Tutorial 2

Problem 2.1

Suppose that the function g is defined for all $x \in R$ and satisfies the following properties for all $a,b>0$:

$$g(8)=0,\ g\left(\frac{8}{b}\right)=g(b)$$

Prove that $f(x)=g\left(x+\sqrt{x^2+8}\right)$ is even.

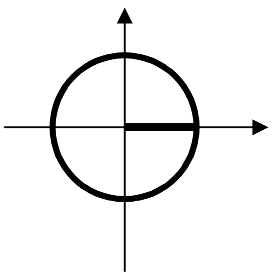
Solution:

$$f(-x)=g\left(-x+\sqrt{x^2+8}\right)=g\left(\frac{8}{x+\sqrt{x^2+8}}\right)=g\left(x+\sqrt{x^2+8}\right)=f(x)$$

Problem 2.2

Solve $\cos(5x)=1$.

Solution:

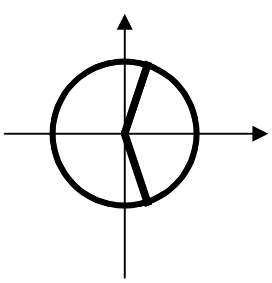


$$5x=2\pi k \Rightarrow x=\frac{2}{5}\pi k, k\in Z$$

Problem 2.3

Solve $\cos(5x)=\frac{1}{2}$.

Solution:



$$5x=\pm\frac{\pi}{3}+2\pi k \Rightarrow x=\pm\frac{\pi}{15}+\frac{2}{5}\pi k, k\in Z$$

Some really important trigonometric identities!!!

1. $\sin 2\theta = 2\sin\theta\cos\theta$
2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
3. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
4. $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
5. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
6. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Problem 2.4

Solve: $2\cos^2 x - \sin x - 1 = 0$

Solution:

$$2\cos^2 x - \sin x - 1 = 0 \Leftrightarrow 2\left(1-\sin^2 x\right) - \sin x - 1 = 0 \Leftrightarrow 2 - 2\sin^2 x - \sin x - 1 = 0 \Leftrightarrow$$

$$2\sin^2 x + \sin x - 1 = 0$$

Substitution: $\sin x = t \Rightarrow 2t^2 + t - 1 = 0 \Leftrightarrow t = \frac{1}{2}; -1$

So,

$$\sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} + 2\pi k; x = \frac{5\pi}{6} + 2\pi k$$

$$\sin x = -1 \Leftrightarrow x = -\frac{\pi}{2} + 2\pi k$$

Problem 2.5

Prove: $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

Solution:

$$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x} \Leftrightarrow 1-\sin^2 x = \cos^2 x \Leftrightarrow \sin^2 x + \cos^2 x = 1$$

Problem 2.6

Prove: $\frac{1+\tan\alpha}{1+\cot\alpha} = \tan\alpha$

Solution:

$$\frac{1+\tan\alpha}{1+\cot\alpha} = \tan\alpha \Leftrightarrow 1+\tan\alpha = \left(1+cot\alpha\right)\tan\alpha \Leftrightarrow 1+\tan\alpha = 1+\tan\alpha$$

Problem 2.7

Find $\cos\theta$ if $\sin\frac{\theta}{2} = 0.2$

Solution:

$$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{2} = 0.92$$

Problem 2.8

Solve $\sin^2\theta + ctg^2\theta = 1$

Solution:

$$\sin^2\theta + cot^2\theta = 1 \iff \sin^2\theta + \frac{\cos^2\theta}{\sin^2\theta} = 1 \iff \sin^4\theta + \cos^2\theta = \sin^2\theta \iff \sin^4\theta - 2\sin^2\theta + 1 = 0 \iff$$

$$\left(\sin^2\theta - 1\right)^2 = 0 \iff \sin\theta = \pm 1 \iff \theta = \frac{\pi}{2} + \pi k$$

Problem 2.9

Solve: $2\sin x + \cos x \cdot ctg\, x + 2 = 0$

Solution:

$$2\sin x + \cos x \cdot \cot x + 2 = 0 \iff 2\sin x + \frac{\cos^2 x}{\sin x} + 2 = 0 \iff 2\sin^2 x + 1 - \sin^2 x + 2\sin x = 0 \iff$$

$$\sin^2 x + 2\sin x + 1 = 0 \iff \left(\sin x + 1\right)^2 = 0 \iff \sin x = -1 \iff x = -\frac{\pi}{2} + 2\pi k$$