# Tutorial 6

***Problem 6.1***

Carry out the differentiation $\frac{d}{dx}\left(\sqrt{\frac{2x+1}{x-5}}\right)$

**Solution:**

$$\frac{d}{dx}\left(\sqrt{\frac{2x+1}{x-5}}\right)=\frac{1}{2\sqrt{\frac{2x+1}{x-5}}}\left(\frac{2x+1}{x-5}\right)'=\frac{\frac{2(x-5)-(2x+1)\cdot 1}{(x-5)^2}}{2\sqrt{\frac{2x+1}{x-5}}}=-\frac{11}{2(x-5)^2\sqrt{\frac{2x+1}{x-5}}}$$

***Problem 6.2***

Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(1;2)$ $x^3+4x^2y+y^2+5y-23=0$

**Solution:**

The first derivative:

$$3x^2+4\left(2xy+x^2y'\right)+2yy'+5y'=0 \Leftrightarrow 3x^2+8xy+4x^2y'+2yy'+5y'=0 \Rightarrow y'=-\frac{3x^2+8xy}{4x^2+2y+5} \Rightarrow y'=-\frac{19}{13}$$

The second derivative:

$$6x+4\left(2y+2xy'+2xy'+x^2y''\right)+2y'y'+2yy''+5y''=0 \Rightarrow y''=-\frac{6x+8y+8xy'+8xy'+2y'y'}{4x^2+2y+5}=\ldots$$

***Problem 6.3***

Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $\left(\pi;-\frac{\pi}{3}\right)$ $\sin(x+3y)=0$.

**Solution:**

The first derivative:

$$(1+3y')\cos(x+3y)=0 \ \left(\cos\left(\pi-3\frac{\pi}{3}\right)\neq 0\right) \Rightarrow y'=-\frac{1}{3}$$

The second derivative: $y''=0$

***Problem 6.4***

Find all points of the curve $x^3y^2+x+xy=2$ , where the slope of the tangent line is -1.

**Solution:**

The first derivative: $3x^2y^2+2x^3yy'+1+y+xy'=0 \;\Rightarrow\; y'\left(2x^3y+x\right)=-\left(1+y\right)-3x^2y^2 \;\Rightarrow$

$$y'=-\frac{\left(1+y\right)+3x^2y^2}{2x^3y+x}.$$

The slope of the tangent line is -1 at $-\frac{\left(1+y\right)+3x^2y^2}{2x^3y+x}=-1$

All the points of the curve are: $\begin{cases}\frac{\left(1+y\right)+3x^2y^2}{2x^3y+x}=1\\ x^3y^2+x+xy=2\end{cases}$

If you could solve the system of equations you should solve it.

***Problem 6.5***

Using $y$ as the independent variable and $x$ as the dependent variable, find $\frac{dx}{dy}$ for the equation

$\left(x^3+y^4\right)^2=4x^2y$, when $x=1$ and $y=1$.

**Solution:**

The first derivative:

$$2\left(x^3+y^4\right)\left(3x^2x'+4y^3\right)=4\left(2xx'y+x^2\right)$$

$$2\left(x^3+y^4\right)3x^2x'+8\left(x^3+y^4\right)y^3=8xx'y+4x^2$$

$$x'\left(2\left(x^3+y^4\right)3x^2-8xy\right)=4x^2-8\left(x^3+y^4\right)y^3$$

$$x'=\frac{4x^2-8\left(x^3+y^4\right)y^3}{2\left(x^3+y^4\right)3x^2-8xy}$$

For the point $\left(1;1\right)$ we have $x'\left(1\right)=\frac{4-16}{12-8}=-1.5$

***Problem 6.6***

Two cars begin their movement, when they begin from $\left(0;0\right)$. The first car is traveling east along the x axis at $70\,km/h$. The second one is traveling east along the line $y=x$ at $90\sqrt{2}km/h$. At what rate the distance between the cars increasing after one hour?

**Solution:**

The position of the first car is $\left(70t;0\right)$ and for the second car is $\left(90t;90t\right)$.

So, the distance is $\sqrt{\left(90t-70t\right)^2+\left(90t-0\right)^2}=\sqrt{400t^2+8100t^2}=10\sqrt{85}t$

So, the rate is $10\sqrt{85}$.

***Problem 6.7***

At what angle the parabolas $y=-x^2+8$ and $y=x^2$ intersect at the point $(2;4)$?

**Solution:**

The angle of $y=x^2$ is $y'=2x \Rightarrow y'(2)=4$, the angle with the x axis is $\alpha_1$

The angle of $y=-x^2+8$ is $y'=-2x \Rightarrow y'(2)=-4$, the angle with the x axis is $\alpha_2$

$\alpha$ is an angle between the curves. Then $\tan\alpha=\tan(\alpha_1-\alpha_2)=\dfrac{\tan\alpha_1-\tan\alpha_2}{1+\tan\alpha_1\tan\alpha_2}=\dfrac{4-(-4)}{1+4(-4)}=-\dfrac{8}{7}$

***Problem 6.8***

Determine the numbers $x$ between $0$ and $2\pi$ where the line tangent to the curve is horizontal:

$y=\sin x+\cos^2 x$.

**Solution:**

We are looking for the values of x, when the $y'(x)=0$.

$y'=\cos x-2\cos x\sin x=0 \Leftrightarrow \cos x=2\cos x\sin x \Leftrightarrow \sin x=\dfrac{1}{2}$ or $\cos x=0$, $x=\dfrac{\pi}{6};\dfrac{5\pi}{6};\dfrac{\pi}{2};\dfrac{3\pi}{2}$