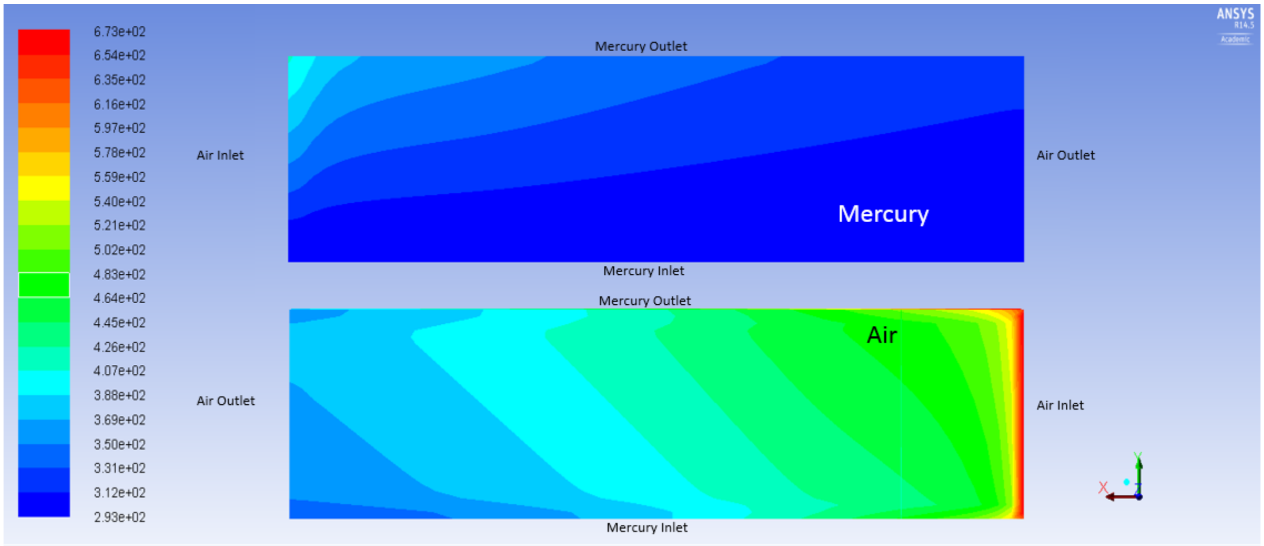
The important parameters for each case tested is determining whether mercury is undergoing turbulent or laminar flow, the chosen inlet velocity of mercury (and therefore mass flow rate) and the wall thickness of steel separating the fluids. In order to find the optimum value the velocity was varied by an order of magnitude until the outlet air temperature came close to the desired result. Then when it was close the value was increased or decreased slowly until the required outlet temperature was reported. As the wall thickness varied an educated guess was made that allowed the required mass flow rate to be obtained sooner. The process was sped up when data from previous results was used as the initial starting condition for the next case. The choice between results to determine the optimal wall thickness and flow rate is discussed above in **Part E**.

Mercury Velocity (m/s)	Turbulence Intensity (%)	Total Mercury Mass Flow Rate (kg/s)	Wall Thickness (mm)	Average Outlet Temp of Air (K)	Average Outlet Temp of Air (C)
1.00E+00	0.028967	5431.60	10	400.4527	127.30
1.00E-01	0.038628	543.16	10	401.906	128.76
1.00E-02	0.051512	54.32	10	406.4571	133.31
1.00E-03	Laminar	5.43	10	426.0903	152.94
1.50E-03	Laminar	8.15	10	420.1456	147.00
1.25E-03	Laminar	6.79	10	422.8539	149.70
1.25E-03	Laminar	6.79	0	420.4268	147.28
1.20E-03	Laminar	6.52	0	421.334	148.18
1.15E-03	Laminar	6.25	0	422.0686	148.92
1.10E-03	Laminar	5.97	0	422.8594	149.71
1.50E-03	Laminar	8.15	20	422.7711	149.62

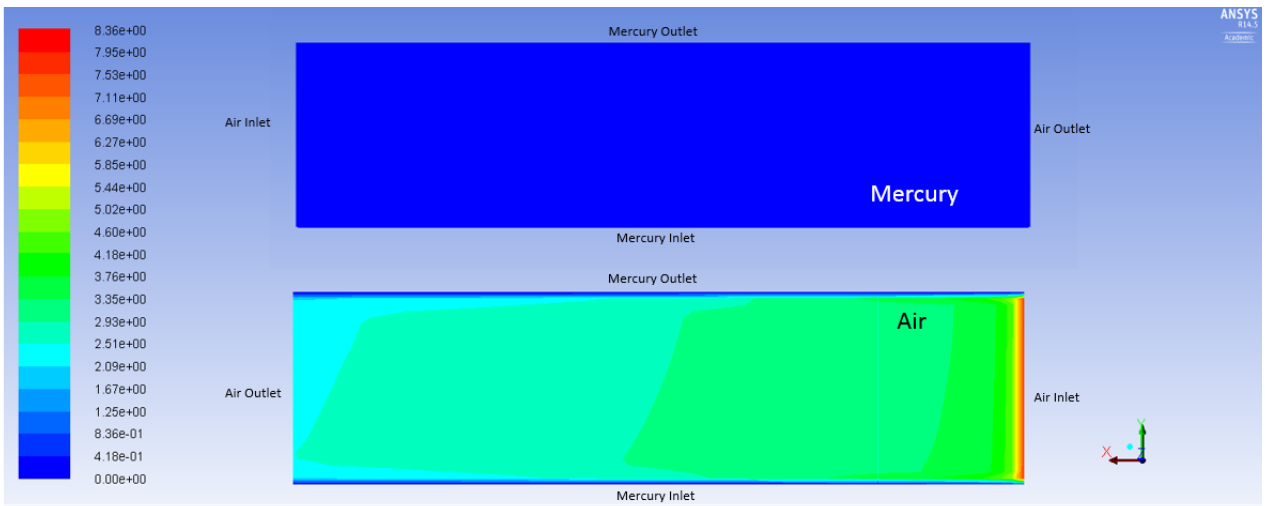
PART G

Plots and Reports

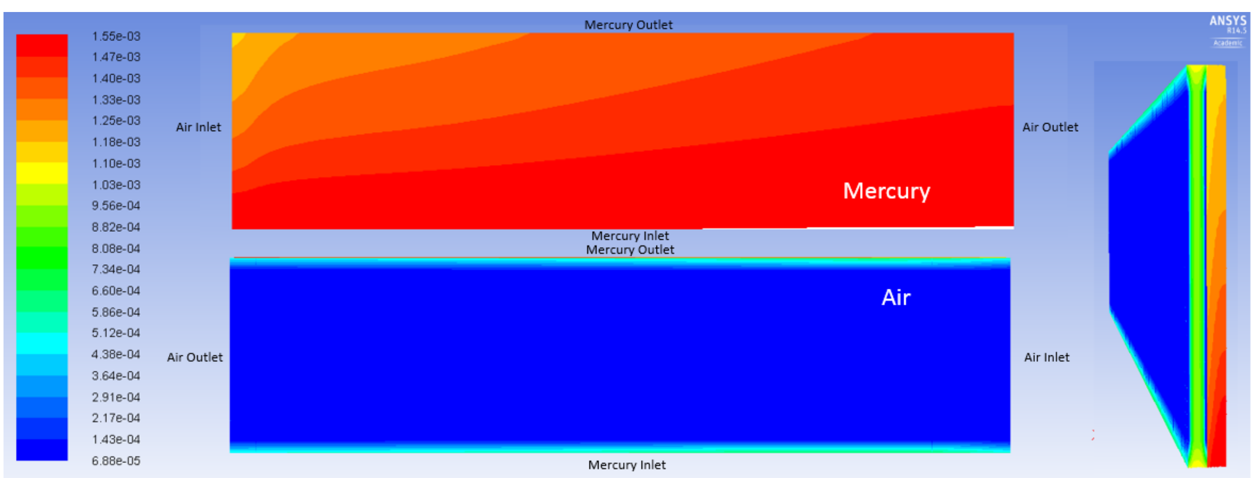




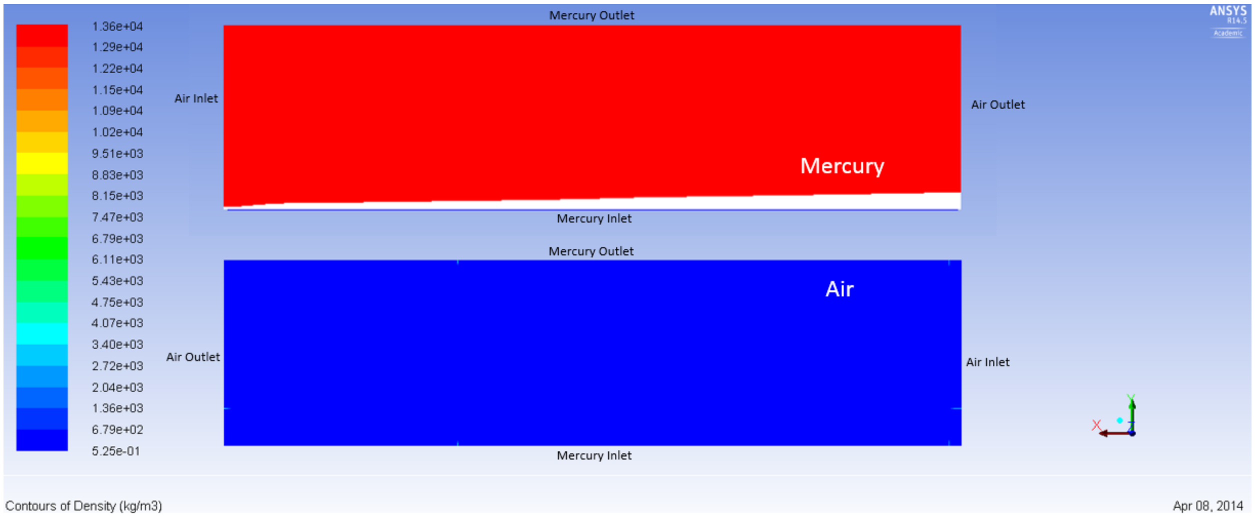
Contours of Static Temperature (k) Apr 08, 2014



Contours of Velocity Magnitude (m/s) Apr 08, 2014



Contours of Effective Viscosity (kg/m-s) Apr 08, 2014



<u>Total Heat Transfer</u>

26082.58 W + 26082.58 W = 52.165 kW

<u>Average Mercury Temperature at Outlet</u>

348.1253 K

<u>Cooling Capacity per Cooling Mercury</u>

$$\frac{\Delta T}{\dot{m}} = \frac{(348.13 - 293.15)}{6.79} = 8.10 \text{ K} \cdot \text{s/kg}$$

<u>Power</u>

$$\Delta PQ = \frac{\Delta P \times \dot{m}}{\rho} = \frac{(16.40) \times 6.79}{13500} = 8.24 \times 10^{-3} W$$

<u>Cooling Capacity per Power</u>

$$\frac{\Delta T}{\Delta PQ} = \frac{\Delta T \rho}{\Delta P \dot{m}} = \frac{(348.13 - 293.15) \times 13500}{(16.40) \times 6.79} = 6665 \text{ K}/W$$

<u>PART H</u>

<u>Fundamental Equation Check</u>

To check that the results are consistent with fundamental heat transfer equations a mass and energy balance can be performed. Since this solution is at steady state then the amount of heat transferred between air and mercury should be equal:

$$\dot{Q}_{air} = \dot{Q}_{mercury}$$

The heat transfer can be described by:

$$\dot{Q} = \dot{m} c_p (T_f - T_i)$$

Where, $\dot{m}$ is the mass flow rate, c_p is the specific heat and T_f is the final temperature and T_i is the initial temperature.

The mass flow rate for air is given at 0.2 kg/s, the initial temperature of air is 673.15 °K, the final desired temperature of air is 423.15 °K and the value for the heat transfer coefficient can be found as the average of the heat transfer coefficients at the initial and final temperatures. The heat transfer coefficient at the initial temperature is 1077.13 J/kg·K and at the final temperature it is 1018.87 J/kg·K.

For mercury the mass flow rate is the unknown quantity to be determined, the initial temperature of mercury is given at 293.15 °K, the final temperature is according to the ANSYS solution is 348.12 °K. The heat transfer coefficient at the initial temperature is 139 J/kg·K and the heat transfer coefficient at the final temperature is 138 J/kg·K. The equation becomes:

$$\dot{m}_{air}\left(\frac{c_{p\_air\_T_f} + c_{p\_air\_T_i}}{2}\right)\left(T_{f\_air} - T_{i\_air}\right) = \dot{m}_{merc}\left(\frac{c_{p\_merc\_T_f} + c_{p\_merc\_T_i}}{2}\right)\left(T_{f\_merc} - T_{i\_merc}\right)$$

$$0.2\left(\frac{1018.87 + 1077.13}{2}\right)(423.15 - 673.15) = \dot{m}_{merc}\left(\frac{138 + 139}{2}\right)(348.12 - 293.15\ )$$

$$\dot{m}_{merc} = 6.88\ kg/s$$

This compares very favourably with the ANSYS result of 6.79 kg/s.

IMPORTANT NOTE

Also since the k-ε turbulence model is used the control volumes should be larger at the walls in order to obtain y+ values above 30 and below 300 and account for the thin turbulent boundary layer. This was realized too late while reviewing lecture notes to redo all of the simulations however if there was time, the following explains the steps that would be taken.

The mesh would be redone to have control volumes of larger size at all the boundaries, then the simulation would be re-run with the same conditions. A y+ plot would be generated and the mesh would be adjusted until the y+ values were in the proper range. Once this was obtained, it is expected that mesh independence would be even more clearly displayed with a variation of less than 1 degree between changing the mesh size.

REFERENCES

[1] http://www.engineeringtoolbox.com/air-properties-d_156.html
[2] http://en.wikipedia.org/wiki/Hydraulic_diameter
[3] http://en.wikipedia.org/wiki/Reynolds_number
[4] http://www.engineeringtoolbox.com/dry-air-properties-d_973.html
[5] http://www.engineeringtoolbox.com/mercury-d_1002.html
[6] http://www.cfd-online.com/Wiki/Turbulence_intensity