Image Segmentation by Histogram Thresholding Using Fuzzy Sets

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***Abstract*—Methods for histogram thresholding based on the minimization of a threshold-dependent criterion function might not work well for images having multimodal histograms. In this paper we propose an approach to threshold the histogram according to the similarity between gray levels. Such a similarity is assessed through a fuzzy measure. In this way, we overcome the local minima that affect most of the conventional methods. The experimental results demonstrate the effectiveness of the proposed approach for both bimodal and multimodal histograms.**

***Index Terms*—Fuzzy measures, fuzzy sets, histogram thresholding, image segmentation.**

I. Introduction

TYPICAL computer vision applications usually require an *image segmentation-preprocessing* algorithm as a first procedure. At the output of this stage, each object of the image, represented by a set of pixels, is isolated from the rest of the scene. The purpose of this step is that objects and background are separated into nonoverlapping sets. Usually, this segmentation process is based on the image gray-level histogram. In that case, the aim is to find a critical value or threshold. Through this threshold, applied to the whole image, pixels whose gray levels exceed this critical value are assigned to one set and the rest to the other. For a well-defined image, its histogram has a deep valley between two peaks. Around these peaks the object and background gray levels are concentrated. Thus, to segment the image using some histogram thresholding technique, the optimum threshold value must be located in the valley region. A myriad of algorithms for histogram thresholding can be found in the literature [1]–[11]. Some algorithms [5] use an iterative scheme to achieve pixel separation. Entropy based algorithms have been proposed in [6], [7]. In general, all histogram thresholding techniques work very well when the image gray-level histogram is bimodal or nearly bimodal. On the other hand, a great deal of images are usually ill defined (corrupted by noise and/or irregularly illuminated) leading to a multimodal histogram (Fig. 1) where, in these cases, the ordinary histogram thresholding techniques perform poorly or even fail. In this class of histograms, unlike the bimodal case, there is no clear separation between object and background pixel occurrences. Thus, to find a reliable threshold, some adequate criterion for splitting the image histogram should be used. A possible one is the use of a measure of similarity or closeness between gray levels. At this point, the question that arose is: "How can this measure be quantified in order to classify a gray level as belonging to a certain set (object or background pixel set)?" The answer to this question is not easy to find by employing conventional thresholding techniques. Since the fuzzy set theory was introduced, it has become a powerful tool to deal with linguistic concepts such as similarity. Several segmentation algorithms based on fuzzy set theory are reported in the literature [8]–[11]. They are based on the optimization of a threshold-dependent criterion function, which is in general a measure of fuzziness (index of fuzziness, compactness, among others). In [8] and [10], approaches based on evaluating a global fuzzy measure for all possible gray levels are presented; hence, the optimum threshold is selected for the gray level that minimizes such a measure. For well-defined images, i.e., having bimodal (or nearly) histograms, such methods work very well. However, for images with very irregular histograms, approaches based on global measures might not work well. As a matter of fact, in these cases the criterion function may have a minimum, corresponding to a histogram local minimum, or even not have any minimum at all.

The proposed method is also based on a fuzzy measure to threshold the image histogram. However, differently of previous approaches, we do not use a criterion function to be minimized. Instead, the image histogram is thresholded based on a criterion of similarity between gray levels. To this end, a measure of fuzziness is used for assessing such a concept. The technique proposed in this work consists in defining two linguistic variables {object, background} modeled by two fuzzy subsets and to establish a *fuzzy region* on the gray level histogram. In a second step, we assign each of the gray levels of the fuzzy region to both defined subsets (object and background) and measure the index of fuzziness (IF) of each of these subsets. Finally, the histogram threshold is determined for the gray level in which the IF's are equal; hence, the gray levels are grouped according to their similarity. It is interesting to point out that the threshold determined in this way may or not correspond to an absolute minimum of the histogram. As a matter of fact, note that as the proposed method is not based on the minimization of a criterion function, the problem of detecting local minima is avoided. This characteristic represents an attractive property of the proposed method.

The outline of the paper is as follows. In Section II some basic definitions about fuzzy sets as well as measures of fuzziness are reviewed. In Section III the proposed algorithm for histogram thresholding is presented. Section IV shows some experimental

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results to illustrate the effectiveness and usefulness of the proposed approach. Section V ends the paper with concluding remarks.

II. Basic Definitions

A. Fuzzy Set Theory

A fuzzy set is a class of points possessing a continuum of membership grades, where there is no sharp boundary among elements that belong to this class and those that do not [12]. We can express this membership grade by a mathematical function called *membership function* or *characteristic function* $\mu_{\mathbf{A}}(x_i)$. This function assigns to each element in the set a membership grade in the interval [0, 1]. Let $\mathbf{X}$ be the universe of discourse, with a generic element denoted by x_i: $\mathbf{X} = \{x_1, x_2, \ldots, x_m\}$. A fuzzy set $\mathbf{A}$ in $\mathbf{X}$ is formally defined as

$$\mathbf{A} = \{(x_i, \mu_{\mathbf{A}}(x_i))\}, \qquad x_i \in \mathbf{X} \tag{1}$$

where $\mathbf{A}$ is characterized by the function $\mu_{\mathbf{A}}(.)$, which associates with each point $x_i \in \mathbf{X}$ a membership grade $\mu_{\mathbf{A}}(x_i) \in [0, 1]$. In this work, the S-function is used for modeling the characteristic function. Such a function is defined as

$$\mu_{\mathbf{A}_S}(x) = S(x; a, b, c) = \begin{cases} 0, & x \le a \\ 2\{(x-a)/(c-a)\}^2, & a \le x \le b \\ 1 - 2\{(x-c)/(c-a)\}^2, & b \le x \le c \\ 1, & x \ge c. \end{cases} \tag{2}$$

The S-function can be controlled by the parameters a and c. b denotes the crossover point, which is given by $b = (a+c)/2$, with $\mu_{\mathbf{A}_S}(b) = 0.5$; the bandwidth of the function is defined as $\Delta b = b - a = c - b$. Herein, we also use the Z-function, which is derived from the S-function as follows:

$$\mu_{\mathbf{A}_Z}(x) = Z(x; a, b, c) = 1 - S(x; a, b, c). \tag{3}$$

B. Measures of Fuzziness

By using the IF introduced by Kaufmann [13], we can determine how compact the set $\mathbf{A}$ is as compared with its nearest ordinary set $\underline{\mathbf{A}}$. This latter set is such that its characteristic function is given by

$$\mu_{\underline{\mathbf{A}}}(x_i) = \begin{cases} 0, & \text{if} \quad \mu_{\mathbf{A}}(x_i) < 0.5 \\ 1, & \text{if} \quad \mu_{\mathbf{A}}(x_i) \ge 0.5. \end{cases} \tag{4}$$

In Kaufmann's definition, this index is obtained by measuring the distance between $\mathbf{A}$ and $\underline{\mathbf{A}}$. Such an index is defined as

$$\psi_k(\mathbf{A}) = \frac{2}{n^{1/k}} d_k(\mathbf{A}, \underline{\mathbf{A}}) \tag{5}$$

where $d_k(\mathbf{A}, \underline{\mathbf{A}})$ is a measure of distance, and n is the number of elements in $\mathbf{A}$. Such a distance is computed according to

$$d_k(\mathbf{A}, \underline{\mathbf{A}}) = \frac{2}{(n)^{1/k}} \left(\sum_{i=1}^{n} \left(\mu_{\mathbf{A}}(x_i) - \mu_{\underline{\mathbf{A}}}(x_i) \right)^k \right)^{1/k}. \tag{6}$$

In this paper, we have used (6) with $k = 1$, such an index is denoted as a linear index of fuzziness [13]. For other measures of fuzziness, which could also be used, the reader is referred to Reference [15]. In [14] it is stated that the IF of a fuzzy set $\mathbf{A}$, having n supporting points, reflects the degree of ambiguity present in it. Note that in our application we use the concept of *similarity*. That is, a fuzzy set having a low index of fuzziness indicates that its elements are very similar, i.e., exists a low *ambiguity* between them.

III. Proposed Method

A. Algorithm

In order to implement the thresholding algorithm on a basis of the concept of similarity between gray levels, we make the following assumptions:

i) there exists a significant contrast between the objects and background;
ii) the gray level is the universe of discourse, a one-dimensional set, denoted by $\mathbf{X}$.

Our purpose is to threshold the gray-level histogram by splitting the image histogram into two crisp subsets, object subset $\mathbf{O}$ and background subset $\mathbf{F}$, using the measure of fuzziness previously defined. Now, based on the assumption i), let us define two linguistic variables {object, background} modeled by two fuzzy subsets of $\mathbf{X}$, denoted by $\mathbf{B}$ and $\mathbf{W}$, respectively. The fuzzy subsets $\mathbf{B}$ and $\mathbf{W}$ are associated with the histogram intervals $[x_{\min}, x_j]$ and $[x_r, x_{\max}]$, respectively, where x_j and x_r are the final and initial gray-level limits for these subsets, and $x_{\min}$ and $x_{\max}$ are the lowest and highest gray levels of the image, respectively. We know that the gray levels in each of these subsets have the intuitive property of belonging with certainty to the final subsets object $(\mathbf{O})$ or background $(\mathbf{F})$. So, $\mathbf{B} \subset \mathbf{O}$ and $\mathbf{W} \subset \mathbf{F}$ or *vice-versa*. Those subsets are located at the beginning and the end regions of the histogram. With these subsets, we have a seed for starting the similarity measure process. Also, we define a *fuzzy region* placed between $\mathbf{B}$ and $\mathbf{W}$, as depicted in Fig. 1. Then, to obtain the segmented version of the gray-level image, we have to classify each gray level of the fuzzy region as being object or background. The classification procedure is as follows. We add to each of the seed subsets $\mathbf{B}$ and $\mathbf{W}$ a gray level x_i picked from the *fuzzy region*. Then, by measuring the IF's of the subsets $\mathbf{B} \cup \{x_i\}$ and $\mathbf{W} \cup \{x_i\}$, we assign x_i to the subset with lower IF (maximum similarity). Finally, applying this procedure for all gray levels of the *fuzzy region*, we can classify them into *object* or *background* subsets. In other words, we observe how the introduction of a gray level of the *fuzzy region* affects the similarity measure among gray levels in each of the

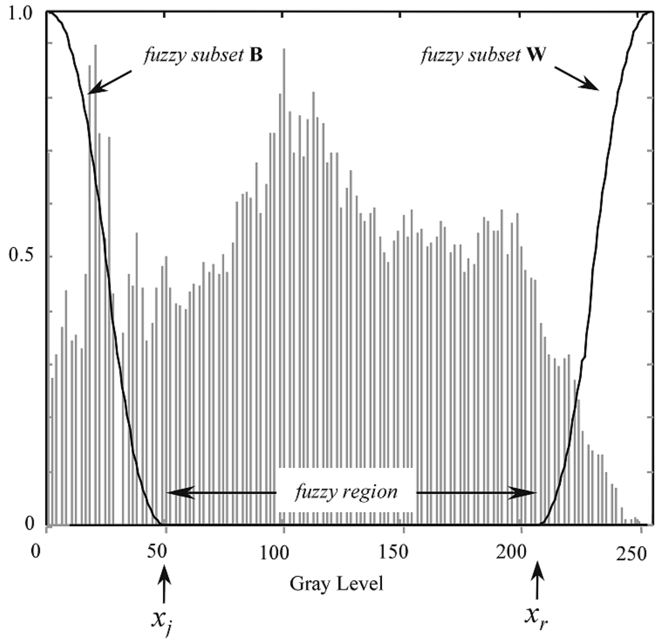

Fig. 1. Multimodal image histogram and the characteristic functions for the seed subsets.

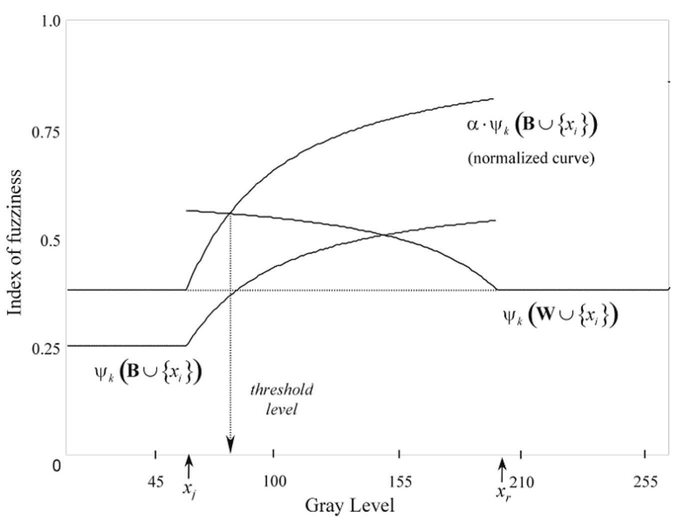

Fig. 2. Normalization step of the indices of fuzziness and determination of the threshold value.

modified fuzzy subsets ($\mathbf{B} \cup \{x_i\}$ and $\mathbf{W} \cup \{x_i\}$). For the defined subsets, the following statements are valid:

$$a)\ \mathbf{O} \cup \mathbf{F} = \mathbf{X};$$

$$b)\ \begin{cases} \mathbf{B} \subset \mathbf{O} \text{ and } \mathbf{W} \subset \mathbf{F} & \text{for light background,} \\ \text{or} & \\ \mathbf{B} \subset \mathbf{F} \text{ and } \mathbf{W} \subset \mathbf{O} & \text{for dark background.} \end{cases}$$

Let us now consider the fuzzy subsets $\mathbf{W}$ and $\mathbf{B}$ with membership functions $\mu_{\mathbf{W}_S}(x)$ and $\mu_{\mathbf{B}_Z}(x)$ modeled by the S-function (2) and Z-function (3), respectively (see Fig. 2). Note that $\mu_{\mathbf{W}_S}(x)$ and $\mu_{\mathbf{B}_Z}(x)$ present higher values of membership when x is near $x_{\max}$ or $x_{\min}$, respectively. Conversely, for values of x near the *fuzzy region* the membership decreases.

Instead of using the S and Z-functions with a fixed bandwidth (Δb) as in [8], let us take the parameters of the S and Z-functions as follows:

$$b = \frac{\sum_{i=p}^{q} x_i h(x_i)}{\sum_{i=p}^{q} h(x_i)} \tag{7}$$

$$c = b + \max\left\{|b - (x_i)_{\max}|,\, |b - (x_i)_{\min}|\right\}, \quad p \le i \le q \tag{8}$$

and

$$a = 2b - c \tag{9}$$

where $h(x_i)$ denotes the image histogram and x_p and x_q are the limits of the subset being considered. The quantities $(x_i)_{\max}$ and $(x_i)_{\min}$ in (8) represent the maximum and minimum gray levels in the current set for which $h((x_i)_{\max}) \neq 0$ and $h((x_i)_{\min}) \neq 0$. Note, that the crossover point b (7) is the mean gray level value of the interval $[x_p, x_q]$. Next, by using (8) and (9) c and a are obtained. With the function parameters computed in this way, we introduce some type of adaptability in the computation of the membership functions. In this way, we permit that the S and Z functions adjust its shape as a function of the set elements. This desired characteristic is not present if we select a fixed bandwidth (Δb). A method for automatic bandwidth selection is given in [9].

Since the key of the proposed classification method is the comparison of IF measures, we have to normalize those measures. This is done by first computing the IF's of the seed subsets $\mathbf{W}$ and $\mathbf{B}$, and by computing a normalization factor α according to the following relation

$$\alpha = \frac{\psi_k(\mathbf{W})}{\psi_k(\mathbf{B})} \tag{10}$$

where $\psi_k(\mathbf{W})$ and $\psi_k(\mathbf{B})$ are the IF's of the subsets $\mathbf{W}$ and $\mathbf{B}$, respectively. Fig. 2 illustrates how the normalization works. Note from this figure, the different threshold that would have been determined without this previous step. The proposed algorithm, for the case $\mathbf{W} \subset \mathbf{F}$ and $\mathbf{B} \subset \mathbf{O}$, can be summarized in the following steps:[1]

step 1: compute the normalization factor α;
step 2:
  for $i = j+1$ to $r-1$;
    compute $\psi_k(\mathbf{B} \cup \{x_i\})$;
    compute $\psi_k(\mathbf{W} \cup \{x_i\})$;
    if $\psi_k(\mathbf{W} \cup \{x_i\})$ is *lower than* $\alpha \cdot \psi_k(\mathbf{B} \cup \{x_i\})$;
    then: x_i is included in set $\mathbf{F}$,
    otherwise: x_i is included in set $\mathbf{O}$,
  end for.

Fig. 2 shows the plots of the functions $\psi_k(\mathbf{W} \cup \{x_i\})$, $\psi_k(\mathbf{B} \cup \{x_i\})$ and $\alpha \cdot \psi_k(\mathbf{B} \cup \{x_i\})$, for $i = j+1$ to $r-1$. The threshold

[1]Here, we suppose that the IF of the fuzzy subset $\mathbf{B}$ needs to be normalized, as is the case of Fig. 2.

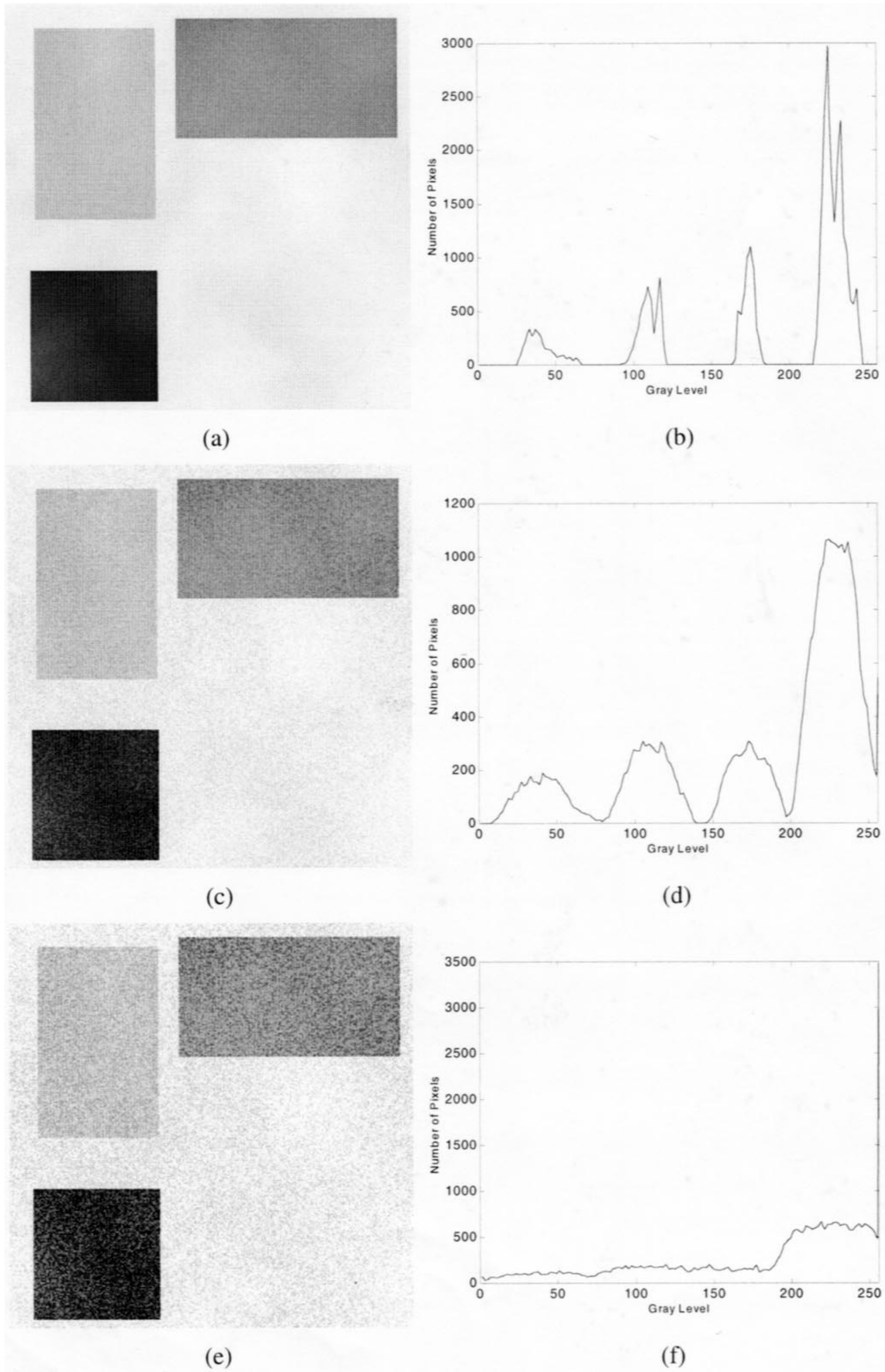

Fig. 3. Performance evaluation of the proposed method. (a) Reference image; (b) reference image histogram; (c) test image #5 (SNR $= 23$ dB); (d) test image #5 histogram; (e) test image #7 (SNR $= 19$ dB); and (f) test image #7 histogram.

level for image segmentation is determined by the intersection of the normalized curves of the indices of fuzziness. The final crisp subset $\mathbf{F}$ is composed of all gray levels above the intersection point and the crisp subset $\mathbf{O}$ by those below it, for a light background case.

B. Performance Evaluation

In order to evaluate the performance of the proposed algorithm, a synthetic reference image was used. It is composed of three objects, having a known number of pixels, on a textured background (cloud-type texture) [Fig. 3(a)]. By adding increasing quantities of noise with uniform distribution, several test images were generated having different SNR [obtained by (12)]. For instance, Fig. 3(c) and (e) depict the test images with SNR of 23 dB and 19 dB, respectively. The histograms of the reference and the above test images are shown in Fig. 3(b), (d) and (f), respectively. To compare the segmentation results a probability of error measure is used. Such a figure of merit is defined as [16]

$$P(\text{error}) = \sum_{j=1}^{N} \sum_{\substack{i=1 \\ i \neq j}}^{N} P(R_i|R_j)P(R_j) \tag{11}$$

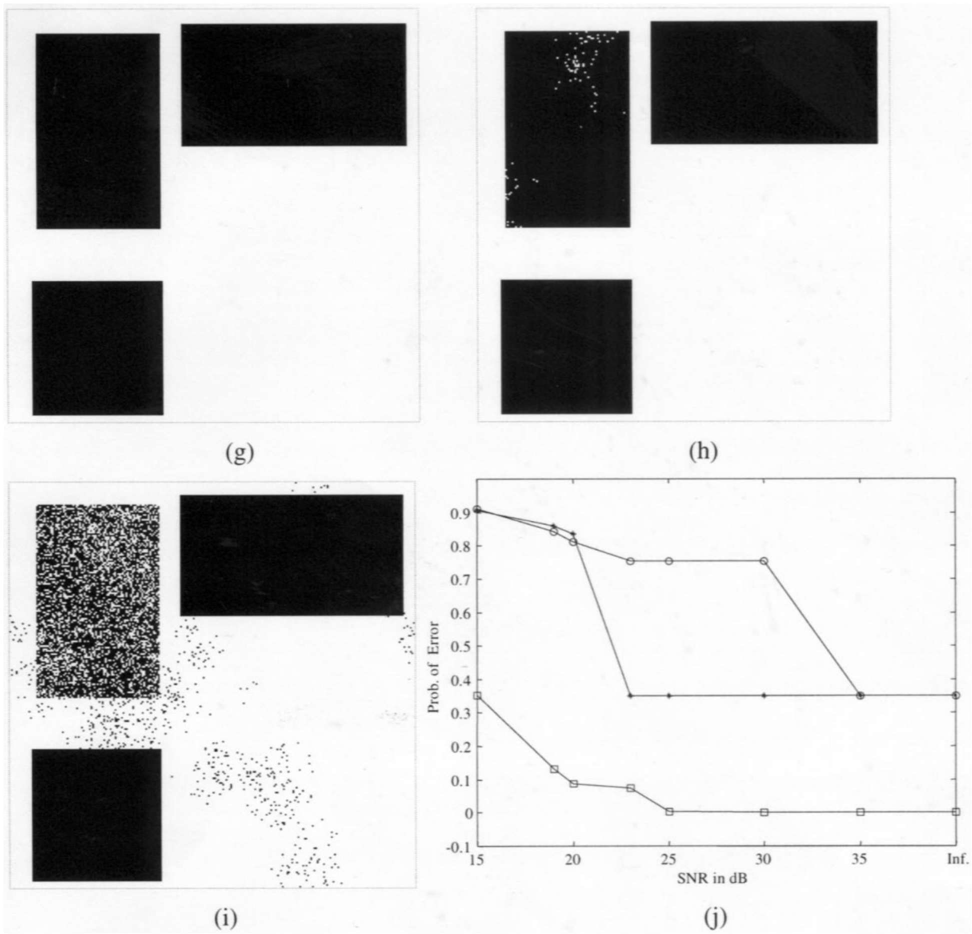

Fig. 3. (*Continued.*) Performance evaluation of the proposed method. (g) Reference image manually segmented; (h) and (i) segmentation results by the proposed method for test images #5 and #7, respectively; and (j) probability of error results. □: proposed method; $\times$: [10]; $*$: [8].

where R_i and R_j are the number of pixels in the ith and jth regions in the image, and N represents the number of regions in the segmented image ($N = 2$, for our case). Equation (11) returns a measure that is a function of the misclassified pixels between the manually segmented reference image [Fig. 3(g)] and the test images segmented using the proposed algorithm. Segmentation results for test images, SNR 23 dB and 19 dB, are illustrated in Fig. 3(h) and (i), respectively. The probability of error $(P(\text{error}))$ *versus* SNR, for the test images using the proposed approach is shown in Fig. 3(j). Due to the nature of the methods presented in [8] and [10], we have used them to compare with our approach for performance evaluation. Before continuing, let us briefly describe such approaches. Both methods are based on a single fuzzy measure by using the standard S-function to represent the membership function. Such a function is computed for all possible gray levels by using a given bandwidth Δb. In [8] the membership function is used to determine the entropy measure, whereas in [10] through that function the index of fuzziness is determined. In addition, due to the fact of using a fixed bandwidth, both methods are performance sensitive regarding the value for this parameter.

In Fig. 3(j), the results obtained by [8] and [10] are also plotted. For that figure, $P(\text{error}) = 1$ corresponds to the situation in which the all pixels are misclassified (all pixels are classified as being object or background). Fig. 3(j) shows that the proposed approach has found the optimum threshold ($P(\text{error}) = 0$) when the SNR is greater or equal than 25 dB. When the SNR is about of 20 dB the $P(\text{error})$ is relatively low, as compared with the results obtained by [8] and [10]. Note that these methods ([8] and [10]) maintain a $P(\text{error}) > 0$ even for higher values of SNR. This behavior is mainly due to the existing local minima in the image histogram. In fact, its presence severely affects the determination of the thresholds obtained by such methods. Thus, Fig. 3(j) clearly illustrates the advantage of a method that does not depend on minimizing a certain criterion function. The signal-to-noise ratio (SNR) used herein is defined as

$$\text{SNR} = 10 \log \left[\frac{\sum_{r=1}^{Nr} \sum_{c=1}^{Nc} I^2(r, c)}{\sum_{r=1}^{Nr} \sum_{c=1}^{Nc} (I(r, c) - I_n(r, c))^2} \right] \tag{12}$$

where $I(r, c)$ and $I_n(r, c)$ represent the intensity of the (r, c)th pixel of the reference and test (noise added) images, respectively. Nr and Nc represent the number of rows and columns of the image, respectively.

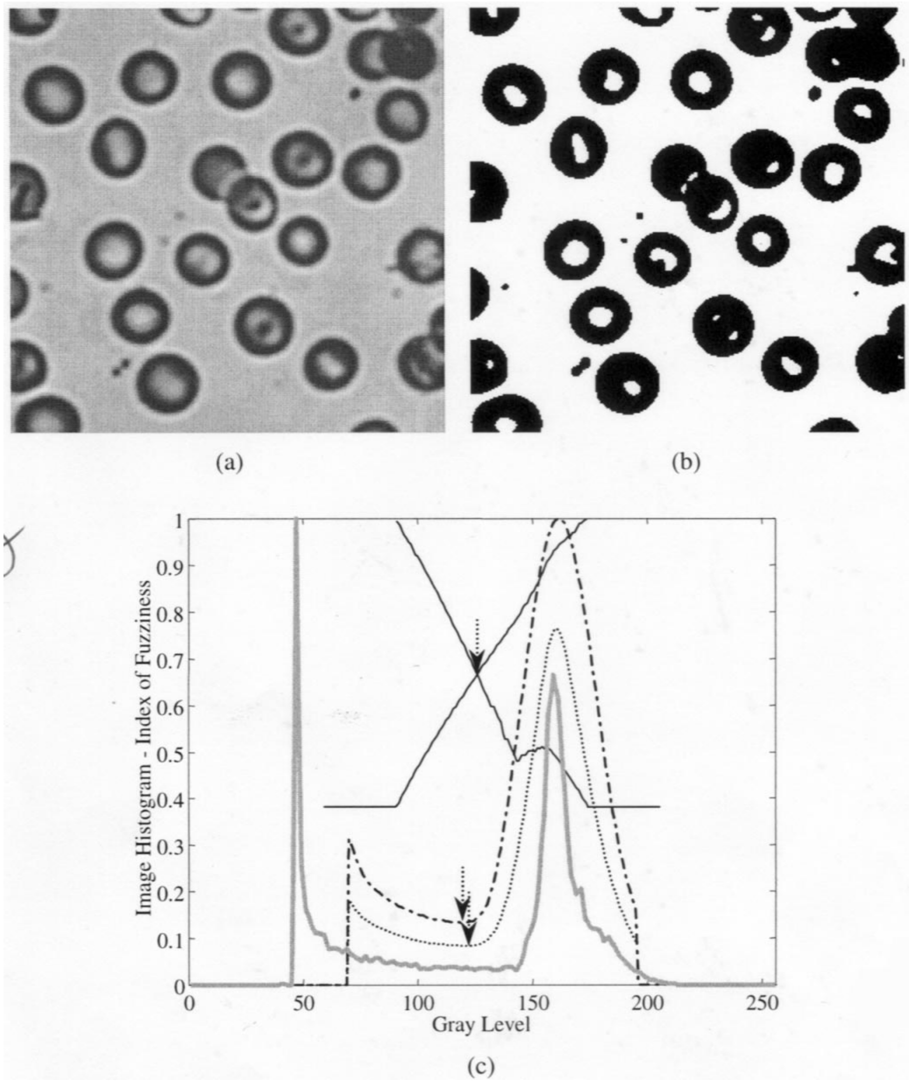

Fig. 4. Histogram thresholding results for an image having bimodal histogram. (a) Blood image; (b) segmented image using the threshold value of 122; (c) gray line: image histogram; solid lines: evolution of the IF's (proposed method); dotted line: criterion function [8]; dash-dotted line: criterion function [10]. Threshold locations are indicated with arrows.

IV. Experimental Results

We have tested the proposed method by applying it to a variety of images having different types of histograms. Figs. 4–6 show comparative results of histogram thresholding of real images performed by [8] and [10] approaches and by the proposed one. For the Blood image [Fig. 4(a)], the curves corresponding to the criterion functions of [8] and [10] as well as the evolution of the IF's obtained by the proposed method are shown in Fig. 4(c). It reveals that the methods [8], [10] and the proposed one perform similarly, which is due to the fact that the image histogram has a global minimum. The obtained thresholds are 120, 122, and 125, respectively. Since the thresholds determined are very close, the segmented image by using any of them will have minor differences. In Fig. 4(b) the segmented image is depicted by using the threshold equal to 122. In contrast, the Bacteria image [Fig. 5(a)] exhibits a histogram having several local minima [Fig. 5(c)]. Thus, we can expect that the methods based on the minimization of a global measure may fail in determining a satisfactory threshold. In Fig. 5(c), we can note that the criterion functions obtained by [8] and [10] do not present any global minimum. Thus, for the Bacteria image, no threshold can be determined from [8] and [10]. Fig. 6 shows an image (Porous Media) in which a slight overlap exists between object and background sets. As can be observed from Fig. 6(a), this is due to the irregular illumination of the image. For this kind of images, better segmentation results can be obtained by the use of a spatial technique [6]. In Fig. 6(b) and (c) the [8], [10] and proposed methods yield somewhat comparable results. In particular, note that the segmented image Fig. 6(c) resulting from the use of the proposed method has the background slightly more perceivable than Fig. 6(b), segmented by [8] and [10].

As can be seen from Figs. 4(c) and 5(c), the shape of the histogram may slightly affect the evolution of the IF's of the seed subsets. However, this fact does not represent a major drawback of the proposed method. The histogram shape also determines the selection of the boundary values x_j and x_r for the seed subsets. We may use different sizes for these subsets, $(x_j - x_{\min})$

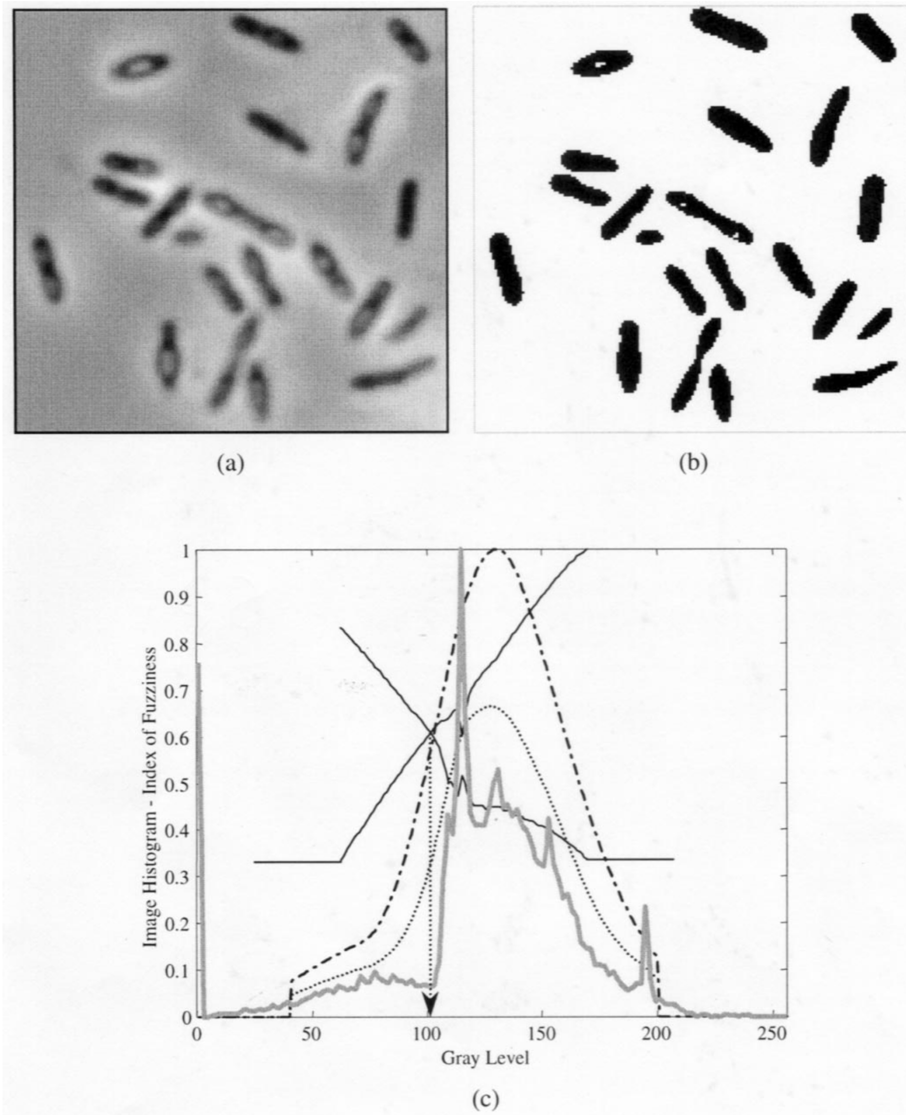

Fig. 5. Histogram thresholding results for an image having a multimodal histogram. (a) Bacteria image; (b) segmented image using the threshold obtained by the proposed method; and (c) gray line: image histogram; solid lines: evolution of the IF's (proposed method); dotted line: criterion function [8]; dash-dotted line: criterion function [10]. The arrow indicates the determined threshold.

and $(x_{\max}-x_r)$, as a function of the number of gray level occurrences in such regions. The condition to be satisfied is to have sufficient information within the seed subsets. Providing this, the values for x_j and x_r will not be critical to the performance of the proposed method. The values used for $\{(x_j - x_{\min}),(x_{\max} - x_r)\}$ are: Fig. 3(d) {40, 40}; Fig. 3(f) {50, 40}; Fig. 4 {30, 30}; Fig. 5 {50, 50} and Fig. 6 {40, 50}.

Also, note that the threshold determined by the proposed method does not correspond to an absolute minimum of the histogram. As mentioned earlier, this behavior is mainly due to the concept of similarity, which does not depend on detecting such a global minimum. In the experimental results obtained with the [8] and [10] methods the optimal value of Δb has been used in order to obtain the best-segmented image for those techniques.

V. Conclusions

In this paper, we introduce a procedure for histogram thresholding which is not based on the minimization of a criterion function. Instead, the histogram threshold is determined according to the similarity between gray levels. The fuzzy framework is used to obtain a mathematical model of such a concept. Through the comparison of results of the proposed approach with the ones obtained by minimizing threshold-dependent criterion functions, we verify that the improvement achieved by our method, for multimodal histograms, is due to the following reason. The presented approach does not attempt to detect a global minimum; hence, the risk of getting blocked in a local minimum is avoided. The threshold determined in this way may or may not correspond to an absolute minimum of the histogram. Because of the used assumption, in which objects and background must occupy nonoverlapping regions of the histogram, the applicability of the proposed method is limited to images that satisfy such a requirement. On the other hand, this does not represent a serious restriction since the number of real images having the required characteristic is very large.

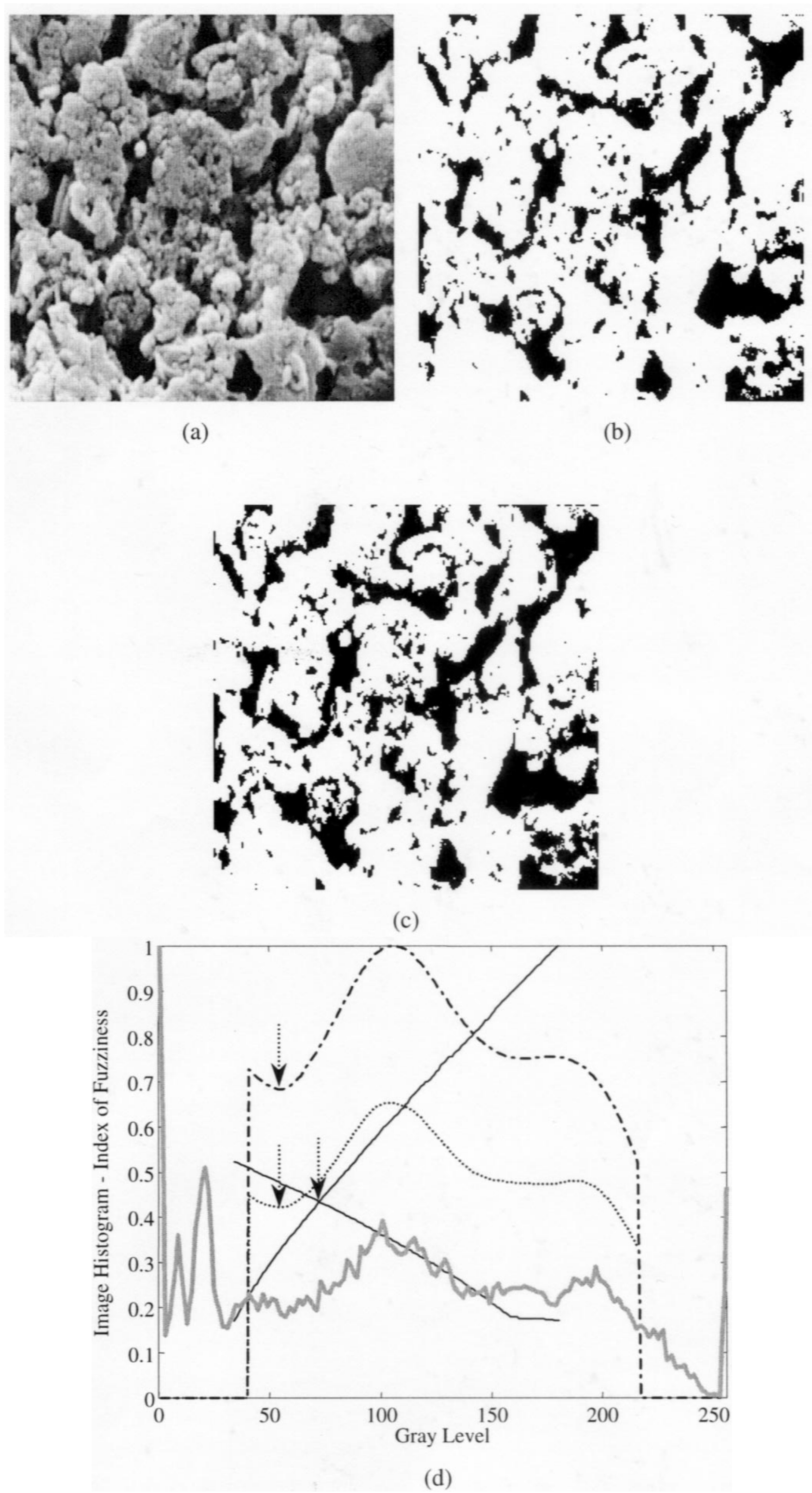

Fig. 6. Histogram thresholding results for an image having a multimodal histogram. (a) Porous Media image; (b) segmented image by [8] and [10]; (c) segmented image using the threshold obtained by the proposed method; and (d) gray line: image histogram; solid lines: evolution of the IF's (proposed method); dotted line: criterion function [8]; dash-dotted line: criterion function [10]. Threshold locations are indicated with arrows.

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