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Image segmentation by three-level thresholding based on maximum fuzzy entropy and genetic algorithm

Wen-Bing Tao^{*}, Jin-Wen Tian, Jian Liu

*State Key Laboratory for Image Processing and Intelligence Control, Huazhong University of Science and Technology,
Wuhan, Hubei 430074, China*

*Institute for Pattern Recognition and Artificial Intelligence, Huazhong University of Science and Technology,
Wuhan, Hubei 430074, China*

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Abstract

In the paper, a three-level thresholding method for image segmentation is presented, based on probability partition, fuzzy partition and entropy theory. A new fuzzy entropy has been defined through probability analysis. The image is divided into three parts, namely, dark, gray and white part, whose member functions of the fuzzy region are Z -function and I -function and S -function, respectively, while the width and attribute of the fuzzy region can be determined by maximizing fuzzy entropy. The procedure for finding the optimal combination of all the fuzzy parameters is implemented by a genetic algorithm with appropriate coding method so as to avoid useless chromosomes. The experiment results show that the proposed method gives good performance.

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Keywords: Image segmentation; Fuzzy entropy; Genetic algorithm; Probability partition

1. Introduction

The goal of image segmentation is to extract meaningful objects from an input image. Image segmentation, with wide recognized significance, is one of the most difficult low-level image analysis tasks, as well as the bottle-neck of the development of image processing technology. All the subse-

quent tasks, including feature extraction, model matching and object recognition, rely heavily on the quality of the image segmentation process. Thresholding is undoubtedly one of the most popular segmentation approaches for the sake of its simplicity. It is based on the assumption that the objects can be distinguished by their gray levels. It is an important issue to find a correct gray level threshold that can separate different objects or separate objects from background.

However, the automatic selection of a robust, optimum threshold has remained a challenge in image segmentation. An early review of thresholding methods was reported (Sahoo et al., 1988;

^{*} Corresponding author. Address: Institute for Pattern Recognition and Artificial Intelligence, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China. Tel.: +86-27-87556301; fax: +86-27-87543594.

E-mail address: wbtiao_hust@163.com (W.-B. Tao).

Pal and Pal, 1993). Pun (1980) described a method that maximizes the upper bound of the posteriori entropy derived from the histogram. Wong and Sahoo's method (Wong and Sahoo, 1989) determines the optimum threshold by maximizing the posteriori entropy subject to certain inequality constraints that characterize the uniformity and shape of the segmented regions. Pal and Pal (1989, 1991) developed another entropy-based method by considering the joint probability distribution of the neighboring pixels, which they further modified with a new definition of entropy. The method of Kapur et al. (1985) selects the optimum threshold by maximizing the sum of entropies of the segmented regions. A similar approach was reported by Abutaleb (1989), which maximizes the 2D entropy. Brink's method (Brink, 1994) maximizes the sum of the entropies computed from two autocorrelation functions of the thresholded image histograms.

A fuzzy entropy is a function on fuzzy sets that becomes smaller when the sharpness of its argument fuzzy set is improved. The notion of entropy, in the theory of fuzzy sets, was first introduced by Luca and Termini (1972). There have been numerous applications of fuzzy entropies in image segmentation. Cheng et al. (1998) proposed fuzzy homogeneity vectors to handle the grayness and spatial uncertainties among pixels, and to perform multilevel thresholding. Cheng et al. (1999) presented a thresholding approach by performing fuzzy partition on a two-dimensional histogram based on fuzzy relation and maximum fuzzy entropy principle. Cheng et al. (2000) defined a new approach to fuzzy entropy, used it to select the fuzzy region of membership function automatically so that an image is able to be transformed into fuzzy domain with maximum fuzzy entropy, and implemented genetic algorithm to find the optimal combination of the fuzzy parameters. Cheng has employed the proposed approach to perform image enhancement and thresholding, and obtained satisfactory results. Zhao et al. (2001) presented an entropy function by the fuzzy *c*-partition (FP) and the probability partition (PP) which was used to measure the compatibility between the PP and the FP. Zhao used the simplest function, that is monotonic, to approximate the

memberships of the bright, the dark and the medium and derived a necessary condition of the entropy function arriving at a maximum, which is $p_d = p_m = p_b = 1/3$, and deduced an algorithm for three-level thresholding.

Based on the idea of Zhao et al. (2001), this paper designs a new three-level thresholding method for image segmentation. The paper defines a new fuzzy entropy through probability analysis, fuzzy partition and entropy theory. The image is partitioned into three parts, namely dark, gray and white part, whose member functions of the fuzzy region are *Z*-function and *II*-function and *S*-function respectively. The width and attribute of the fuzzy region can be decided by maximum fuzzy entropy, in turn the thresholds can be decided by the fuzzy parameters. For getting optimal thresholds, we must find the optimal combination of all the fuzzy parameters. Thus, the segmentation problem can be formulated as an optimal problem. The fuzzy entropy of the image has been chosen as the objective function. We must design an effective search strategy, in which we can find the optimal combination of all the parameters quickly. We have reviewed many techniques, commonly used for function optimization, in the view of determining their usefulness for this particular task. Genetic algorithm (GA) (Schmitt, 2001; Goldberg, 1989) is able to overcome many of the defects in other optimization techniques such as exhaustive techniques, calculus-based techniques, partial knowledge (hill climbing, beam search, best first, branch and bound, dynamic programming, A*), knowledge based techniques (production rule systems, heuristic methods). They search from a population of individuals (search points), which make them ideal candidates for parallel architecture implementation, and are far more efficient than exhaustive techniques. Since they use simple recombination of existing high quality individuals as well as a method of measuring current performance, they do not require complex surface descriptions, domain specific knowledge, or measures of goal distance. Further more, due to the generality of the genetic process, they are independent of the segmentation technique used, requiring only a measure of performance, which is referred to segmentation quality, for any given

parameter combination. GA has been used to solve various problems in computer vision, including image segmentation (Cheng et al., 2000; Bhanu et al., 1995; Bhandarkar and Zhang, 1999), feature selection (Roth and Levine, 1994), image matching (Ansari et al., 1992) and object recognition (Bebis et al., 2002; Soodamani and Liu, 2000; Minami et al., 2001), to name a few. In this paper, we propose using GA in finding the optimal combination of all the fuzzy parameters efficiently. The experiment results show that the proposed method gives better performance.

2. Maximum fuzzy entropy principle based on probability partition

Let $D = \{(i, j) : i = 0, 1, \dots, M-1; j = 0, 1, \dots, N-1\}$, $G = \{0, 1, \dots, l-1\}$, where M, N and l are three positive integers. Thus a digitized image defines a mapping $I : D \rightarrow G$. Let $I(x, y)$ be the gray level value of the image at the pixel (x, y) :

$$\begin{aligned} I(x, y) \in G \quad \forall (x, y) \in D \\ D_k = \{(x, y) : I(x, y) = k, (x, y) \in D\}, \\ k = 0, 1, \dots, l-1 \end{aligned} \quad (1)$$

$$h_k = \frac{n_k}{N * M}, \quad k = 0, 1, \dots, l-1 \quad (2)$$

where n_k denotes the number of pixels in D_k . The following conclusions can be easily formed:

$$\bigcup_{k=0}^{l-1} D_k = D, \quad D_j \cap D_k = \Phi \quad (k \neq j)$$

$$0 \leq h_k \leq 1, \quad \sum_{k=0}^{l-1} h_k = 1, \quad k = 0, 1, \dots, l-1$$

$H = \{h_0, h_1, \dots, h_{l-1}\}$ is the histogram of the image, $\prod_l = \{D_0, D_1, \dots, D_{l-1}\}$ is a probability partition of D with a probabilistic distribution

$$p_k = P(D_k) = h_k, \quad k = 0, 1, \dots, l-1 \quad (3)$$

The gray levels l of the image in the paper is 256.

In this paper three-level thresholding is used to segment the image. Name the two thresholds t_1

and t_2 . Then they segment the image into three gray levels. In this gray levels image, the domain D of the original image is classified into three parts: E_d, E_m and E_b . E_d is composed of pixels with low gray levels, E_b is composed of pixels with high gray levels, E_m is composed of pixels with medium gray levels. $\prod_3 = \{E_d, E_m, E_b\}$ is an unknown probabilistic partition of D , whose probability distribution is given below:

$$p_d = P(E_d), \quad p_m = P(E_m), \quad p_b = P(E_b) \quad (4)$$

A classical set is normally defined as a collection of element that can either belong to a set or not. A fuzzy set is an extension of a classical set in which an element may partially belongs to a set. Let A be a fuzzy set, where $A \subset X$ is defined as $A = \{(x, \mu_A(x)) | x \in X\}$ where $0 \leq \mu_A(x) \leq 1$ is called the membership function. The value of $\mu_A(x)$ is the grade of x belonging to A .

We use the $Z(k, a, b, c)$ -function, $\Pi(k, a, b, c)$ -function and $S(k, a, b, c)$ -function (see Fig. 2) to approximate the memberships of μ_d, μ_m and μ_b of an image with 256 gray levels. The membership functions have six parameters, namely $a_1, b_1, c_1, a_2, b_2, c_2$. In other words, the two thresholds t_1, t_2 , for three-level thresholding are depended on $a_1, b_1, c_1, a_2, b_2, c_2$. And the following conditions are satisfied:

$0 < a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2 < 255$. For each $k = 0, 1, \dots, 255$, let

$$D_{kd} = \{(x, y) : I(x, y) \leq t_1, (x, y) \in D_k\}$$

$$D_{km} = \{(x, y) : t_1 < I(x, y) \leq t_2, (x, y) \in D_k\}$$

$$D_{kb} = \{(x, y) : I(x, y) > t_2, (x, y) \in D_k\}$$

Then the following conditions can be satisfied:

$$\begin{aligned} p_{kd} &= P(D_{kd}) = p_k * p_{d|k} \\ p_{km} &= P(D_{km}) = p_k * p_{m|k} \\ p_{kb} &= P(D_{kb}) = p_k * p_{b|k} \end{aligned} \quad (5)$$

When the pixel belongs to D_k , it is evident that $p_{d|k}, p_{m|k}, p_{b|k}$ are the conditional probability of a pixel when it is classified into the class “d” (dark), “m” (medium) and “b” (bright) respectively, with a constraint that $p_{d|k} + p_{m|k} + p_{b|k} = 1$ ($k = 0, 1, \dots, 255$).

$$\begin{aligned}
p_d &= P(E_d) = \sum_{k=0}^{255} P(D_{kd}) \\
&= \sum_{k=0}^{255} P(D_k) * P(E_d|D_k) = \sum_{k=0}^{255} p_k * p_{d|k} \\
p_m &= P(E_m) \\
&= \sum_{k=0}^{255} P(D_{km}) = \sum_{k=0}^{255} P(D_k) * P(E_m|D_k) \\
&= \sum_{k=0}^{255} p_k * p_{m|k} \\
p_b &= P(E_b) = \sum_{k=0}^{255} P(D_{kb}) \\
&= \sum_{k=0}^{255} P(D_k) * P(E_b|D_k) = \sum_{k=0}^{255} p_k * p_{b|k}
\end{aligned} \tag{6}$$

Let the grades of a pixel belonging to the class d (dark), m (medium) and b (bright), whose gray level value is k , be equal to its conditional probability $p_{d|k}$, $p_{m|k}$, $p_{b|k}$, respectively. Then the following conclusion can be gained:

$$\begin{aligned}
p_d &= \sum_{k=0}^{255} p_k * \mu_d(k) \\
p_m &= \sum_{k=0}^{255} p_k * \mu_m(k) \\
p_b &= \sum_{k=0}^{255} p_k * \mu_b(k)
\end{aligned} \tag{7}$$

The three membership functions are shown in Fig. 1. We choose the $Z(k, a, b, c)$ -function as the membership function $\mu_d(k)$ of the class d (dark), the $\Pi(k, a, b, c)$ -function as the membership function $\mu_m(k)$ of the class m (medium) and the $S(k, a, b, c)$ -function as the membership function $\mu_b(k)$ of the class b (bright). Formulas (8)–(10) give detail information.

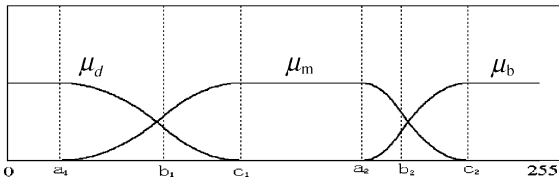


Fig. 1. Membership function graph.

$$\mu_d(k) = \begin{cases} 1 & k \leq a_1 \\ 1 - \frac{(k - a_1)^2}{(c_1 - a_1) * (b_1 - a_1)} & a_1 < k \leq b_1 \\ \frac{(k - c_1)^2}{(c_1 - a_1) * (c_1 - b_1)} & b_1 < k \leq c_1 \\ 0 & k > c_1 \end{cases} \tag{8}$$

$$\mu_m(k) = \begin{cases} 0 & k \leq a_1 \\ \frac{(k - a_1)^2}{(c_1 - a_1) * (b_1 - a_1)} & a_1 < k \leq b_1 \\ 1 - \frac{(k - c_1)^2}{(c_1 - a_1) * (c_1 - b_1)} & b_1 < k \leq c_1 \\ 1 & c_1 < k \leq a_2 \\ 1 - \frac{(k - a_2)^2}{(c_2 - a_2) * (b_2 - a_2)} & a_2 < k \leq b_2 \\ \frac{(k - c_2)^2}{(c_2 - a_2) * (c_2 - b_2)} & b_2 < k \leq c_2 \\ 0 & k > c_2 \end{cases} \tag{9}$$

$$\mu_b(k) = \begin{cases} 0 & k \leq a_2 \\ \frac{(k - a_2)^2}{(c_2 - a_2) * (b_2 - a_2)} & a_2 < k \leq b_2 \\ 1 - \frac{(k - c_2)^2}{(c_2 - a_2) * (c_2 - b_2)} & b_2 < k \leq c_2 \\ 1 & k > c_2 \end{cases} \tag{10}$$

where the six parameters $a_1, b_1, c_1, a_2, b_2, c_2$ satisfy the following condition:

$$0 < a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2 < 255$$

The fuzzy entropy function of each class is given below:

$$\begin{aligned}
H_d &= - \sum_{k=0}^{255} \frac{p_k * \mu_d(k)}{p_d} * \ln \left(\frac{p_k * \mu_d(k)}{p_d} \right) \\
H_m &= - \sum_{k=0}^{255} \frac{p_k * \mu_m(k)}{p_m} * \ln \left(\frac{p_k * \mu_m(k)}{p_m} \right) \\
H_b &= - \sum_{k=0}^{255} \frac{p_k * \mu_b(k)}{p_b} * \ln \left(\frac{p_k * \mu_b(k)}{p_b} \right)
\end{aligned} \tag{11}$$

Then the total fuzzy entropy function is given as following:

$$H(a_1, b_1, c_1, a_2, b_2, c_2) = H_d + H_m + H_b \quad (12)$$

The total fuzzy entropy varies along with six variables $a_1, b_1, c_1, a_2, b_2, c_2$. We can find an optimal combination of $(a_1, b_1, c_1, a_2, b_2, c_2)$ so that the total fuzzy entropy $H(a_1, b_1, c_1, a_2, b_2, c_2)$ has the maximum value. Then the most appropriate combination of thresholds, by which the image is segmented into three classes, can be computed as following:

$$\begin{aligned} \mu_d(t_1) &= \mu_m(t_1) = 0.5 \\ \mu_m(t_2) &= \mu_b(t_2) = 0.5 \end{aligned} \quad (13)$$

As is shown in Fig. 1, threshold t_1 is the point of intersection of $\mu_d(k)$ -curve and $\mu_m(k)$ -curve, while threshold t_2 is the point of intersection of $\mu_m(k)$ -curve and $\mu_b(k)$ -curve. Based on formulas (8)–(10) the solution can be given below:

$a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2$. In our experiments, each image has 256 gray levels, i.e., the maximum value of c_2 is 255. Therefore, the chromosome of the genetic algorithm in our experiment is encoded as six 8-bits strings that represent the value of all the parameters, respectively. If all the parameters $a_1, b_1, c_1, a_2, b_2, c_2$ are generated randomly, it is possible that $a_1, b_1, c_1, a_2, b_2, c_2$ do not satisfy the criteria $a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2$. In such a case, we can assign zero to the value of object function for illegal chromosomes. It will not participate the reproduction of next generation. The drawback of this method is that there are too many useless chromosomes in the searching space. Here we use some mathematical processing method to make all chromosomes legal. That is to say, every chromosome will satisfy the criteria $a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2$. The method is given below:

-
- (1) If $(a_1 + c_1)/2 \leq b_1 \leq c_1$, then $t_1 = a_1 + \sqrt{(c_1 - a_1) * (b_1 - a_1)/2}$,
 - (2) If $a_1 \leq b_1 < (a_1 + c_1)/2$, then $t_1 = c_1 - \sqrt{(c_1 - a_1) * (c_1 - b_1)/2}$,
 - (3) If $(a_2 + c_2)/2 \leq b_2 \leq c_2$, then $t_2 = a_2 + \sqrt{(c_2 - a_2) * (b_2 - a_2)/2}$,
 - (4) If $a_2 \leq b_2 < (a_2 + c_2)/2$, then $t_2 = c_2 - \sqrt{(c_2 - a_2) * (c_2 - b_2)/2}$.
-

3. Genetic algorithm

In this paper, the procedure for finding the optimal combination of all the fuzzy parameter is implemented by genetic algorithms. To solve an optimization problem, GA needs to maximize a fitness function depending on the nature of the problem. The user is required to specify the following parts for using GA: coding method, object function, the population size PoP, the cross-over probability P_c , the mutation probability P_m and the maximal number of generations MaxGen. So we should present the encoding mechanism, the selection scheme, genetic operators, and the fitness function used.

The first step is to encode the parameters $a_1, b_1, c_1, a_2, b_2, c_2$ into an alphabet string. Notice that $a_1, b_1, c_1, a_2, b_2, c_2$ have to follow the increasing order

$$\begin{aligned} c_1^1 &= c_1 \\ b_1^1 &= c_1^1 * (b_1/255) \\ a_1^1 &= b_1^1 * (a_1/255) \\ a_2^1 &= c_1^1 + (255 - c_1^1) * (a_2/255) \\ b_2^1 &= a_2^1 + (255 - a_2^1) * (b_2/255) \\ c_2^1 &= b_2^1 + (255 - b_2^1) * (c_2/255) \end{aligned} \quad (14)$$

Then the following condition is satisfied:

$$0 < a_1^1 \leq b_1^1 \leq c_1^1 \leq a_2^1 \leq b_2^1 \leq c_2^1 < 255$$

We can compute fitness function value by the six parameters $a_1^1, b_1^1, c_1^1, a_2^1, b_2^1, c_2^1$.

We choose the entropy function in Eqs. (11) and (12) as the fitness function and the parameters of the genetic algorithm are set as following by experiments on many images.

MaxGen (maximal generation number) = 300;
 PoPs (population size) = 100;
 P_c (probability of cross-over) = 0.5;
 P_m (probability of mutation) = 0.01.

After determining the coding method, object function and all GA parameters, GA can operate and find the solution $(a_1, b_1, c_1, a_2, b_2, c_2)$ with the maximal entropy $H(a_1, b_1, c_1, a_2, b_2, c_2)$. Finally, the two fuzzy region can be determined by the interval $[a_1, c_1]$ and $[a_2, c_2]$ from the final result, then determined the thresholds by Eq. (13).

4. Experimental results

To verify the efficiency of the method, experiments have been carried on many gray level images. In this paper, we use three typical images to illustrate the effect of the method. Figs. 2–4 show the results, with (a) showing the original image, (b) showing the result using the proposed method, (c) showing the result using Zhao's method, (d) showing genetic evolution plot (the horizontal axis corresponds to the number of generations, while

the vertical one to the fitness) and (e) showing the member function plot. The original image is partitioned to (divided into) three parts (dark, gray and bright). The gray levels of the three parts are: 0, 128 and 255. The experiment results show that the proposed three-level thresholding method gives good performance.

Fig. 2(a) is a gray scale ship in the ocean image and (b) is the three-level thresholding image. We can see that the main features of the image, such as the ship, ocean, sky and cloud, are also well preserved. Fig. 2(c) is the result using Zhao's method, where the three parts (dark, gray and bright) satisfy $p_d = p_m = p_b = 1/3$. So the number of the pixels in those parts should be nearly equivalent, which leads to the hopeless loss of some main features. Fig. 2(d) is the genetic evolution plot. The maximal fuzzy entropy 13.2568 is gained in 294 generation and the correspond fuzzy 3-partition for the image is $(a_1, b_1, c_1, a_2, b_2, c_2) = (42, 46, 73, 95, 242, 253)$, while the thresholds are also $(t_1, t_2) = (53, 202)$. Fig. 2(e) is the member function plot of the image, which shows that the width of the fuzzy region between the dark part and the gray part is narrow and the width of the fuzzy

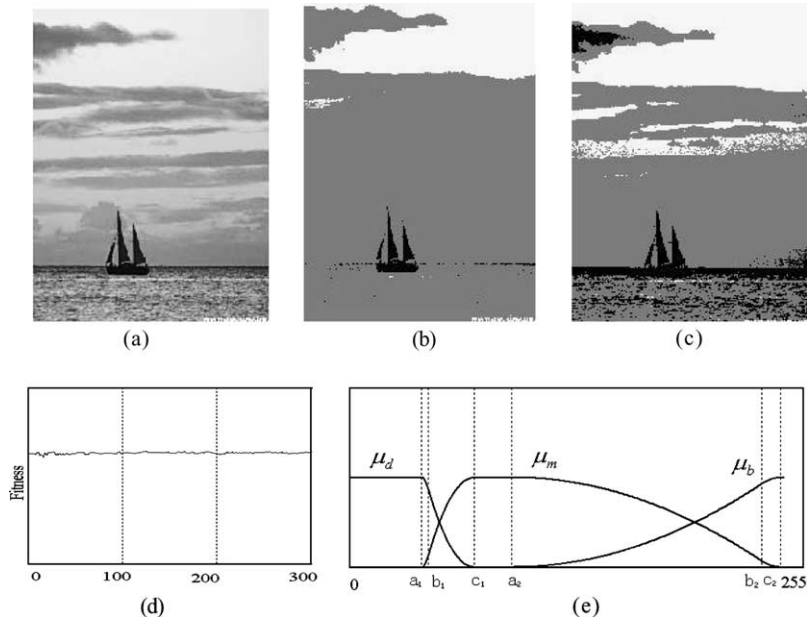


Fig. 2. (a) Original image; (b) result image using proposed fuzzy approach, $(t_1, t_2) = (53, 202)$; (c) result image using Zhao's method; (d) fitness results of GA; (e) member function plot, $(a_1, b_1, c_1, a_2, b_2, c_2) = (42, 46, 73, 95, 242, 253)$.

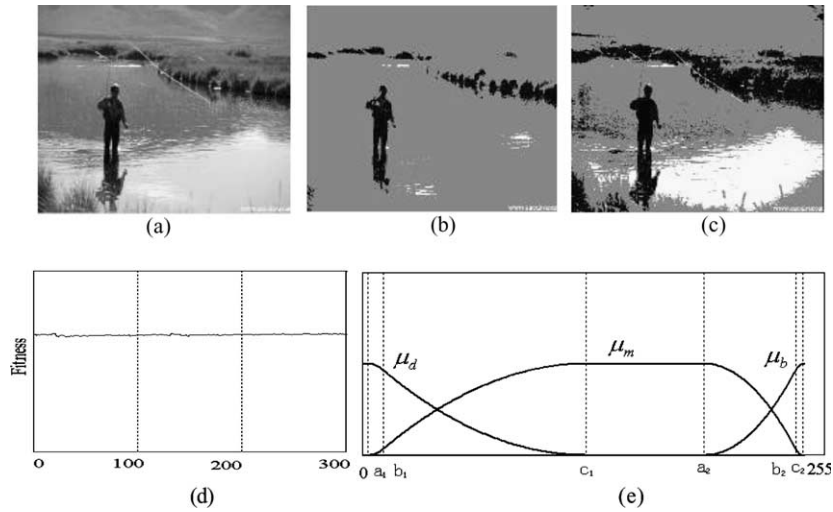


Fig. 3. (a) Original image; (b) result image using proposed fuzzy approach, $(t_1, t_2) = (44, 235)$; (c) result image using Zhao's method; (d) fitness results of GA; (e) member function plot, $(a_1, b_1, c_1, a_2, b_2, c_2) = (3, 12, 129, 197, 250, 254)$.

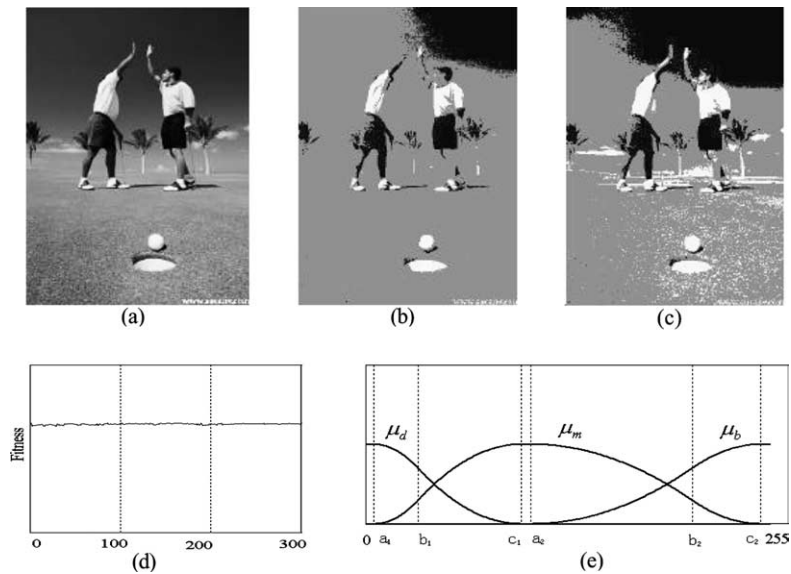


Fig. 4. (a) Original image; (b) result image using proposed fuzzy approach, $(t_1, t_2) = (44, 189)$; (c) result image using Zhao's method; (d) fitness results of GA; (e) member function plot, $(a_1, b_1, c_1, a_2, b_2, c_2) = (5, 33, 98, 104, 206, 249)$.

region between the gray part and the bright is broad.

Fig. 3(a) is a gray scale fisherman image and (b) is the three-level-thresholded image. The results show that by the proposed method the fisherman and the float grass is divided into the dark part, a

majority part of the water surface and the land is divided into gray part, while a little into bright part. The main features of the image, such as the fisherman, the water surface and the float grass, are again well preserved. Fig. 3(c) is the result using Zhao's method. Fig. 3(d) is the genetic evolution

plot. The maximal fuzzy entropy 13.6247 is gained in 196 generation and the correspond fuzzy 3-partition for the image is $(a_1, b_1, c_1, a_2, b_2, c_2) = (3, 12, 129, 197, 250, 254)$, while the thresholds are $(t_1, t_2) = (44, 235)$. Fig. 3(e) is the member function plot of the image that shows that the width of the fuzzy region between the dark part and the gray part is broad while the width of the fuzzy region between the gray part and the bright part is narrow.

Fig. 4(a) is a gray scale linksman image and (b) is the thresholded image using the proposed method. We can see that the main features of the image, such as the linksman, golf, hole and the tree, are well preserved. Fig. 4(c) is the result using Zhao's method, where some clutter is produced. Fig. 4(d) is the genetic evolution plot. The maximal fuzzy entropy 13.6426 is gained in 275 generation, and the corresponding fuzzy 3-partition for the image is $(a_1, b_1, c_1, a_2, b_2, c_2) = (5, 33, 98, 104, 206, 249)$, with the thresholds being $(t_1, t_2) = (44, 189)$. Fig. 4(e) is the member function plot of the image, which shows that the width of the fuzzy region between the dark part and the gray part and the width of the fuzzy region between the gray part and the bright are both very broad, nearly covering the entire distribution.

5. Comparison and discussion

5.1. Comparison

Some ideas in Zhao's method (Zhao et al., 2001) are adopted in the paper, so it is necessary to state the differences between Zhao's method and the proposed method. The main differences are:

1. *Different membership function.* Zhao used the simplest function, which is monotonic, to approximate the membership of bright μ_b and dark μ_d , while the membership of medium μ_m is dependent on μ_b and μ_d . The membership functions have four parameters a_1, c_1, a_2, c_2 . In other words, the two thresholds t_1 and t_2 , for three-level thresholding, are dependent on a_1, c_1, a_2, c_2 . The proposed method in this paper used *S*-function, *II*-function and *Z*-function to approximate the membership of bright μ_b , medium μ_m and dark μ_d . Although the membership functions used in (Zhao

et al., 2001) are succinct, when compared with the proposed method, it is less appropriate in describing the fuzzy of one image.

2. *Different fuzzy entropy principle.* The fuzzy entropy principle in (Zhao et al., 2001) is

$$H(a_1, c_1, a_2, c_2) = -p_d \ln p_d - p_m \ln p_m - p_b \ln p_b$$

According to the maximum fuzzy entropy principle, $p_d = p_m = p_b = 1/3$ should be satisfied. The number of the pixels in those parts should be nearly equivalent. However, sometime the classification that satisfied this condition is not always the best result.

The fuzzy entropy principle proposed in this paper is formulas (11) and (12).

Moreover, we could find that the conclusion is just the same as the 1-D entropy principle when the following condition is satisfied:

$$\mu_d(k) = \begin{cases} 1 & 0 < k \leq t_1 \\ 0 & \text{else} \end{cases}$$

$$\mu_m(k) = \begin{cases} 1 & t_1 \leq k < t_2 \\ 0 & \text{else} \end{cases}$$

$$\mu_b(k) = \begin{cases} 1 & t_2 \leq k < 255 \\ 0 & \text{else} \end{cases}$$

So the fuzzy entropy principle proposed in this paper is more universal.

3. *Different optimal algorithm.* Generally, the exhaustive techniques can be chosen to locate the global maximum if the search space is small enough. But in many cases, the complexity of the parameter search space will lead to computational prohibition of the exhaustive techniques. So some fast optimizing algorithm, such as genetic algorithm, can be adopted to overcome the computational complexity problem.

The fuzzy entropy principle in (Zhao et al., 2001) has only four parameters a_1, c_1, a_2, c_2 with the condition $p_d = p_m = p_b = 1/3$ to simplify the computation, so Zhao in his paper may use exhaustive techniques to maximize the fuzzy entropy. In this paper, the fuzzy entropy principle has six parameters $a_1, b_1, c_1, a_2, b_2, c_2$ with the condition $0 < a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2 < 255$, so it is almost impossible to use exhaustive technique for finding the optimal combination of all the fuzzy

parameters. We implement genetic algorithm to maximize the fuzzy entropy and then to get the optimal combination of the parameters $a_1, b_1, c_1, a_2, b_2, c_2$, thanks to the efficient and collateral global searching ability of the genetic algorithm, the performance is satisfactory. The experiment results show that our proposed method gives better performance.

It should be pointed out that the parameters a_1, c_1, a_2, c_2 in (Zhao et al., 2001) may not satisfy the condition $c_1 \leq a_2$, which does not lead to computational complexity because the membership functions used in (Zhao et al., 2001) are the simplest monotonic function. Whereas, because of the complexity of the membership function used in this paper, the condition $0 < a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2 < 255$ is necessary, which can simplify the description of the membership function and the optimizing process.

5.2. Discussion

The proposed method can be easily expanded to N -level ($N > 3$) thresholding. For N -level thresholding, there will be N membership functions $\mu_1, \mu_2, \dots, \mu_N$. We can obtain N membership functions and the maximal fuzzy entropy function, which includes $3N$ parameters $a_1, b_1, c_1, a_2, b_2, c_2, \dots, a_N, b_N, c_N$, by using the same method as described above. These parameters satisfy the following condition:

$$0 < a_1 \leq b_1 \leq c_1 \leq a_2 \leq b_2 \leq c_2 \leq \dots \leq a_N \leq b_N \leq c_N < 255 \quad (15)$$

For multiple thresholding method, we can also choose the appropriate thresholds by using the proposed maximum fuzzy entropy. We can find the optimal combination of all the $3 * N$ fuzzy parameters by genetic algorithm, and then the appropriate thresholds. Therefore, the chromosome of the genetic algorithm is encoded as $3 * N * 8$ -bits binary strings of 0's and 1's. The more the number of the thresholds is, the longer the chromosome string will be. But the excessive long chromosome string will lead to bad performance of the genetic algorithm. So we must choose other appropriate code method when N is too big

in the N -level thresholding method. It is the next work that we will do.

6. Conclusion

In the paper, we present a three-level thresholding method for image segmentation based on probability partition and fuzzy partition and entropy theory. We defined a new fuzzy entropy through probability analysis. The image is segmented to three parts, including dark, gray and white part, whose member functions of the fuzzy region are Z -function, Π -function and S -function respectively. There are six parameters in those member functions, determining the widths and the attributes of two fuzzy region of the image. The fuzzy region is found by genetic algorithm based on the maximum fuzzy entropy principle. The image can keep as much information as possible when the image is transformed from the intensity domain to the fuzzy domain. In most of the existing methods, the cross-over point of S -function, Z -function and Π -function is set as the mid-point of the fuzzy region $[a, c]$, i.e., $\Delta b = b - a = c - b$. However, the mid-point strategy does not promise that the fuzzy entropy is the global maximum. In this paper, when searching for the fuzzy region $[a, c]$, the appropriate determination of the parameter b , parameter of the asymmetrical S -function, Z -function and Π -function, can make the fuzzy entropy higher than that using the symmetrical S -function, Z -function and Π -function. The procedure for finding the optimal combination of all the fuzzy parameter is implemented by genetic algorithm with appropriate coding method to avoid useless chromosomes. The experiment results show that our proposed method gives good performance.

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