A Robust Power Control Method for Cognitive Radio Networks

N. Movahhedinia[*1], A. Tizghadam[**], A. Leon-Garcia[**]

[*]Computer Engineering Dept., Faculty of Eng., University of Isfahan, Isfahan, Iran
Email: naserm@eng.ui.ac.ir
[**]School of Electrical and Computer Engineering, University of Toronto, Toronto, Canada
Email: {alberto.leongarcia, ali.tizghadam}@utoronto.ca

Abstract- **One of the issues in Cognitive Radio Networks (CRNs) is how to manage the transmit powers of the secondary users to achieve the best network performance while preserving the interference on the primary users below an acceptable level. Link failures, node mobility and changing interference level make robustness a challenging issue in CRN. In this paper, considering robustness as the optimization criterion for power management, we minimize Network Criticality, a recently developed robustness metric, with primary users' interference requirements and total capacity concavity as the constraints. Our method is carried out on a decentralized strategy distributed in secondary nodes. We show that the proposed method not only provides stability to the power control mechanism but also improves network performance in terms of capacity increase.**

I. INTRODUCTION

Boosted by Software Defined Radio (SDR), Cognitive Radio technology is anticipated to enhance spectrum utilization by leveraging spectrum holes. The objective of CRN is to promote spectrum utilization enhancement through opportunistic or cooperative relay networking. In such a network the spectrum which is likely licensed to some Primary Users (PUs) (legacy users such as TV broadcast stations), is shared by unlicensed or Secondary Users (SUs) opportunistically (Opportunistic Spectrum Allocation (OSA)), or on a leasing basis abiding by the constraints imposed by the PUs, provided that no interference or disturbance is to be caused to the PUs. The SUs may relay the PUs' signal to provide transmission as well as capacity gain for the system as a whole. To achieve higher bandwidth utilization and reliability, another approach for spectrum sharing between PUs and SUs is to allow CR to transmit even when the PR link is active, provided that the interference to the primary transmissions remains below an acceptable threshold.

In typical CR systems the PUs' unpredictable presence, the contention among SUs to seize the vacant bandwidth, the traffic variations, and the interference caused by rivals result in fluctuating capacity provided to SUs, requiring enhanced power control mechanisms. On other hand, unequal PUs' channel bandwidth, different interference levels and dissimilar assigned power for SUs may cause poor throughput or unfairness. Per se, inappropriate power assignment to SUs brings about reduced system efficiency. Therefore, there should be a centralized or distributed (more consistent with ad hoc networks) mechanism to manage the transmission powers in a collaborative fashion to decrease level of interference and improve utilization while maintain robustness.

As link outage due to node movements and unreliable wireless media may cause network break down especially in bottlenecks, robustness is one of the main concerns in CRNs. In this paper our target is to optimize the power control mechanism in terms of network wide robustness criterion while the PU's interference levels are preserved below the acceptable thresholds. Our approach is based on a concept developed for network resource management which is referred to as Network Criticality [1], to achieve highest level of robustness as well as fairness and network capacity increase.

The rest of this paper is organized as follows. In section II, we review some of the previous research in CR power control. In section III our approach based on network criticality is described. In section IV we show how to deploy relaxation method to solve a nonlinear optimization problem. Section V is dedicated to simulation results and the paper in concluded in section VI.

II. PREVIOUS RESEARCH

Power management in the context of CR networks has been considered in recent literature. To control the secondary transmitter transmit power, [2] considers the peak power and the average interference constraints at the primary receivers to characterize the power adaptation strategies that maximize the secondary user SNR and capacity. In [3] a power control strategy for the cognitive users is proposed where by opportunistically adapting its transmit power, the cognitive user can maximize its achievable transmission rate without degrading the outage probability of the primary user. Reference [4] considers the case when global knowledge of all active subscribers is available for making control decisions. Solving a mixed-integer linear programming for downlink channel/power allocation, they propose a scheme that maximizes the number of supported subscribers. For local knowledge of active subscribers within each cell they propose a scalable two-phase channel/power allocation scheme.

In [5] a method, called PCLC (Primary-Capacity-Loss Constraint), is proposed to protect the primary transmission by ensuring that the maximum Ergodic capacity loss of the PR link due to the CR transmission is not greater than some predefined value. The new CR power control policy is shown

[1] The first author has been on sabbatical at the Electrical and Computer Engineering department of the University of Toronto, Canada. His sabbatical is partly supported by Iranian Education &Research Institute for ICT.

to be superior over the conventional one based on PIPC in terms of the achievable ergodic capacities of both the PR and CR links. Reference [6] considers the transmit-power control in a non-cooperative framework, using control theory tools to study both the equilibrium and transient behaviors of the network under dynamically varying conditions. The iterative water-filling algorithm (IWFA) is formulated for transmit power control. They propose an optimization problem in which the capacity of the links are maximized, subject to preserving the interference level of PUs bellow an acceptable level. As link capacity is not always concave function of transmit powers, their method tends to be unstable during simulations. To maintain stability a Max-Min procedure is applied which brings about stability, however it is suboptimal in maximizing the overall capacity.

Applying game theory for cognitive radio power adjustment so that some utility function is maximized has been the focus of a number of papers. To ensure a minimum required QoS (interference) for PUs and to achieve energy efficient transmissions for SUs, [7] proposes a primary-secondary power control game with an exponential pricing term incorporated into the utility function. In [8], non-cooperative, finite repeated game, based on incomplete information of the system is used for power control of two users in a CR system. Reference [9] proposes a non-cooperative primary-secondary user power control game for dynamic spectrum leasing, to primary utility, defining a utility function that is proportional to the amount of primary user tolerable interference level, and a utility for secondary users to enhance their throughput per unit power. Taking into consideration both network efficiency and user fairness, in [10] a cooperative Nash bargaining power control game called NBPCG in which interference power constraints are imposed to protect the primary users' transmissions is proposed. A utility function based on signal to interference plus noise ratio is employed to provide reliable transmission opportunities to secondary cognitive users.

Although disapproved by the FCC in May 2007, the notice to the interference temperature limit for PUs has attracted much attention by researchers. Reference [11] tries to jointly adjust SUs' rates, frequencies, and power resources, under the constraints of multiple PUs' interference temperatures to maximize multiple weighted SUs by a nonlinear and non-convex optimization problem. Considering interference temperature constraints, in [12] the optimal power control in CR network is modeled as a concave maximization problem of the total capacity. Based on that, an improved branch and bound algorithm is proposed for power control optimization.

III. THE PROPOSED NETWORK CRITICALITY APPROACH

As we pointed out, our approach is based on the network criticality concept which results in a set of nonlinear optimization equations. To solve this set of equations, we deploy a method called Relaxation. This method results in a Simultaneous Iterative Water Filling (SIWF) approach, in which each SU updates its power using the most recent transmission parameters of the others.

We assume that a spectrum pool comprising slightly-overlapping channels is available to be assigned to SUs. The communication system is assumed to consist of some number of Primary Transmitters (such as TV stations) and Primary Receivers, and a mobile mesh network of Secondary Transmitters/Receivers which are using the primary licensed spectrum band. The primary spectrum band is assumed to be divided into a number of OFDM channels which have been allocated to the secondary transmitters. Each channel includes n subcarriers to accommodate OFDM symbols. The locations of primary systems are assumed to be known by all of the secondary nodes. Moreover, at the end of each execution of our distributed algorithm, the secondary nodes exchange their location and transmission parameter information over a common control channel or by broadcasting on all frequency bands. The secondary terminals are considered to be equipped with single antennas to transmit on the assigned frequency bands. The SUs' mobile mesh network topology is assumed to change slowly relative to one iteration of our algorithm so that the situation of the network can be considered constant with respect to the power control mechanism. The additive noise and interference at PU's and SU's are assumed to be independent Gaussian white noise (AWGN assumption).

As an introduction to our optimization problem, in the next sub-section we present the definition of network criticality and some of its characteristics. Then we define our cost function and introduce the optimization problem which provides our power control mechanism.

A. Network Criticality Concept

Network criticality is a global measure on a graph which quantifies the robustness of a network graph to the environmental changes, mainly traffic shifts, topology modifications, and changes in the origin and destination for traffic. Network criticality derives its roots from the definition of random-walk betweenness in graphs. Consider a set of trajectories walked by a random walker, starting at s and terminating when the walk arrives at destination d for the first time. The random walk betweenness of a node k for the set of trajectories from s to d is defined as the average number of visits to node k. the total random walk betweenness of node k is the sum of the contributions for all s-d trajectories.

It has been shown that for a weighted random walk, where the probability of transitioning along a link to a neighbor node is proportional to the weight of that link, the normalized random walk betweenness of a node (i.e. the node betweenness divided by the node weight) is a global measure on the graph and it is independent of the node location [1]. This global graph metric is referred to as network criticality which has some nice properties and interpretations, describing why it is an important robustness measure on graphs. Network criticality can be interpreted as the total resistance of a network if we view the network as an electrical circuit. Consider an electrical circuit with the same graph as our original network graph, and with link resistances equal to the reciprocal of link weights. It can be shown that network criticality numerically equals the total resistance distance (effective resistance) seen between different pairs of nodes in the electrical circuit. A high network criticality is an indication of high resistance in the equivalent electrical circuit, therefore, in two networks with the same number of nodes, the one with lower network criticality is better connected, hence better positioned to accommodate network flows. Furthermore, network criticality quantifies the sensitivity of a network to the environmental changes. It has been shown that network criticality equals the average of link betweenness sensitivities, where link betweenness sensitivity is defined as the partial derivative of link betweenness with respect to the corresponding link weight [1]:

$$\tau = \frac{1}{m-1}\sum_{(i,j)\epsilon E}\frac{\partial b_{ij}}{\partial w_{ij}} \tag{1}$$

where E, m, b_{ij} and w_{ij} denote the set of links, the number of links, the betweenness of link *(i,j)*, and the weight of link *(i,j)* respectively. Equation (1) states that minimization network criticality results in minimizing the average sensitivity of link betweennesses with respect to the changes in link weights (which in turn captures environmental changes). In fact, designing a control algorithm for network criticality balances the betweenness of the links in such a way to keep the average sensitivity below a desired level. From another point of view, the lower the criticality, the better distributed is the traffic between all the links of a network, and the better balanced the load of the traffic among all active links. This implies better fairness in routing the traffic in the nodes of the network.

Another advantage of having low network criticality is the robustness enhancement of the network. Suppose that a node is failing or becoming inaccessible so that it is unable to route the traffic passing through it. Minimizing network criticality adaptively results in adjusting the betweenness in such a way that traffic is re-routed to other nodes instead of the impaired one and that brings about higher robustness against unpredictable deleterious situations.

In conclusion, since the wireless networks have intrinsic dynamism, minimizing the criticality adaptively yields higher utilization of resources, better fairness and increased robustness of the network.

In our approach network criticality minimization is performed under the constraint of having limited interference on PU bands. Denoting the transmission power of secondary user j (SU_j) in subcarrier k as p_k^j, the received interference at primary user i (PU_i) is given by the following equation:

$$I_{k,j}^{(PU)} = \sum_{j=1}^{N} p_k^j \cdot h_k^{ij} \tag{2}$$

h_k^{ij} is defined as:

$$h_k^{ij} = \frac{\gamma_k^{ij} \cdot G_k^{ij}}{{d_{ij}}^{\beta}} \tag{3}$$

where G_k^{ij} is the average channel gain between SU_j and PU_i in subcarrier k, γ_k^{ij} is the coefficient representing the overlapped bandwidth of SU_j and PU_i causing interference at the primary user, d_{ij} is the distance between SU_j and PU_i, β is the path fading factor and N is the number of active secondary users. Having equations (5), the interference power at the primary users should satisfy the following inequality:

$$I_k^{(PU)} = \sum_k H_k \cdot P_k \leq T_k^{(PU)} \tag{4}$$

$I^{(PU)}$, P and $T^{(PU)}$ are the primary users interference, the secondary users transmit power and the primary users acceptable interference threshold vectors which are defined in the following set of equations as well as $N \times M$ matrix H_k:

$$\begin{gathered} I_k^{(PU)} = [I_{k,1}^{(PU)}, \dots, I_{k,M}^{(PU)}]^t \\ P_k = [p_k^1, \dots, p_k^N]^t \\ T_k^{(PU)} = [T_{k,1}^{(PU)}, \dots, T_{k,M}^{(PU)}]^t \\ H_k = [h_k^{ij}] \quad i = 1, \dots, M;\ j = 1, \dots, N;\ k = 1, \dots, n \end{gathered} \tag{5}$$

where $T_{k,i}^{(PU)}$ is the acceptable interference power at the primary user i. These equations can be solved to find the maximum allowed power for each secondary node in each subcarrier such that:

$$P_k - P_k^{(max)} \leq 0 \; for \; k = 1, \dots, n \tag{6}$$

Letting the bandwidth dedicated to the secondary node i in subcarrier k be B_k^i, the weight (capacity) of the radio link between the secondary node i as the transmitter and node j as the receiver considering the interference caused by other nodes transmissions can be written as:

$$w_{ij} = C_{ij} = \sum_k B_k^i \cdot log_2\left(1 + \frac{S_k^{ij}}{\sigma_k^j + I_k^j}\right) \tag{7}$$

where σ_k^j is the background Gaussian white noise power at the receiver node j in subcarrier k, and S_k^{ij} and I_k^j are the signal power received form node i, and interference power at that receiver j which are given by:

$$S_k^{ij} = \frac{p_k^i \cdot G_k^{ij}}{{d_{ij}}^{\beta}} = A_k^{ij} \cdot p_k^i \tag{8}$$

$$I_j = \sum_{\substack{l=1 \\ l \neq i}}^{N} \frac{p_k^j \cdot G_k^{lj} \cdot \zeta_k^{il}}{{d_{lj}}^{\beta}} = \sum_{\substack{l=1 \\ l \neq i}}^{N} \alpha_k^{il} \cdot p_k^l \tag{9}$$

In the above equation G_k^{ij} is the average channel gain between the transmitting node SU_i and the receiving node SU_j, G_k^{lj} is the average channel gain between interfering node SU_l and the receiving node SU_j, and ζ_k^{il} is the coefficient representing the bandwidth overlap between the transmitting and the interfering SUs, all for subcarrier k.

B. *Cost Function Definition*

Our objective in this work is to minimize the criticality of the network to achieve the highest robustness, while keeping the resulting interference on each of the PUs below the acceptable threshold.

As we need τ minimization to coincide with total capacity increase, we add another constraint that necessitates the total capacity of all the SUs to be concave with respect to $q_k^l = log\,(p_k^l)$. As network criticality is a monotone decreasing function of weights (link capacities) [1], this constraint guarantees that each node may increase its power only when the total capacity is increased thereafter. Proscribing the nodes to be selfish, we drive them not only think of their own good, but also to consider the benefit of the whole network. The first order of conditions for concavity of the total capacity (denoted by $\psi = \sum_j \sum_i w_{ij}$), with respect to each node transmit power logarithm can be stated as:

$$\nabla\psi(q_k^l) \cdot (\hat{q}_k^l - q_k^l) + \psi(q_k^l) - \psi(\hat{q}_k^l) \leq 0 \;\; for\ all\ k\ and\ l \quad (11)$$

where $q_k^l = log\ (p_k^l)$ and $\hat{q}_k^l = log\ (\hat{p}_k^l)$ are two values of the log power of the node *l* transmitting in subcarrier *k* during the course of optimization, and $\nabla\psi(q_k^l)$ is the gradient of the total capacity with respect to q_k^l which is given by the following equation:

$$\nabla\psi(q_k^l) = \frac{\partial\psi}{\partial q_k^l} = \sum_j \sum_i \frac{\partial w_{ij}}{\partial q_k^l} = \sum_j \left(\frac{\partial w_{lj}}{\partial q_k^l} + \sum_{i \neq l} \frac{\partial w_{ij}}{\partial q_k^l} \right) \quad (12)$$

To maintain the convexity of our cost function, we assume the SINRs at the receiving nodes are relatively high (e.g. $\frac{S_k^{ij}}{\sigma_k^j + I_k^{ij}} \geq 5$) so the approximation $log\left(1 + \frac{S_k^{ij}}{\sigma_k^j + I_k^{ij}}\right) \cong log\left(\frac{S_k^{ij}}{\sigma_k^j + I_k^{ij}}\right)$ can be used. Having this approximation, it is proved in [13] that the link capacities $w_{ij}(Q_k)$ are concave in variable Q_k.

To continue with convergence of τ minimization with respect to p_l, we refer to the following inequality which has been proven in [1]:

$$\frac{\partial\tau}{\partial w_{ij}} < 0\ , \frac{\partial\tau}{\partial w_{ij}} \neq 0 \quad (13)$$

$$w_{ij} > \widehat{w}_{ij} \Rightarrow \tau(w_{ij}) < \tau(\widehat{w}_{ij}) \quad (14)$$

In the above inequalities, $w_{ij}(Q_k)$ and $\widehat{w}_{ij} = w_{ij}(\hat{Q}_k)$ are two values for the capacity of link *i* to *j*. Considering the concavity of $w_{ij}(Q_k)$ we may write:

$$w_{ij}(\alpha . q_k^l + (1-\alpha)\hat{q}_k^l) \geq \alpha . w_{ij}(q_k^l) + (1-\alpha) w_{ij}(\hat{q}_k^l) \quad (15)$$

Putting inequalities (14) and (15) together, one can conclude:

$$\tau(\alpha . q_k^l + (1-\alpha)\hat{q}_k^l) \leq \alpha . \tau((q_k^l)) + (1-\alpha)\tau((\hat{q}_k^l)) \quad (16)$$

which is the required condition for convexity of τ with respect to the transmit powers.

IV. Power Optimization Using Relaxation Method

In this section, using the proposed cost function, our optimization problem is defined and then an iterative solution based on relaxation method is presented.

A. *Optimization Statement*

Now, having τ as the cost function and inequalities (7) and (11) as constraints, our optimization problem would be:

For each subcarrier *k* Minimize: $\tau(Q_k)$

$$\text{Subject to:} \quad \begin{aligned} f_0(Q_k) &\leq \underline{0} \\ f_1(Q_k) &\leq \underline{0} \end{aligned} \quad (17)$$

where $f_0(Q_k) = e^{Q_k} - P_k^{(max)}$ and $f_1(Q_k) = \nabla\psi(Q_k) \vdots (\hat{Q}_k - Q_k) + \psi(Q_k) - \psi(\hat{Q}_k)$, ($\nabla\psi(Q_k) \vdots (\hat{Q}_k - Q_k)$ is the component wise product for two vectors $\nabla\psi(Q_k)$ and $(\hat{Q}_k - Q_k)$).

The Lagrangian function associated with our cost function and its constraints can be defined for each subcarrier k as:

$$L(Q_k, \lambda_k^0, \lambda_k^1) = \tau(Q_k) + {\lambda_k^0}^t \cdot f_0(Q_k) + {\lambda_k^1}^t \cdot f_1(Q_k) \quad (18)$$

where vectors λ_k^0, λ_k^1 are the Lagrange multipliers associated with our constraints. For our nonlinear optimization problem, the optimum solution ${Q_k}^* = log\ ({P_k}^*)$ satisfies our constraints, while there should exist ${\lambda_k^0}^*$, ${\lambda_k^1}^*$ such that:

$$\left.\frac{\partial L}{\partial Q_k}\right|_{{Q_k}^*, {\lambda_k^0}^*, {\lambda_k^1}^*} = \underline{0}\ for\ all\ k \quad (19)$$

The optimality conditions can be restated as the following:

$$\begin{aligned} &\left(\frac{\partial f_0}{\partial {Q_k}^*}\right)^t \cdot {\lambda_k^0}^* + \left(\frac{\partial f_1}{\partial {Q_k}^*}\right)^t \cdot {\lambda_k^1}^* + \frac{\partial\tau}{\partial {Q_k}^*} = \underline{0} \\ &f_0({Q_k}^*) \leq \underline{0}\ ,\ f_1({Q_k}^*) \leq \underline{0} \\ &{\lambda_k^0}^{*t} \cdot f_0({Q_k}^*) = \underline{0}\ ,\ \ {\lambda_k^1}^{*\,t} \cdot f_1({Q_k}^*) = \underline{0} \\ &{\lambda_k^0}^*\ ,\ \ {\lambda_k^1}^*\, and\ {Q_k}^* \geq \underline{0} \end{aligned} \qquad k = 1\ to\ n \quad (20)$$

B. *Solution Using Relaxation Method*

To solve the optimization problem based on the relaxation method an iterative partitioning algorithm is performed in which each secondary user carries out its own part in a distributed SIWF approach. In each iteration, the active nodes adjust their transmit power in turn and update their powers simultaneously using the most recent transmission parameters (power values, location coordinates, destination nodes, transmission gains, path fading factors and frequency band overlaps). When a node is in relaxation, it assumes the other parts of the network are doing perfectly and the only problem to be solved is optimizing its own power level as the source of transmission to the sink which is its destination. The interactions between this node and other parts of the network are considered in the interferences that active nodes cause for each other and for primary systems.

V. Simulation Results

To evaluate the performance of our proposed scheme, a set of simulations are performed similar to [6] and the results are compared to those obtained using the method proposed therein. In [6] the utility function is considered to be the data rate of each user; however since this is not a convex function of transmit powers, the mechanism is not stable causing severe fluctuations in power control and user data rates. Thus, the authors propose a Max-Min solution that is suboptimal but robust. We are interested to evaluate the gains that can be achieved using a network criticality cost function.

The simulation scenario, as depicted in Fig. 1, contains three overlapping cluster of eight SUs. The background noise levels σ_k^j, the interference coefficients α_k^{ij}, the propagation attenuations A_k^{ij} for the nodes in each cluster, and the maximum transmit power vector P_{max} are chosen as uniform random values in (0,0.1/(m-1)), (0,1/(m-1)), (0.6,0.9), (n/2,n) intervals respectively. As in [6] two subcarriers (n=2) are assumed to be used by each CR node. The exclusive members of two neighboring clusters are supposed to be roughly twice further apart compared to the nodes belonging to same cluster, so α_k^{ij} and A_k^{ij} are divided by four for them which corresponds to path fading factor, β=2. So the distances between the distant nodes are assumed to be 4 times of the distances between the nodes in same cluster.

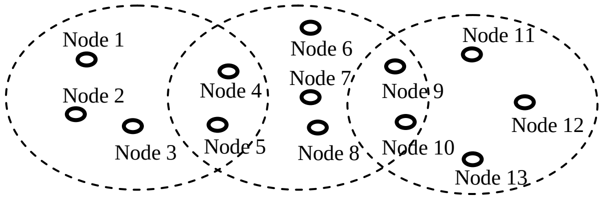

Figure 1. The scenario used for simulations.

The spectral efficiency for the nodes 1, 4, 7, 9 and 12 for rate based utility function, and network criticality cost function methods are illustrated in Fig. 2-a and 2-b. In both of the methods the link capacities between node 2 and the far end nodes 9 and 12 are very low due to the further distances and lower allocated transmit powers to reduce the interference imposed on other nodes transmissions. The fluctuations at the first iteration of Fig. 2-(a) are due to having zero interference because of zero powers at initiation of the simulations, so theses values are transient and invalid. Comparing these two figures, one can infer that although the power assignments to nodes are different for the two schemes, but the network criticality minimization leads to higher overall capacity.

Fig. 3 depicts the total bit rate per Hertz for all the possible communications between all the nodes using the two methods. This figure shows the improved performance of our proposed method based on criticality minimization which is about 22% higher than the Max-Min rate maximization method. Thus the criticality minimization does not incur a "robustness penalty" as the Max-Min method does. Furthermore, we note that in addition to maximizing user rate performance locally, network criticality minimization method considers the entire network to also achieve higher performance in terms of robustness and stability.

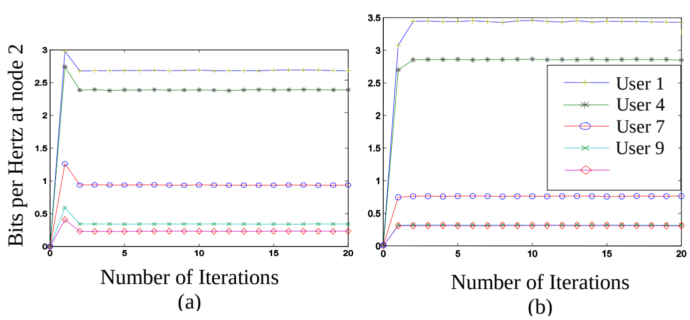

Figure 2. The spectral efficiency for users 1, 4, 7, 9 and 12 communicating with node 2 using (a): Rate Maximization, and (b): Network Criticality Minimization methods.

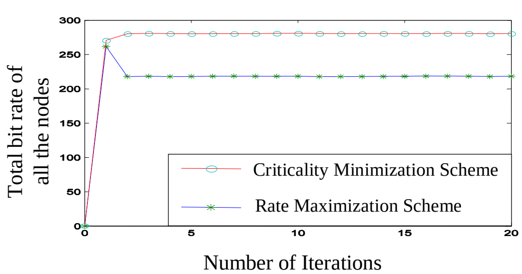

Figure 3. The total bit rate using Rate Maximization and Network Criticality Minimization methods.

VI. CONCLUSION

Cognitive Radio performance strongly depends on how well the power control is carried out. To maximize the robustness of CRN, in this paper an approach is proposed based on minimizing network criticality which is performed under two constraints: increasing the total capacity of the network, and preserving the total interference imposed on each of the PUs below an acceptable threshold. The lower network criticality results in more robustness against shortage in network connectivity, and better distributing traffic among all the available links. Moreover, the relaxation method is employed to solve the power control optimization problem in a distributed Simultaneous Iterative Water Filling fashion. In addition to robustness and stability enhancement, the simulation results showed improved performance for the proposed method in terms of achieved total capacity, compared to a previous method based on users' rates maximization with Max-Min scheme.

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