On tests of context

Benj Hellie

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The semantical device of the ‘test of context’ may be of considerable philosophical significance.

In a nutshell, the idea is this. Perhaps ‘I believe that $P$’ is a test of context. What that would mean is that the possible-worlds content of ‘I believe that $P$’, against a context $c$, is never a locality in modal space. Instead, the content represents the result of a ‘test’ of whether the subject of $c$ really does believe that $P$: if they do, this success is represented with the content being the trivially true ‘necessary proposition’; if they don’t, this failure is represented with the trivially false ‘impossible proposition’. This contrasts sharply with sentences with a more familiar descriptive meaning, which (perhaps relative to a given context) encode a certain region of modal space; in default cases, asserting such a sentence can therefore transmit the information that actuality is in the encoded region.

These ‘test’ semantic properties would give ‘I believe that $P$’ a striking pragmatic profile. Everyone believes the necessary proposition, so whenever a sentence has as its semantic value the necessary proposition against $c$, anyone in $c$ endorses the sentence, at least implicitly. Accordingly, one believes that $P$ just if one (at least implicitly) endorses ‘I believe that $P$’. Consequently, if one externalizes this implicit endorsement by asserting ‘I believe that $P$’, one thereby displays or expresses the presence of this belief.

Despite being meaningful in this way, relative to $c$, ‘I believe that $P$’ has no independent power to distinguish possibilities: on its own, it has no descriptive meaning. Once one has committed to $P$, one neither need nor can commit to or settle anything else to commit to or settle one’s believing that $P$. Accordingly, it gets the ‘logic’ of belief wrong to worry, metaphysically, whether beliefs clutter up the world (Lewis 1994); or to worry, epistemologically, through what mechanism one might acquire justification about one’s beliefs (Byrne 2005).

The test of context, therefore, advances the traditional promise of expressivist views, of dissolving philosophical perplexity by introducing nuance to the theory of meaning (Gibbard 1990, Blackburn 1993). But it does so without the revisionist semantical baggage of traditional expressivist approaches, which founder under the demand to explain embedding under Boolean operators or serving as a premiss or conclusion to an entailment relationship (Geach 1960). For the test of context buoys up unproblematically under this burden: though uninformative, the necessary and impossible propositions remain propositions, and can therefore em-

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bed or entail or be entailed as well as anything else. Ta-da—the solution to the ‘Frege-Geach problem’ for expressivism.\(^1\)

A further alluring feature of tests is their self-restraint. Just as a hammer makes everything look like a nail, a novel device encourages its indiscriminate application to everything that has been vexing us. But tests exhibit distinctive non-classical logical behavior: certain symptomatic patterns of interaction with Boolean connectives;\(^2\) entailment relationships among sentences that, classically, shouldn’t be able to enter such relationships.\(^3\) Such behaviors give empirical substance to the claim that a certain construction is a test of context. So tests of context offer not just a way of implementing expressivism, but an actual methodology for discriminating discourse that is expressive from discourse with a more familiarly ‘descriptive’ meaning.

Emerging from the programs of dynamic semantics (Heim 1983) and nonmonotonic logic (Fuhrmann 1989), the test of context was initially canvased as a device for representing epistemic modals and default reasoning (Veltman 1996). However, more recently, a range of philosophically interesting constructions have been hypothesized to be tests of context. The potential scope of application is of sufficient breadth as to reshape much of the philosophical landscape.\(^4\)

But this promise may be a mirage. Schroeder (2011), for example, thinks friends of tests have mistaken a mere ‘formal trick’ for a solution to a ‘philosophical problem’. More substantively, Cian Dorr and Geoff Lee have each objected as follows:\(^5\) if the content of ‘I believe that \(P\)’ is a noncontingent proposition, that seems to mean that it is not contingent whether I believe that \(P\); and yet it clearly is contingent whether I believe that \(P\).\(^6\) And although the direct target of the ‘Ramsey–Moore–God paradox’ (Chalmers and Hájek 2007) is a set of ‘Ramseyan’ and ‘Moorean’ rules of inference allegedly governing conditionals and belief-avowals, test-semantic approaches to these constructions do in fact validate these rules, and with them the problematic apriority of ‘\(P\) just if I believe that \(P\)’.

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\(^1\)The connection between tests and expressivism is, to my knowledge, first drawn in remarks by Yalcin (2007, 1021). The test–expressivism link advances to a more prominent position in Yalcin 2011. ‘Test’-based implementations of expressivism about various domains are also developed by Hellie (2011c,a,b, 2014, 2015) and Charlow (2011, 2014).

\(^2\)Namely, failures of the ‘suppositional’ rules of reductio and dilemma: Veltman 1996 already recognizes the strange behavior of negation with his dynamic epistemic possibility operator; Yalcin (2007, 988) exploits the incoherence of \(P \land \Diamond \neg P\) together with the nonentailment of \(\neg P\) by \(\Diamond P\) in defense of his approach; Yalcin (2012) notes the entailment of ‘I know that \(P\)’ by \(P\) but the failure of the contraposited entailment. An initial stab at the systematic mapping of the Boolean–test relationship is undertaken in Hellie 2011b; see also Hellie 2015, 2.1.

\(^3\)Cross-categorical’ entailment patterns have been hypothesized between: imperatives and ‘ought’-declaratives (Charlow 2011); interrogatives and declarative statements of determinacy (Hellie 2011b); imperatives, deontic modals, and expressions of intention (Hellie 2015); the latter apparatus could be easily extended to expressions of ‘wonderment’, which test for the endorsement of interrogative sentences.

\(^4\)Regions of discourse for which test-based semantic analyses have been (or should be!) advanced include: epistemic modals (Veltman 1996, Gillies 2004, Yalcin 2007, 2011; section 3.2.2, this article); expressions of presumption (Veltman 1996); ‘avowals’ of belief (namely, first-person statements like ‘I believe that \(P\)’) (Gillies 2001); ‘Ramsey-test’ conditionals (Gillies 2004, 2009, 2010, Yalcin 2007; section 2.4, this article); avowals of subjective credence (Yalcin 2007); ‘counterfactual’ conditionals (Veltman 2005, Gillies 2007; section 2.4.3, this article); avowals of consciousness like ‘here’s what it’s like: \(P\)’ (Hellie 2011c, 2014; section 3.2.2, this article); expressions of sensory qualification (‘the glass looks empty’) (Hellie 2011c); deontic modals (Charlow 2011, 2014, Helie 2015); practical conditionals (Charlow 2011, 2014); claims of metaphysical indeterminacy (Hellie 2011b; section 3.2.3, this article); knowledge ascriptions (Helie 2011a, Yalcin 2012); expressions of intention (Hellie 2015); ascriptions of these various psychological states (Hellie 2011c; section 5, this article); ‘objective’ counterparts of all these, such as statements of objective chance, metaphysical modality, subjunctive conditionals (for instance, by using the ‘vertical modality’ apparatus of this article together with restriction to a class of ‘elite’ contexts).

\(^5\)Dorr in Q&A at the 2011 ISCSS; Lee in comments on my ‘Out of this world’ at the 2013 Pacific APA.

\(^6\)Directly following this worry is the concern that a test is a posteriori, eliding the vast distinction between self-knowledge and empirical knowledge.
It is evidently a worthwhile endeavor, then, to take the measure of Schroeder’s arched eyebrow, Dorr’s and Lee’s objection, and Chalmers and Hájek’s paradox. On behalf of this endeavor, this article surveys some of the terrain around tests of context—specifically, the region inhabited by the main players in Naming and Necessity (Kripke 1972/1980), as abetted by the more formal perspectives of Kaplan (1977/1989) and Stalnaker (particularly his 1975): metaphysical contingency and noncontingency; apriority and aposteriority; rigidification and the affiliated cross-cutting of the former categorizations; the nature and role of context; the prospects for a ‘Golden Triangle’ (Chalmers 2005) of analytic linkages among meaning, modality, and rationality.

The large scope of this project compels us to sharply restrict our focus: empirical matters will receive extremely short shrift; and the only application we target directly for study is the informational test imagined in the opening paragraphs, with all but the dialectically indispensable among those discussed in footnote 4 abandoned by the wayside.

The nucleus of our interpretation of the informational test treats it as a rigidifying operator. Its famous ancestor is therefore the operator A (‘actually’) (Kaplan 1977/1989), envisaged to map an operand ϕ to a context-dependent sentence which is necessary if ϕ is true at the context, impossible if ϕ is false at the context. Section 1 rehearses the interaction of A with modals and Boolean connectives, in a classical framework assigning sentences truth-values relative to contexts and indices.

A sentence is true at a context just if the content of the sentence there includes the world of the context; accordingly, A fits well with a ‘Kaplanean’ conception of context as a location in modal space. But on the contrasting ‘Stalnakean’ conception (Stalnaker 1975), context is a mental state. Where A rigidifies on Kaplanean contexts, we may equally well imagine a rigidifier on Stalnakean contexts. Such an operator would map an operand ϕ to a context-dependent sentence which is necessary if ϕ is endorsed in the context, impossible if ϕ is not endorsed in the context. That would be the informational test.7

Section 2 seeks to put the informational test qua ‘Stalnakean rigidifier’ on a sound formal footing by assembling around it a congenial framework within which to locate the related phenomena of modals, conditionals, and entailment. I call this framework mindset semantics: very roughly, mindset semantics ‘hacks’ the classical Kaplanean theory of these phenomena by replacing every appeal to truth at a possible episode of language use (qua Kaplanean context) with appeal to support (‘implicit endorsement’) by a mental state (qua Stalnakean context). The classical rigidifier A targets the actual world; on mindset semantics, the informational test targets a proposition representing information in hand. Modals and conditionals are classically understood as quantifiers over worlds; on mindset semantics, they quantify instead over states of mind. Structures akin to the classical ‘double-indexing’ are available in mindset semantics. Where these classically generate the familiar ‘necessary aposteriority’ and ‘contingent apriority’ effects associated with A, the analogous effects in mindset semantics concern relations among a complex of perspectives contexts may represent: of utterance; of interpretation; of evaluation.

Section 3 takes a brief detour through the fundamental significance of the apparatus. The first subsection asks how strong an apparatus is necessary to generate the nonclassical interaction with Boolean connectives allegedly symptomatic of tests. Unlike the informational test, A does not do it. This asymmetry is explained

7Tests of context are frequently associated with a ‘dynamic’ approach to semantics, in light of the dynamism of their originator (Veltman 1996). As Yalcin (2007, fn. 22) recognized early on, nothing is essentially dynamic about the elementary familiar applications of tests; according to the result of Rothschild and Yalcin (MS), dynamism is optional for a range of applications sufficiently broad as to include all those listed in footnote 4.
as an effect of a fundamental contrast between set-membership and set-inclusion, in essential cooperation with a class of sentence not much larger than the rigidifier. The second subsection reflects on the first, asking why this nonclassical interaction took so long to come to light, whether it has an empirical basis, and what it signifies for expressive forms of meaning.

Chapter 4 returns to the main line of the dialectic, addressing the question of what in the formal framework represents the meaning of a sentence. In broad relief, the meaning I ascribe a sentence includes both what it takes for me to endorse it and what I expect of others who endorse it. Relativization of semantic value to an interpretive perspective is assigned the job of harmonizing these demands. Having prised propositions out of their classical job as the objects of modal evaluation (and therefore as quasi-metaphysical representatives of ‘ways a world might be’), they are free to do purely epistemological/psychological work. Their new job is measuring positions one might (intelligibly) occupy in the course of inquiry; what we intend by assigning \( \varphi \) the semantic value \( p \) is that anyone in a position at least as advanced as that measured by \( p \) implicitly endorses \( \varphi \). Assigning the ‘necessary’ or ‘impossible’ proposition, therefore, represents obligatory or forbidden endorsement, respectively.

Section 5 draws on the formalism of section 2 and the interpretation of section 4 to interpret the formalism as applied to modals and conditionals scoping over rigidifiers. At center stage are the epistemic interpretation of propositional content and the perspective-shifting conception of modality. The ‘metaphysical necessity’ that worries Dorr and Lee is unmasked as a strong but indispensable sort of first-person authority; the godlike self-image worrying Chalmers and Hájek vanishes as soon as we get out of our own heads.

The reader may appreciate a handy list of the pillars of the framework articulated in the paper:

(A) Context as psychological
(B) Modality as perspectival
(C) Propositions as epistemic
(D) Meanings as endorsement-conditions

Pillars (A) and (B) are introduced in section 2, pillars (C) and (D) in section 4.

1 Rigidification classically

The nucleus of our interpretation is that the informational test is a kind of rigidifier. Our formal story substantiating this claim appears in section 2. But to make clear why the label is apposite, we get there in stages. In this first section, we begin by rehearsing in a familiar form the classical story about rigidification (more or less the story presented in Kaplan 1977/1989), in connection with the related phenomena of context, content, entailment, and modality. Section 2 then presents a sort of ‘way-station’ between the classical theory and ours: a theory with the content of the classical theory but expressed in a form more closely resembling ours. Our theory will then emerge by making one substantive change to the way-station theory.

1.1 Origins of rigidification

In the beginning was Frege, whose semantics was extensional, absolute, and hermetic: with only extensions (The True and The False) available as candidate semantic values for sentences; with only nonparametric or absolute semantic-valuation functions to assign them; and with the determination of semantic values a hermetic enterprise, unaffected by distinctive circumstances of language use.
An ancien regime emerged from Frege’s primordial approach, out of the observation that representing such pervasive linguistic phenomena as modality and tense requires a richer apparatus. The strategy of such ancients as Carnap and Reichenbach preserved extensionality and hermeticism but abandoned absoluteness. Their one-dimensionally parametric semantic-valuation functions assigned extensions to sentences relative to indices: worlds, for modals (Carnap 1947/1956); times, for tense (Reichenbach 1947); and perhaps others (Lewis 1970).

A later classical era eventually cast off hermeticism, acknowledging the incursion of extralinguistic ‘contextual’ circumstances into meaning. The founding moment of the classical semantics was Hans Kamp’s observation (Kamp 1968) that ‘a child was born who would be king’ and ‘a child was born who will be king’ contrast in meaning: on the issue whether the child has yet become king, the latter is negative where the former is neutral. Kamp modeled this contrast by two-dimensionalizing: by adding to the pre-existing index parameter a parameter for context.

In Kamp’s story, the futurity associated with ‘would’ is index-sensitive whereas the futurity associated with ‘will’ is context-sensitive. The past-tense operator adjusts the index in such a way that the futurity pertains to some time after the birth of the child— which, being in the past, leaves open whether the crowning has yet happened. But no operator can adjust the context, the extralinguistic circumstances in which the sentence is uttered: consequently, the futurity of the crowning is mandatorily with regard to the time of utterance.

As with tense, so, it was predicted (Segerberg 1973), with modality. In particular, the contrast in (1) was argued to justify a comparable incursion of extralinguistic circumstance into meaning:

1. (a) Everything must be as it is
   (b) Everything must be as it actually is

The former seems more easily read as an uninteresting triviality, the latter as a preposterously strong necessitarian metaphysics. We will call this the Kamp-Segerberg Contrast.

It is common practice to regiment (1) as in (2):

2. (a) □(P ≡ P)
   (b) □(P ≡ AP)

In this regimentation, A is the rigidifying operator actually, while □ and ≡ are of course the necessity operator and the material biconditional. While □ quantifies over the indices relative to which P is assigned a truth-value, A saturates that index with the unbindable world of the context. Accordingly, whenever P is contingent, there is at least one world in agreement, and at least one in disagreement, with the world of the context on whether P— contravening the universal demand of □.

1.2 The classical treatment of actuality

We now review the classical explanation of the Kamp-Segerberg Contrast, with the objective of introducing the rudiments of the classical understanding of rigidification, modality, and entailment, and the links among them.

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8Or so Dowty (2007, 51) claims.
9Kamp was confused. ‘Would’ and ‘will’ are both modals (Kratzer 1981) rather than somehow ‘tense operators’. Assigning the occurrences in these examples a meaning essentially linked to the temporal would require postulating a lexical ambiguity for explicitly modal occurrences like ‘dogs will be friendly if raised well’ and ‘if kangaroos had no tails, they would topple over’.
1.2.1 Horizontal modality and actuality

What is characteristically classical is the relativization of extensional semantic values to both index and context. The True is represented here as the number 1 and The False represented as −1; while both the indices I and the contexts C are drawn from a set W of possible worlds:

3. (a) Parameters
   i. I = W
   ii. C = W

(b) Structural semantics
   i. Sentences
      • For i ∈ I, c ∈ C, [x]i,c ∈ {1, −1}
   ii. Zero-dimensional operators
      • [O(φ, ψ)]i,c = [O]i,c ([φ]i,c, [ψ]i,c)
   iii. One-dimensional operators
      • [O(φ)]i,c = [O]i,c (λt′ : [φ]i,c′ (t′) = [φ]i,c′(t′))
   iv. Two-dimensional operators
      • [O(φ)]i,c = [O]i,c (λi′, c′ : [φ]i,c′

(c) Lexical semantics: zero-dimensional operators
   i. [∧]i,c = (λt, t′ : min{t, t′})
   ii. [∨]i,c = (λt, t′ : max{t, t′})
   iii. [¬]i,c = (λt : t · −1)
   iv. [≡]i,c = (λt, t′ : t · t′)

(d) Lexical semantics: one-dimensional operators

• Here we introduce □ and ◊ as symbols for the familiar one-dimensional ‘horizontal’ modals, to be distinguished shortly from the two-dimensional ‘diagonal’ modals

   i. [□]i,c = (λF : minF ′(i′))12
   ii. [◊]i,c = (λF : maxF ′(i′))

(e) Lexical semantics: two-dimensional operators

   i. [A]i,c = (λR : R(c, c))

The contrast between (3d) and (3e)i is what implements the Kamp-Segerberg Contrast. Because A binds both index and context, it is able in (2b) to saturate its index parameter with the context of utterance before that parameter can be bound by □. That has the following effect:

4. Structural/lexical lemmas

(a) [□φ]i,c
   = [□]i,c (λi′ : [φ]i′,c)
   = (λF : minF ′(i′))(λi′ : [φ]i′,c)
   = minF ′(i′)
   = minF ′ [φ]i′,c

(b) [φ ≡ ψ]i,c
   = [≡]i,c ([φ]i,c, [ψ]i,c)
   = (λt, t′ : t · t′)([φ]i,c, [ψ]i,c)
   = [φ]i,c · [ψ]i,c
other is the to be always false—in accord with the Kamp-Segerberg Contrast. Returning to (1), we see that ‘everything must be as it is’ is ruled to be always true, while ‘everything must be as it actually is’ is ruled to be always false—in accord with the Kamp-Segerberg Contrast.

But this is not the only notable feature of the apparatus. Another is the noncontingency of actuality:

5. Horizontal–diagonal lemma

- Let \( P \) be contingent and context-independent. Then for any \( c \), for some \( i' \), \( \llbracket P \rrbracket^{i',c} \neq \llbracket P \rrbracket^{c,c} \) so that \( \llbracket P \rrbracket^{i',c} \cdot \llbracket P \rrbracket^{c,c} = -1 \); and for some \( i'' \), \( \llbracket P \rrbracket^{i'',c} = \llbracket P \rrbracket^{c,c} \) so that \( \llbracket P \rrbracket^{i'',c} \cdot \llbracket P \rrbracket^{c,c} = 1 \).

6. Semantic values for (2)

(a) \( \llbracket \Box (P \equiv P) \rrbracket^{i,c} \)

\[
= \min_{\nu} \llbracket P \equiv P \rrbracket^{\nu,i} \]
\[
= \min_{\nu}(\llbracket P \rrbracket^{\nu,i} \cdot \llbracket P \rrbracket^{\nu,c}) \]
\[
= \min\{1\}
\]
\[
= 1 \quad (4a)
\]

(b) \( \llbracket \Box (P \equiv AP) \rrbracket^{i,c} \)

\[
= \min_{\nu}(\llbracket P \rrbracket^{\nu,i} \cdot \llbracket AP \rrbracket^{\nu,c}) \quad (4a, 4b)
\]
\[
= \min_{\nu}(\llbracket P \rrbracket^{\nu,i} \cdot \llbracket P \rrbracket^{\nu,c}) \quad (4c)
\]
\[
= \min\{1\} \quad (5)
\]
\[
= 1
\]

Under any circumstances, ‘either it must actually be that \( P \) or it must fail actually to be that \( P' \) is true.

1.2.2 Entailment

A semantic theory seeks, inter alia, to predict/explain patterns of entailment. Intuitively, if one endorses the sentences in the set \( \Delta \) and those sentences entail \( \varphi \) (\( \Delta \vdash \varphi \)), one in some sense or other ‘should’ endorse \( \varphi \). That accounts for our ordinary use of entailment as a ‘dialectical sword and shield’: those who endorse \( \Delta \) but neglect to endorse \( \varphi \) are ruled somehow in violation of some norm;
when we face protest at our endorsement of $\varphi$ but not at our endorsement of $\Delta$, we rest content that the protest is misplaced.

The classical ‘Fregean’ conception of entailment identifies it with ‘truth-preservation’: when $\Delta \vdash \varphi$, it is in some way impossible for the $\Delta$ to all be true unless $\varphi$ is. This goes some way toward explaining the intuitive ‘normativity’ of entailment, perhaps along the following lines. Suppose that our ‘goal’ in endorsing sentences is to build a stock of truths while avoiding building a stock of falsehoods, that $\Delta \vdash \varphi$, and that we endorse all the $\Delta$. If $\varphi$ is true, then in failing to endorse it, we fall short of our goal; while if $\varphi$ is false, then even if we endorse it, we do not thereby endorse any added increment of falsehood not already present in our endorsement of $\Delta$; so endorsing $\varphi$ is secure; so in failing to act under security, we haven’t done what we ‘should’.

The ‘impossibility’ that $\varphi$ is false unless one of the $\Delta$ is suggests a generality of some sort; with both $i$ and $c$ parameters available, several options for specifying this generality are available. The standard implementation is this ‘diagonal’ analysis:

9. Let $\text{acceptance}$ be a theory-neutral term for whatever relation between a context and a sentence is thought to be ‘preserved’ on any given theory of entailment

(a) $\Psi(c, \sigma) := c \text{ accepts } \sigma$
(b) The $\text{acceptance-condition}$ of $\sigma := \{c : \Psi(c, \sigma)\}$, namely the set of contexts accepting $\sigma$
(c) $\Delta \vdash \varphi$ just if the acceptance-condition of $\varphi$ includes the intersection of the acceptance-conditions of the members of $\Delta$

The classical theory of entailment then emerges by identifying acceptance with what we will call ‘(diagonal) verification’:

10. $c$ accepts a declarative sentence $\varphi$ just if $c$ (diagonally) verifies $\varphi$ ($c \vdash \varphi$), where

(a) $c \vdash \varphi$ just if $\llbracket \varphi \rrbracket^{c,c} = 1$
(b) $\llbracket \varphi \rrbracket = \{c : c \vdash \varphi\}$
(c) $\Delta \vdash \varphi$ just if $\bigcap_{\delta \in \Delta} \llbracket \delta \rrbracket \subseteq \llbracket \varphi \rrbracket$

11. Theoremhood

$\bullet \vdash \varphi$ just if $\llbracket \varphi \rrbracket = W$

According to (10), entailment is preservation of (diagonal) verification: whenever the premisses are all true at a given context and its index, so is the conclusion. Similarly, according to (11), a theorem has an inescapable (diagonal) verification-condition.

This definition, arguably, provides what we want out of the notion of ‘impossibility’ at the heart of truth-preservation. Perhaps it is in some sense baldly possible that the $\Delta$ are all true but $\varphi$ is
false. But if that is a mere abstract possibility, one we can determine up front we are not in, who cares? What truth-preservation is for is security: the assurance that we, now, in endorsing \( \varphi \) will not thereby make any mistake we have not already made. This localized and location-sensitive security is exactly what the diagonal analysis affords.

Theoremhood is a plausible sufficient condition for ‘apriority’: for if we possess a guarantee of meaning that, no matter what, our circumstances are not those in which \( \varphi \) is false, \( \varphi \) is plausibly known a priori to be true (at least implicitly); and if so, its content is plausibly known a priori by all who understand it. Conversely, perhaps this is the only route to a priori knowledge, and any knowledge which isn’t a priori is a posteriori; if so, any sentence which is neither a theorem nor the negation of a theorem (an antitheorem) is ‘a posteriori’.

The verification-preservation analysis validates the following transparency schemata for \( A \):

12. \( \vdash \varphi \equiv A \varphi \)
   - By (4b), for any \( c \), \( \llbracket \varphi \equiv A \varphi \rrbracket^{c.e} = \llbracket A \varphi \rrbracket^{c.e} \cdot \llbracket \varphi \rrbracket^{c.e} \); as above, for any \( c \), \( \llbracket A \varphi \rrbracket^{c.e} = \llbracket \varphi \rrbracket^{c.e} \); so for any \( c \), \( \llbracket \varphi \equiv A \varphi \rrbracket^{c.e} \) is either \( 1 \cdot 1 = 1 \) or \( -1 \cdot -1 = 1 \); so \( \vdash \varphi \equiv A \varphi \).

13. (a) \( \varphi \vdash A \varphi \)
   (b) \( A \varphi \vdash \varphi \)
   - By (4c), for any \( i, c \), \( \llbracket A \varphi \rrbracket^{i.e} = \llbracket \varphi \rrbracket^{i.e} \); so in particular, for any \( c \), \( \llbracket A \varphi \rrbracket^{c.e} = \llbracket \varphi \rrbracket^{c.e} \); so for any \( c \), \( \llbracket A \varphi \rrbracket^{c.e} \leq \llbracket \varphi \rrbracket^{c.e} \) and \( \llbracket A \varphi \rrbracket^{c.e} \geq \llbracket \varphi \rrbracket^{c.e} \); so \( A \varphi \vdash \varphi \) and \( \varphi \vdash A \varphi \), as desired.

14. \( \vdash \varphi \) just if \( \vdash A \varphi \)

15. (a) \( \vdash \varphi \) just if \( \vdash \Box \varphi \)
   - Left to right: if \( \vdash \varphi \), at all \( c \), \( \llbracket \varphi \rrbracket^{c.e} = 1 \); in which case for any \( c \), \( \llbracket \Box \varphi \rrbracket^{c.e} = \min_w \llbracket \varphi \rrbracket^{w.e} = 1 \)—and so \( \vdash \Box \varphi \).

   Right to left: if \( \vdash \Box \varphi \), for any \( c \), \( \llbracket \Box \varphi \rrbracket^{c.e} = \min_w \llbracket \varphi \rrbracket^{w.e} = 1 \); in which case for any \( c \), \( \llbracket \varphi \rrbracket^{c.e} = 1 \)—and so \( \vdash \varphi \).

(Allowing \( i \) and \( c \) to vary independently of one another in the definition of entailment allows the semantic values of \( A \varphi \) and \( \varphi \) to diverge, which would predict nonentailment; transparency and diagonal verification are closely bound to one another.)

1.2.3 Diagonal modality and actuality

It was recognized early on (Stalnaker 1970, Segerberg 1973, Kaplan 1977/1989) that two features of this approach generate a second ‘dimension’ of modality: the identification (3a) \( I = C = W \) of the set of indices, the set of contexts, and the set of worlds; and the understanding of modals as ‘quantifiers over worlds’ as formalized in (3d). As a result, it is possible to define ‘diagonal’ modals as further two-dimensional operators alongside \( A \):

(3e) Lexical semantics: two-dimensional operators

   ii. \( \llbracket \Box \rrbracket^{i.e} = (\lambda R : \min_w R(w, w)) \)
   iii. \( \llbracket \Diamond \rrbracket^{i.e} = (\lambda R : \max_w R(w, w)) \)

The symbols \( \Box \) and \( \Diamond \) can be read ‘diagonally necessary’ and ‘diagonally possible’, to display the contrast with the ‘horizontal’, one-dimensional modals in (3d).

Diagonal necessity is of interest because it tracks theoremhood and therefore, arguably, apriority.

15. (a) \( \vdash \varphi \) just if \( \vdash \Box \varphi \)
\[ \neg \varphi \text{ just if } \vdash \neg \square \varphi \]

- Left to right: if \( \neg \varphi \), for some \( c \), \( \square \varphi \)^{c} \equiv \neg 1 \); in which case for any \( c \), \( \square \varphi \)^{c} \equiv \min \{ \neg \varphi \}^{w} \equiv -1 \); in which case for any \( c \), \( \neg \square \varphi \)^{c} \equiv 1 \)—and so \( \vdash \neg \square \varphi \)

- Right to left: if \( \vdash \neg \square \varphi \), for any \( c \), \( \neg \square \varphi \)^{c} \equiv 1 \); in which case, for any \( c \), \( \square \varphi \)^{c} \equiv -1 \); in which case for some \( c \), \( \neg \square \varphi \)^{c} \equiv -1 \); in which case \( \neg \varphi \)

Conversely, diagonal contingency tracks the conjunction of non-theoremhood and non-antitheoremhood and therefore, arguably, aposteriority.

Horizontal and diagonal necessity contrast in respect of context-sensitivity:

16. (a) It is not generally the case that \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'}

- \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'}; whenever \( \neg \varphi \), for some \( c \), \( c' \), \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'} just if \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'} just if \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'}

(b) \( \square \varphi \)^{i,c} \equiv \square \varphi \)^{i',c'}

- \( \square \varphi \)^{i,c} \equiv \min \{ \square \varphi \}^{i',c'} = \min \{ \square \varphi \}^{i',c'} \), a value invariant with respect to \( c \)

In application to context-independent expressions, horizontal and diagonal modality are equivalent:

17. When \( \square \varphi \)^{i,c} = \square \varphi \)^{i',c'} for all \( i, c, c' \), \( \square \varphi \)

(a) Left to right: Let \( \square \varphi \)^{i,c} = 1 \); then by (4a), \( \min \{ \square \varphi \}^{i',c'} = 1 \); then by context-independence, for all \( c' \), \( \min \{ \square \varphi \}^{i',c'} = 1 \); so for all \( i, c \), \( \square \varphi \)^{i,c} = 1 \); so for all \( c \), \( \square \varphi \)^{c} = 1 \); so \( \square \varphi \)^{c} \equiv 1 \); so \( \square \varphi \)^{c} \equiv 1 \); as desired.

(b) Right to left: Let \( \square \varphi \)^{i,c} = 1 \); then for all \( c' \), \( \square \varphi \)^{i',c'} = 1 \); then by context-independence, for all \( i' \), \( \square \varphi \)^{i',c'} = 1 \); so for all \( i' \), \( \square \varphi \)^{i',c'} = 1 \); so \( \square \varphi \)^{c} = 1 \); as desired.

However, \( A \) prises horizontal and diagonal modality apart:

18. Whether \( P \equiv AP \) is contingent a priori

(a) Contingency: by (6b),

\[ \vdash \neg (P \equiv AP) \land \neg (P \equiv AP) \]

(b) Apriority: by (12) and (15a),

\[ \vdash (P \equiv AP) \]

The Kamp-Segerberg Contrast does not apply for diagonal modality

19. Whether \( AP \) is noncontingent a posteriori

(a) Noncontingency: by (7c),

\[ \vdash (P \equiv AP) \land \neg (P \equiv AP) \]

(b) Aposteriority: by (14) and (15b),

\[ \vdash (P \equiv AP) \land \neg (P \equiv AP) \]

2 Rigidification in standard-set semantics

As we saw, the classical era rejected Fregean hermeneutism by parametrizing semantic-value assignments to contexts. Semantic-value assignments then became doubly parametrized, for the ancients had already rejected Fregean absolutism by parametrizing semantic-value assignments to indices. But Fregean extensionalism remained in place: it was still The True and The False that were assigned.
It is unclear why this should be, for extensionalism has little or no allure. Almost nothing can be done in an extensional semantics without adding ancient bells and/or classical whistles; better to think of what can be done as an abstraction from something more powerful than as a ‘best-case scenario’, departure from which requires special pleading.\footnote{I allude here to Partee’s well-known slam against Montague’s general adoption of intensional semantic values even for ostensibly extensional expressions.} The mathematics of information is the elegant and powerful system of Boolean abstract algebra, bypassing entirely the distracting complexities of doing referential semantics ‘in all possible worlds’ \citep{Lewis1969}. We have no grip on individual possible worlds: they are too complex.\footnote{I have this on good authority: ‘the thirty-six possible states of the dice are literally thirty-six ‘possible worlds’, as long as we (fictively) ignore everything about the world except the two dice and what they show’ \citep{Kripke1972/Kripke1980,16}. I find this comprehensible only if the ignoring is understood as introducing partitions of modal space, with the members of each cell differing only in respects that are ignored.} Tense and modality are very different animals; lumping them together as instances of ‘index-relativity’ is prima facie an odd choice. The truth-preservation theory of entailment is unattractive (section 2.1.2). On grounds of utility, mathematical elegance, conformity to the underlying psychology of meaning, cutting the varieties of meaning themselves at their joints, and accommodating the normativity of meaning, therefore, best then to abandon allegiance to extensionalism.

A natural next step after adding the context parameter would therefore be to assign propositions rather than truth-values as sentential semantic values (section 2.1.1). Even this initial manoeuvre reshapes the theory of modals and rigidifiers, with the categorial ambiguity between one-dimensional modals and two-dimensional modals and rigidifiers replaced with a single-category approach (section 2.2). A complete de-extensionalization clears out all parametrization to worlds, with contexts ‘lifted’ from worlds to propositions—the formal aspect of pillar (A) of the framework, the psychological conception of context. Entailment is then lifted to meet context (section 2.1.2). But this lifting brings in its wake a lifting of modals and rigidifiers (section 2.3), and a consequent lifting of conditionals (section 2.4). Part of the lifting in both cases includes the kicking out of the familiar horizontal modals/conditionals and their replacement with what we will call ‘vertical’ modals/conditionals (more on which very shortly).

The result is an across-the-board lifting of the network among entailment, rigidification, modality, and conditionality—the promised ‘hacking’ of the classical theory by altering the form and interpretation of the acceptance-relation. Tests of contextual information are then located in this lifted network as rigidifiers, in the position classically occupied by \(A \). Predictably, therefore, these tests generate effects analogous to both ‘noncontingent aposteriority’ (section 2.3.2) and ‘contingent apriority’ (section 2.4.4).

Our destination is pillar (B) of the framework, the perspectival conception of modals, according to which rigidifiers and diagonal and vertical modals are distinguished from one another by how they juggle contexts representing states of mind in three different relations to their operand: \textit{uttering} the sentence; \textit{interpreting} it to fix a propositional content; and \textit{evaluating} the sentence so interpreted for belief, disbelief, or uncertainty. A rigidifier and a diagonal modal coindex the ‘contexts of’ evaluation and interpretation; a rigidifier and a vertical modal leave the context of interpretation unbound, retaining compatibility with later saturation by the context of utterance; and a vertical or diagonal modal binds the context of evaluation to a universal quantifier. Having chosen one of these strategies for coordinating argument places, each then goes on to test for whether the context of evaluation (or: each context of evaluation) possesses the information encoded by the sentence against an appropriately related context of interpretation.

\footnotesize
\begin{itemize}
\item It is unclear why this should be, for extensionalism has little or no allure. Almost nothing can be done in an extensional semantics without adding ancient bells and/or classical whistles; better to think of what can be done as an abstraction from something more powerful than as a ‘best-case scenario’, departure from which requires special pleading.\footnote{I allude here to Partee’s well-known slam against Montague’s general adoption of intensional semantic values even for ostensibly extensional expressions.} The mathematics of information is the elegant and powerful system of Boolean abstract algebra, bypassing entirely the distracting complexities of doing referential semantics ‘in all possible worlds’ \citep{Lewis1969}. We have no grip on individual possible worlds: they are too complex.\footnote{I have this on good authority: ‘the thirty-six possible states of the dice are literally thirty-six ‘possible worlds’, as long as we (fictively) ignore everything about the world except the two dice and what they show’ \citep{Kripke1972/Kripke1980,16}. I find this comprehensible only if the ignoring is understood as introducing partitions of modal space, with the members of each cell differing only in respects that are ignored.} Tense and modality are very different animals; lumping them together as instances of ‘index-relativity’ is prima facie an odd choice. The truth-preservation theory of entailment is unattractive (section 2.1.2). On grounds of utility, mathematical elegance, conformity to the underlying psychology of meaning, cutting the varieties of meaning themselves at their joints, and accommodating the normativity of meaning, therefore, best then to abandon allegiance to extensionalism.

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2.1 Standard-set semantics

2.1.1 Schmindices eliminated

The classical understanding of the job of indices appears to be inconsistent:

20. ‘Grass is green’ and ‘snow is white’ are both true and, let us suppose, context-independent. Does either entail the other? No it does not. So, classically, the semantic-value assignment for English applied to these sentences relativizes differently across indices.

According to Lewis (1980, 32) (as good a spokesman for the classical understanding of the job of indices as any), ‘index-dependence is needed only for the treatment of shiftiness’. So if we decided to stop using modals, there would be nothing to shift the world-coordinate in indices, so that coordinate would be unneeded; continuing in our practice of ignoring all other coordinates, we can simplify: indices would be unneeded.

Now, if we decided to stop using modals, would ‘grass is green’ and ‘snow is white’ entail one another? No they would not. But then indices are needed and unneeded. Contradiction.

I propose to take Lewis at his word: in the simple-sentence and Boolean-connective fragment, there is no shiftiness, and therefore no need for indices. But to avoid predicting the equivalence of all its truths and of all its falsehoods, our theory for this fragment cannot fall back into Fregean absolutist extensionalism.

What do do? The most straightforward approach bifurcates the role of indices into indices proper, in charge of shiftability and with no role in our theory for this fragment, and schmindices, used to set up interesting entailment relations for this fragment. This fragment requires not triple-indexing, because of its lack of shiftability, but just indexing to contexts and schminidices. Schmindices are worlds, so that the characteristic set relative to $c$ of the semantic value of $\varphi$ across schmindices, $\chi(\varphi)^c = \{ i : [\varphi]^i.e = 1 \}$, is a set of worlds. Sets of worlds are propositions. So the (repaired) classical semantics for the simple-sentence and Boolean-connective fragment can be equivalently rephrased as a context-sensitive propositional semantics.

More formally,

21. Structural semantics

(a) Sentences

- $[\varphi]^c \subseteq W$

We use letters like $P$ and $Q$ throughout to represent context-independent sentences with semantic values neither vacuous nor trivial and letters like $p$ and $q$ to represent nonextremal propositions—those for which $\emptyset \not\subseteq p \not\subseteq W$

With truth-values out of the picture, the semantic values of Boolean connectives cannot be extensional operations on truth-values. Good: they should be Boolean operations on sets.

More formally,

21. Structural semantics

(b) Booleans

- $[O(\varphi_1, \ldots, \varphi_n)]^c = [O]^c([\varphi_1]^c, \ldots, [\varphi_n]^c)$

22. Lexical semantics: zero-dimensional operators

(a) Booleans

i. $[\land]^c = (\lambda s, s' : s \cap s')$

ii. $[\lor]^c = (\lambda s, s' : s \cup s')$

iii. $[\neg]^c = (\lambda s : W \setminus s) = (\lambda s : \overline{s})$
iv. \[\equiv^c = (\lambda s, s' : s \otimes s') = (\lambda s, s' : (s \cap s') \cup (s \cup s'))\]

Two remarks. First, those wishing to do so may read our discussion of propositions as covertly pertaining to schmindexrelative truth-values. Second, we haven’t yet eliminated indices.

2.1.2 Contexts lifted

Contexts, while retained, are lifted. On the classical approach, contexts are members of \(W\): individual worlds. Our approach replaces these with subsets of \(W\): sets of worlds; propositions.

More formally:

23. Contexts

(a) \(C = \mathcal{P}(W)\)

The set of contexts is the set of subsets of \(W\), aka the set of propositions.

Strictly speaking, the relation between contexts and propositions is not identity: instead, contexts are entities of sufficient richness to determine whatever is needed for the job at hand. For the job currently at hand, a context needs to determine a proposition: the standard of the context \(c\) (nomenclature to be explained shortly), notated \(s(c)\). Still, to reduce the complexity of the notation, we elide the distinction between a context \(c\) and its standard-set \(s(c)\) whenever possible.

With indices neglected and contexts lifted, the relation filling the role of acceptance in (9) is adjusted appropriately. Rephrased in \(\chi\)-notation, the account of acceptance as verification (10a) reads \(c \Vdash \varphi\) just if \(c \in \chi(\varphi)^c\): \(c\) accepts \(\varphi\) when \(c\) is a member of the propositional semantic value of \(\varphi\) against \(c\)—acceptance is set-membership. So when we replace \(\text{worlds}\) in the role of contexts with \(\text{sets of worlds}\), we correlative adjust the acceptance relation from membership to inclusion: \(c\) accepts \(\varphi\) just when \(c\) is a subset of the propositional semantic value of \(\varphi\) against \(c\).

We label the relationship of acceptance so understood as support, and notate the modularized support-preservation analysis of entailment as follows:

24. Entailment

(a) \(c \Vdash \varphi := c \subseteq \llbracket \varphi \rrbracket^c\)

- \(c \Vdash \varphi\) is read ‘\(c\) supports \(\varphi\)’
- Strictly speaking, \(c \Vdash \varphi := s(c) \subseteq \llbracket \varphi \rrbracket^c\)

(b) \(\llbracket \varphi \rrbracket := \{c : c \Vdash \varphi\}\)

- \(\llbracket \varphi \rrbracket\) is read ‘the support-condition of \(\varphi\)’

(c) \(\Delta \vdash \varphi := \bigcap_{\delta \in \Delta} \llbracket \delta \rrbracket \subseteq \llbracket \varphi \rrbracket\)

- With \(\psi \vdash \varphi := \{\psi\} \vdash \varphi\) just if \(\llbracket \psi \rrbracket \subseteq \llbracket \varphi \rrbracket\)

25. Theoremhood

- \(\vdash \varphi\) just if \(\llbracket \varphi \rrbracket = C\)

A semantic theory is a standard-set semantics just if it extends the clauses (21)—(25).

On our favored mentalistic interpretation, a context \(c\) represents a mental state (individual or collective), and \(s(c)\) represents the information possessed in that mental state. Clause (a) then characterizes a relation in which a mental state already possesses the information encoded by a sentence, when that sentence is interpreted in line with the circumstances of that very mental state. Presumably, when that relation between a mental state and a sentence is in place, the mental state already endorses the sentence, if only implicitly.

Let mentalistic standard-set semantics—for short, mindset semantics—be standard-set semantics, mentalistically interpreted.
According to mindset semantics, then, the $\Delta$ entail $\varphi$ when implicit endorsement of all the $\Delta$ is thereby implicit endorsement of $\varphi$. That provides a pleasingly direct route to the ‘normativity’ of entailment: when one endorses all the $\Delta$ and $\Delta \vdash \varphi$, one also does in fact endorse $\varphi$. This endorsement may be merely implicit. But if one then moreover explicitly remains neutral regarding $\varphi$, one’s implicit and explicit attitudes toward $\varphi$ are in conflict with one another: one’s sentential attitudes in totality fail to characterize a coherent picture of the world. Perhaps avoiding incoherence is enough of a ‘should’ that we can end the story right here. This mindset-semantic explanation of the normative force of entailment is much nicer, more direct, and cleaner than the Fregan explanation; to my mind, a notable advantage.

2.1.3 What’s the big deal?

The big deal, put roughly, is that a standard-set semantics will not generally project an entailment structure projected by any possible classical semantics. Entailment is characterized in terms of acceptance: on the classical approach, acceptance is $\models$, verification; on our approach, acceptance is $\vdash$, support. Either way, acceptance is a relation of the following form:

$$\text{26. } (\lambda e, \varphi : (\lambda e, n : \triangledown(e, V(\varphi, n)))(c, c));$$

- $V$ maps a sentence $\varphi$ and a context of interpretation $n$ into a proposition (for example: $\llbracket(\cdot)\rrbracket$ or $\chi(\cdot)$)
- $e$ is a context of evaluation
- $\triangledown$ is the fundamental semantic relation

Namely, a relation holding between a context of utterance and a sentence just when, identifying the contexts of interpretation and evaluation and setting each to that context of utterance, the context of evaluation stands in the fundamental semantic relation to the propositional content of the sentence against the context of interpretation.

The difference between the classical and standard-set approaches, then, is over whether the fundamental semantic relation is membership or inclusion: whether $\triangledown$ is $\in$ or $\subseteq$. Membership and inclusion, in turn, exhibit a fundamental contrast.

Put vividly if crudely, this contrast is that while $\in$ is ‘bivalent’, $\subseteq$ is ‘trivalent’: while for any point $w$ and any set $B$, either $w \in B$ or $w \notin B$, it is not the case that for any sets $A$ and $B$, either $A \subseteq B$ or $A \nsubseteq B$: rather, sometimes, $A$ overlaps $B$ (so that $A \nsubseteq B$) even though $A \notin B$.

The contrast is put more elegantly (and more accurately) in terms of duality. Familiarly, $\square$ and $\lozenge$ are dual in the sense that $\square = \neg \lozenge \neg$, the same goes for $\forall$ and $\exists$, and for $\land$ and $\lor$ (via the DeMorgan laws). The contrast is then that $\in$, but not $\subseteq$, is self-dual.\(^{17}\)

That is to say that, while $w \notin B$ just if $w \in B$, it is not the case that $A \nsubseteq B$ just if $A \subseteq B$.

An immediate consequence is that, in contrast with the identity between $\llbracket \neg \varphi \rrbracket$ and $\overline{\llbracket \varphi \rrbracket}$, while $[\neg \varphi] \subseteq [\varphi]$, the converse inclusion is not generally in place. We will argue later for thinking of a rigidifier $R$ as an ‘object-language projection’ of the acceptance-relation: in particular, this means that $\llbracket \varphi \rrbracket = \llbracket R \varphi \rrbracket$ and $[\varphi] = [R \varphi]$; and it means that $\overline{\llbracket \varphi \rrbracket} = \overline{\llbracket \neg R \varphi \rrbracket}$ and $[\varphi] = [\neg R \varphi]$. But then while $\llbracket \neg \varphi \rrbracket = \overline{\llbracket \neg \varphi \rrbracket}$, generally at most $[\neg \varphi] \subseteq [\neg R \varphi]$. But together with the identity $[\varphi] = [R \varphi]$, that invalidates the classical introduction rule for negation, reductio: for while the latter identity shows that $\varphi \vdash R \varphi$, the former lack of general inclusion shows it not generally to be the case that $\neg R \varphi \vdash \neg \varphi$. (By DeMorgan, the classical elimination rule for disjunction, dilemma, is also

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\(^{16}\)Thanks to Nate Charlow for this terminology.

\(^{17}\)Unsurprisingly, inasmuch as $S \subseteq T := (\forall x \in S) (x \in T)$: inclusion is quantificational, and quantification is self-dual-only over singleton domains.
invalidated.) Section 3 sharpens this argument and addresses its philosophical consequences.

A further consequence of non-self-duality is a revision of the traditional binary distinction between a priori and a posteriori. For if contexts are sets rather than points, we can distinguish two ways for the diagonal to be ‘nontrivial’: by never corresponding to a trivial proposition; by always doing so, but not always to the same one. Section 5.2.2 expands on these considerations.

2.2 Modals without indices

I advance the bold conjecture at this point that the classical theory of modals does not need to relativize to indices either: superficial appearances to the contrary, what is classically intended is relativization in fact only to schmindices.

I have this on good authority. According to Lewis, ‘the list of shiftable features of context may be quite short. I have suggested that the list should include time, place, world and standards of precision’ (27). Why world? ‘Contingency is a kind of indexicality’. Why think that? ‘As you see, I presuppose a metaphysics of modal realism’. So much the worse for including world on the short list? Not in Lewis’s view, anyway: ‘I reject the popular presumption that modal realism stands in need of justification’ (25).

I rest my case.18

2.2.1 Characteristic sets for classical modals

According to the classical approach, modals aggregate truth-values over schmindex-context pairs. A horizontal necessity sentence □\varphi is true at a schmindex-context pair \langle i, c \rangle just if its pre-jacent, \varphi, is true at all schmindex-context pairs \langle i', c \rangle. That is to say, its characteristic set \chi(\square \varphi)^c = W just if the characteristic set \chi(\varphi)^c = W, while if \chi(\varphi)^c is anything else, \chi(\square \varphi)^c = \emptyset.

A diagonal necessity sentence □\varphi, by contrast, is true at \langle i, c \rangle just if \varphi is true at all pairs \langle w, w \rangle. In characteristic-set terms, \chi(\square \varphi)^c = W just if always, \varphi \in \chi(\varphi)^c, and otherwise determines \emptyset. Note that whereas \chi(\square \varphi)^c is potentially inconstant with varying \varphi, \chi(\square \varphi)^c is de jure invariant.

This pattern of determining either W or \emptyset depending on whether some ‘global’ condition is met is characteristic of tests of context. Some notation substantially compresses the description of this pattern:

27. A metalanguage test operator

- Where occurrences of ‘σ’ are substitutable with a sentence of the metalanguage, ‘T[σ]’ is a nonrigid metalanguage name for a proposition, stipulated to behave as follows:
  (a) T[σ] = W just if σ
  (b) T[σ] = \emptyset just if it is not the case that σ

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18This argument is, of course, intended to be tongue in cheek. Whether modals involve shiftiness is an empirical question. We will see, however, that within the scope of this article, there is no need for index-dependence. That does not rule out the emergence later of embeddings of the constructions discussed here which do require it.

19Do not confuse T with object-language tests of context. We have stipulated T into existence as an artificial expression to help us express our theories more conveniently. There can be no question whether T is part of some language, for we have just expanded our language to include it. But there can also be no question whether T is part of ordinary language, for we have just stipulated it into existence. Moreover, T converts a sentence into a name for a proposition.

In all these respects, T is unlike the alleged tests of context that are the subject-matter of this paper. If there are any, that is to be settled empirically. That in turn is because they are allegedly part of ordinary language. And their role in ordinary language is alleged to be that of converting a sentence into a sentence: not itself a name for a proposition, but an entity with a proposition as its semantic value nonetheless.
For example, \( T[\text{snow is white}] = W \), while \( T[\text{snow is black}] = \emptyset \).\(^{19}\)

Using \( T \), we can restate our descriptions of the characteristic sets of \( \Box \varphi \) and \( \lozenge \varphi \) as follows:

28. (a) \( \chi(\Box \varphi)^c = T[(\forall i)(i \in \chi(\varphi)^c)] = T[W \subseteq \chi(\varphi)^c] \)

(b) \( \chi(\lozenge \varphi)^c = T[(\forall c')(c' \in \chi(\varphi)^c)] \)

Horizontal necessitation against \( c \) tests for whether every index is a member of the characteristic-set of the prejacent at \( c \). Diagonal necessitation (against \( c \)) tests for whether every context is a member of the characteristic-set of the prejacent at \( c \).

2.2.2 Modals, acceptance, and contexts of evaluation and interpretation

Put this way, we can recognize at least a notional distinction between our horizontal necessity operator \( \Box \) and a ‘vertical’ necessity operator \( \lozenge \):

28. (c) \( \chi(\lozenge \varphi)^c = T[(\forall c')(c' \in \chi(\varphi)^c)] \)

\( \vdash \Box \varphi \vdash^c = \min\{c' : \vardashv \varphi \vdash^c \} \)

Vertical necessitation against \( c \) checks for whether every context is a member of the characteristic of the prejacent at \( c \). Unlike horizontal necessitation, vertical necessitation is about contexts rather than schmindices. Unlike diagonal necessitation, vertical necessitation does not bind the context variable in the characteristic function of the prejacent, but instead leaves it fixed to the context set by the necessitated sentence.

Unlike horizontal modality, vertical modality is affiliated with diagonal modality, and therefore in turn with acceptance. The classical characterization of the latter as \textit{verification} is expressed in \( \chi \) notation as follows:

\[(10a-\chi) \ c \vdash \varphi \text{ just if } c \in \chi(\varphi)^c \]

This sets the stage for mapping out the relations among vertical and diagonal modality and acceptance.

In (26), we introduced the distinction between contexts of evaluation and interpretation: the former is the first argument of the fundamental semantic relation \( \mathfrak{R} \), the latter is the context against which the second argument of \( \mathfrak{R} \) is determined. Each of vertical and diagonal modality and acceptance pertains to a different relationship among the context of evaluation, the context of interpretation, and the context of \textit{utterance}:

29-A. (a) Acceptance and diagonal modality (but not vertical modality) coindex the contexts of evaluation and interpretation

(b) Diagonal and vertical modality (but not acceptance) bind the context of evaluation

(c) Acceptance and (unbound) vertical modality (but not diagonal modality) index the context of interpretation to the context of utterance

This system of overlapping similarity and contrast among our three phenomena underlies familiar contrasts between diagonal necessity and vertical necessity (familiar for the latter under the guise of horizontal necessity):

30. By comparison with (16), in light of (29-A(c)), vertical and diagonal necessity contrast in respect of context-invariance of acceptance, where

(a) In light of (29-A(b)), sometimes, though there is some \( c \) for which \( c \vdash \Box \varphi \), yet \( \not\vdash \lozenge \varphi \)

\( \bullet \) Suppose \( \chi(\varphi)^c = W \) but \( \chi(\varphi)^c \subseteq W \); then \( c \vdash \Box \varphi \) while \( c^* \vdash \lozenge \varphi \), so that both \( \emptyset \subseteq \lozenge \varphi \subseteq C \) and \( \emptyset \subseteq \lozenge \varphi \subseteq C \)
(b) In light of (29-A(a)), there is some \( c \) for which \( c \not\Vdash \Box \varphi \) just if \( \vdash \Box \varphi \)

- Suppose that \( c \not\Vdash \Box \varphi \); that is so just if \( c \in \chi(\Box \varphi)^c \)
  just if \( c \in T[(\forall c')(c' \in \chi(\varphi)^c)] \) just if, for arbitrary
  \( c', c' \in T[(\forall c')(c' \in \chi(\varphi)^c)] \) just if, for arbitrary
  \( c', c' \not\Vdash \Box \varphi \) just if \( \vdash \Box \varphi \)

31. By comparison with (15), in light of its similarity to acceptance (29-A(a)), over which theoremhood, like diagonal necessity, universally quantifies, diagonal necessity is associated with theoremhood—\( \vdash \Box \varphi \)

- Suppose \( \vdash \Box \varphi \); that is so just if \( \|\Box \varphi\| = C \)
  just if, for every \( c, c \in \chi(\Box \varphi)^c \)
  just if, for every \( c, c \in T[(\forall c')(c' \in \chi(\varphi)^c)] \)
  = \( W \)
  just if \( (\forall c')(c' \in \chi(\varphi)^c) \) just if \( \|\varphi\| = C \)
  just if \( \vdash \varphi \)

In light of its contrast with acceptance, vertical necessity fails to be associated in this way with theoremhood.

2.2.3 Characteristic sets for actually

Rigidification clicks neatly into this schematism:

28. (d) \( \chi(A \varphi)^c = T[c \in \chi(\varphi)^c] \)
- \( \|A \varphi\|^c = \|\varphi\|^c \)

The affinity between rigidification and acceptance is especially evident: \( A \varphi \) tests for whether the context of utterance (considered as both interpreter and evaluator) accepts \( \varphi \).

We may therefore rewrite the system in (29-A) as follows:

29. (a) Rigidification and diagonal modality (but not vertical modality) coindest the contexts of evaluation and interpretation

(b) Diagonal and vertical modality (but not rigidification) bind the context of evaluation

(c) Vertical modality and rigidification (but not diagonal modality) leave the context of interpretation unbound and available for eventual saturation by the context of utterance

These patterns explain a number of familiar relationships among the three sorts of operator:

32. By comparison with (19), in light of (29c), vertical and diagonal necessity contrast in whether \( A \varphi \) ‘necessitates’:

(a) In light of (29b), \( A \varphi \vdash \Box A \varphi \) and \( \neg A \varphi \vdash \Box \neg A \varphi \) (and of course conversely)

i. \( c \vdash A \varphi \) just if \( c \in \chi(A \varphi)^c \)
   just if \( c \in T[c \in \chi(\varphi)^c] \)
   just if, for all \( c', c' \in T[c \in \chi(\varphi)^c] \)
   = \( W \)
   just if \( c \in T[(\forall c')(c' \in T[c \in \chi(\varphi)^c])] \) just if \( c \in \chi(\Box A \varphi)^c \)
   just if \( c \vdash \Box A \varphi \)

ii. \( c \vdash \neg A \varphi \) just if \( c \in \chi(A \varphi)^c \)
   just if \( c \in T[c \in \chi(\varphi)^c] \)
   just if, for all \( c', c' \in T[c \in \chi(\varphi)^c] \)
   = \( W \)
   just if \( c \in T[(\forall c')(c' \in T[c \in \chi(\varphi)^c])] \) just if \( c \in \chi(\Box \neg A \varphi)^c \)
   just if \( c \vdash \Box \neg A \varphi \)

This result says that \( A \varphi \) is ‘metaphysically noncontingent’

(b) In light of (29a), \( A \varphi \not\vdash \Box A \varphi \)

- Let \( \emptyset \not\subseteq P \subseteq W \), for all \( c' \), \( \chi(P)^c = p \), \( c \in p \), and
  \( c' \not\in p \); then \( c \vdash A P \) but \( c' \not\vdash A P \), in which case \( A P \)
  is not a theorem; but then, by (31), \( c \not\Vdash \Box A P \)

This result says that \( A \varphi \) is not ‘a priori’
By comparison with (13), in light of (29c) and (29a), rigidification both freely ‘introduces’ (φ ⊑ Aφ) and ‘eliminates’ (Aφ ⊑ φ):

- c ⊑ φ just if c ∈ χ(φ)c just if T[c ∈ χ(φ)c] = W just if c ∈ T[c ∈ χ(φ)c] just if c ∈ χ(Aφ)c just if c ⊑ Aφ

This result says that A is a ‘transparent’ operator.

We will hold off on discussing the ‘contingent apriority’ of φ just if Aφ for the moment, for as we will see, the conditional cannot be treated on standard-set semantics as a mere material conditional.

Let us sum up. The notation from (26) provides a general schematism for acceptance, modals, and rigidification:

(a) Acceptance
- (λc, φ : (λe, n : 〚φ(e, V(φ, n)))c, c))

(b) i. Vertical necessity
- (λc, φ : T[(∀c′)(λe, n : 〚φ(e, V(φ, n)))c′, c)])

ii. Diagonal necessity
- (λc, φ : T[(∀c′)(λe, n : 〚φ(e, V(φ, n)))c′, c′)])

iii. Rigidification
- (λc, φ : T[(λe, n : 〚φ(e, V(φ, n)))c, c)])

In this subsection, we have seen that the classical treatment of these phenomena identifies (in the schematism in (26)) V with χ and 〚 with ∈. We now discuss the lifted standard-set semantics operators, generated by identifying V with 〚(·)〛c and 〚 with ⊆.

2.3 Modals (and rigidifiers) on standard-set semantics

Having eliminated indices, it is no longer possible to aggregate over them—perhaps a matter of small consequence, because aggregating over schindices remains a prospect, as follows:

22. Lexical semantics: zero-dimensional operators

(b) Horizontal modals

i. 〚□〛c = (λp : {w : (∀w′)(w′ ∈ p)})
   = (λp : T[W ⊆ p])

ii. 〚◇〛c = (λp : {w : (∀w′)(w′ ∈ p)})
   = (λp : T[W ⊆ p])

2.3.1 Operators lifted

The lifting of contexts, by contrast, puts the horizontal/vertical distinction to work.

For this lifting is affiliated with a compensatory lifting of the fundamental semantic relationship 〚 from ∈ (as involved in ✺) to ⊆ (as involved in ⊄). In light of the linkage between acceptance and modals (29-A), this requires in turn a compensatory lifting of the relation between the evaluating context and the propositional content of the prejacent at the interpreting context: the membership relationship on display in (28) is replaced with the subset relation. Vertical necessitation against c then checks for whether every context c′ is a subset of the content of the prejacent at c; diagonal necessitation against c checks for whether every context c′ is a subset of the content of the prejacent at itself, c′.

More formally:

21. Structural semantics

(c) One-dimensional operators
- 〚O(φ)〛c = 〚O〛c′(λc′ : 〚φ〛c′)

35. Lexical semantics: one-dimensional operators

(a) Vertical modals
i. \( [\Box]c := (\lambda S : T[(\forall c')(c' \subseteq S(c))] \)
ii. \( [\Diamond]c := (\lambda S : T[(\forall c')(c' \subseteq \overline{S(c)})] \)

(b) Diagonal modals
i. \( [\Box]c := (\lambda S : T[(\forall c')(c' \subseteq S(c))] \)
ii. \( [\Diamond]c := (\lambda S : T[(\forall c')(c' \subseteq \overline{S(c)})] \)

(c) Rigidifiers
i. \( [\bigtriangledown]c := (\lambda S : T[c \subseteq S(c)]) \)
ii. \( [\Delta]c := (\lambda S : T[c \subseteq \overline{S(c)}]) \)

(The variable ‘\( S \)’ here is intended to suggest the familiar (Lewis 1975, Kratzer 1991) terminology of the (nuclear) scope of a modal operator; in the next subsection, we will go on the hunt for its usual companion, the restrictor.)

Or, simplifying by collecting together the structural and lexical specifications:

36. (a) i. \( [\Box] [\varphi]c = T[(\forall c')(c' \subseteq [\varphi]c')] \)
ii. \( [\Diamond] [\varphi]c = T[(\forall c')(c' \subseteq [\varphi]c')] \)
(b) i. \( [\Box] [\varphi]c = T[(\forall c')(c' \subseteq [\varphi]c')] \)
ii. \( [\Diamond] [\varphi]c = T[(\forall c')(c' \subseteq [\varphi]c')] \)
(c) i. \( [\bigtriangledown] [\varphi]c = T[c \subseteq [\varphi]c] \)
ii. \( [\Delta] [\varphi]c = T[c \subseteq [\varphi]c] \)

2.3.2 Similarity and difference

Despite the exchange of \( e \) for \( \subseteq \) in the role of \( \Box \), a great deal is left intact. For consider the central results of the previous section, reinterpreted or rephrased where necessary. These include:

37. (a) The context-sensitivity of \( \Box [\varphi] \) and insensitivity of \( \Box [\varphi] \); compare (16), (30)

- It is not generally the case that \( [\Box [\varphi]]c = [\Box [\varphi]]c' \)
- Always, \( [\Box [\varphi]]c = [\Box [\varphi]]c' \)

(b) The equivalence between the theoremhood of \( \varphi \) and that of \( \Box [\varphi] \); compare (15), (31)
- \( \vdash [\varphi] \) just if \( \vdash [\varphi] \)

(c) The ‘metaphysical noncontingency’ of \( [\Box] P \) and its ‘non-a-priority’; compare (19), (32)
- \( \vdash [\Box] P \lor [\Box] \neg P \)
- \( \vdash [\Box] P \land [\Box] \neg P \)

(d) The second and third forms of ‘transparency’ of \( [\Box] \); compare (13) and (14), and (33)
- \( \varphi \vdash [\Box] \varphi \)
- \( \vdash [\Box] \varphi \)

None of the proofs of these results makes an essential appeal to the identification of \( \Box \) with \( \subseteq \); each can be mechanically converted into a valid proof of an equivalent result for lifted modals and rigidifiers by using (34) as a translation manual.

One place where the substitution of a non-self-dual relation for \( \Box \) makes a notable difference, however, is in permitting distinct non-self-dual rigidifiers \( \bigtriangledown \) and \( \Delta \) (36c)—in contrast with the unique self-dual classical rigidifier A (with \( \neg A \varphi \vdash \vdash A \varphi \)). As can be easily seen, \( \bigtriangledown \) and \( \Delta \) are dual: \( \neg \bigtriangledown \varphi \vdash \Delta \varphi \). And yet they are distinct in meaning: while \( \varphi \vdash \Delta \varphi \), \( \Delta \varphi \neq \bigtriangledown \varphi \). This relies, of course, on the non-self-duality of \( \subseteq \); for \( [\Delta [\varphi]]c = \overline{T[c \subseteq [\varphi]c]} \) = \( T[c \subseteq [\varphi]c] \). And while \( c \subseteq [\varphi]c \) is generally at least as weak a condition than \( c \subseteq [\varphi]c \), thanks to non-self-duality, it is not generally also at least as strong a condition. Accordingly, sometimes \( [\Delta [\varphi]]c = T[c \subseteq [\varphi]c] \) = \( W \) even though \( [\bigtriangledown [\varphi]]c = T[c \subseteq [\varphi]c] \) = \( \emptyset \).

Another, as we will now go on to discuss, is the unavailability of the first form of ‘transparency’ of A, (12): it is not generally
the case that \( \vdash \varphi \equiv \neg \varphi \). As a consequence, ‘contingent apriority’ effects take a different form on the standard-set approach.

### 2.4 Conditionals

I wanted to avoid discussing conditionals in this paper: conditionals have an internal complexity well exceeding that of modals, which in turn generates data of exponentially greater complexity. It was therefore with mounting dismay that an unfortunate fact dawned on me: the simple \emph{strict biconditional} approach to the contingent apriority of ‘\( \varphi \) just if actually \( \varphi \)’ (18) would not transfer over to an equivalent treatment of ‘\( \varphi \) just if \( \downslice \varphi \)’. As we will see in section 3.1.3, this turns out to be a casualty of the non-self-duality of support. In the interest of efficiency, our discussion of conditionals will be as lean as is compatible with the pedagogical and dialectical aims of this article.

#### 2.4.1 Updating contexts of evaluation

Our approach amalgamates three sources: the schema for modality from the previous subsection; a quasi-Kratzeresque (Kratzer 1981) approach on which conditionals are something like explicitly restricted modals; and the ‘update’ approach to the indicative conditional advanced by several authors (Gillies 2004, Yalcin 2007).

Our schema for modality sees a modal as testing for a certain relationship between contexts of evaluation and the content of the prejacent or \emph{scope} of the modal against a context of interpretation: relations of binding or coindexation between these contexts and the context of utterance discriminate the diagonal and vertical modals and the rigidifier. If a modal is to be restricted, the only room for doing so under this schema is via the range of contexts of evaluation. We will endorse the idea that a conditional explicitly manipulates the range of contexts of evaluation, but not that it does so by introducing explicit restrictions on that range. Instead, we endorse the conception of a conditional \( \rho, \varphi \) as introducing an \emph{update} to certain information with the information in \( \rho \), and then assessing the updated information for support of \( \varphi \).

Collecting these ideas together, if \( \rho, \varphi \) is a modal with each context of evaluation updated with the information in \( \rho \), which then tests each updated context for support of \( \varphi \).

More formally,

**21. Semantic categories**

- **(d) Conditionals**
  
  \[
  \llbracket \text{if} \rho, \varphi \rrbracket^c = \left( \lambda c' : \llbracket \varphi \rrbracket^c \right) \left( \lambda c : \llbracket \rho \rrbracket^c \right)
  \]

  (Here we observe the variable ‘\( R \)’ being used to suggest the familiar terminology of the ‘restrictor’ of a conditional—something akin to the incredibly famous terminology of its ‘antecedent’.)

20Empirically, if \( P, \neg P \) is very awkward to validate otherwise. It should be valid, because it implements the ‘Ramsey Test’. The Ramsey Test, in turn, reflects the fact that we want those who are uncertain whether \( \rho \) to still care whether \( \text{if} \rho, \varphi \). If the conditional simply neglected all non-\( \rho \)-supporting contexts, testing the remainder for whether \( \varphi \), it would not have this effect. What conditionals are useful for is getting us to commit to what we will believe \emph{if} we learn what is now being just hypothesized. That requires the range of contexts of evaluation to remain unrestricted, while each is as it were represented by proxy: our formalism reflects this idea by running contexts of evaluation through a functor.
39. (a) \[\rho \rightarrow \varphi\] 
   \[= T[(\forall c')(c' \cap [\rho]^c \subseteq [\varphi]^c)]\]

(b) \[\rho \rightarrow \varphi\] 
   \[= T[(\forall c')(c' \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c')]\]

(c) \[\rho \rightarrow \varphi\] 
   \[= T[c \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c]\]

These definitions can easily be seen to be continuous with our schematicism for modals, because they preserve the relationships surveyed in (29):

29-C. All conditionals update the context of evaluation, with the restrictor interpreted against the context of interpretation

(a) Rigidifying and diagonal conditionals coindex the contexts of evaluation and interpretation, and therefore interpret the scope of the conditional against the updated context of evaluation; vertical conditionals, failing to do the former, fail also to do the latter

(b) Diagonal and vertical conditionals (but not rigidifying conditionals) bind the context of evaluation

(c) Vertical and rigidifying conditionals (but not diagonal conditionals) leave the (pre-update) context of interpretation unbound, leaving it available for saturation by to the context of utterance

2.4.2 Inferential role

Modus ponens All these conditionals broadly validate modus ponens:21 \[\square \rightarrow \varphi\] and \[\rightarrow \varphi\] for all sincere contexts (where \(c\) is sincere just if \(c\) is in the range of the quantifiers over contexts used in interpreting modals); \[\Diamond\] for all contexts:22

40. Modus ponens := \{if \(\rho, \varphi; \rho\) \[\rightarrow \varphi\]

(a) Let (i) \(c \models \rho \rightarrow \varphi\) and (ii) \(c \models \rho\).

   By (i), \(T[(\forall c') (c' \cap [\rho]^c \subseteq [\varphi]^c)] = W\), so \((\forall c') (c' \cap [\rho]^c \subseteq [\varphi]^c)\). Because \(c\) is sincere, \(c \cap [\rho]^c \subseteq [\varphi]^c\).

   By (ii), \(c \subseteq [\rho]^c\); so \(c \cap [\rho]^c = c\). But then \(c \subseteq [\varphi]^c\)—and so \(c \models \varphi\).

(b) Let (i) \(c \models \rho \rightarrow \varphi\) and (ii) \(c \models \rho\).

   By (i), \(T[(\forall c') (c' \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c')] = W\), so \((\forall c') (c' \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c')\). Because \(c\) is sincere, \(c \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c\).

   By (ii), \(c \subseteq [\rho]^c\); so \(c \cap [\rho]^c = c\). But then \(c \cap [\rho]^c = c \subseteq [\varphi]^c \cap [\varphi]^c = [\varphi]^c\)—and so \(c \models \varphi\).

(c) Let (i) \(c \models \rho \rightarrow \varphi\) and (ii) \(c \models \rho\).

   By (i), \(T[c \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c]\) = \(W\), so \(c \cap [\rho]^c \subseteq [\varphi]^c \cap [\varphi]^c\).

   By (ii), \(c \subseteq [\rho]^c\); so \(c \cap [\rho]^c = c\). But then \(c \cap [\rho]^c = c \subseteq [\varphi]^c \cap [\varphi]^c = [\varphi]^c\)—and so \(c \models \varphi\).

Conditional proof By contrast, \(\Diamond\) alone among these conditionals validates conditional proof; establishing this left as an exercise.

---

21Contra Kolodny and MacFarlane (2010); for more detailed discussion of that paper, see Hellie 2015.

22If contexts represent mental states, sincerity plausibly represents an internally supervenient property of mental states, and therefore if MP fails short of perfect validity, it does so in a way which cannot lead anyone astray.
2.4.3 Flavors of conditional

Affinity to entailment  The diagonal conditional affiliates with entailment—in effect, a generalization of the affinity between diagonal necessity and theoremhood:

41. If all standard-sensitive sentences are tests,
   
   \[\vdash \rho \rightarrow \varphi \text{ just if } \rho \vdash \varphi\]
   
   (lemma i) \[c \vdash \rho \rightarrow \varphi \text{ just if } c \subseteq [\rho \rightarrow \varphi]^c \text{ just if } T[(\forall c')(c' \cap [\rho]^c) \subseteq [\varphi]^c] = W \text{ just if } (\forall c')(c' \cap [\rho]^c) \subseteq [\varphi]^c\]
   
   (lemma ii) \[\rho \vdash \varphi \text{ just if whenever } c \vdash \rho, c \vdash \varphi \text{ just if whenever } c \subseteq [\rho]^c, c \subseteq [\varphi]^c\]

(a) Left to right. Assume \(\vdash \rho \rightarrow \varphi\). Then by lemma i, \((\forall c)(c \cap [\rho]^c) \subseteq [\varphi]^c)\). So assume \(c \subseteq [\rho]^c\). In that case, \(c \cap [\rho]^c = c\), so that \(c \subseteq [\varphi]^c\). But by lemma ii, that is so just if \(\rho \vdash \varphi\)—as desired.

(b) Right to left. Assume \(\rho \vdash \varphi\). Then by lemma ii, \((\forall c : c \subseteq [\rho]^c)(c \subseteq [\varphi]^c)\). For arbitrary \(c^*\), \((c^* \cap [\rho]^c) \subseteq [\rho]^c\), and therefore \((c^* \cap [\rho]^c) \subseteq [\varphi]^c\).

So if \(c^*\) is a counterexample to \((\forall c')(c' \cap [\rho]^c) \subseteq [\varphi]^c)\), \([\varphi]^c \not\subseteq [\varphi]^c\). Let \(r = c^* \cap [\rho]^c\).

If all standard-sensitive sentences are tests, this means that \([\varphi]^c = W\) and \([\varphi]^c = \emptyset\).

But then, because if \(r \subseteq [\rho]^c\), \(r \subseteq [\varphi]^c = \emptyset\), if \(r \subseteq [\rho]^c\), \(r = \emptyset\).

Accordingly, either (A) \(c^* \cap [\rho]^c = \emptyset\) or (B) \(c^* \cap [\rho]^c \not\subseteq [\rho]^c\).

Option (A) is not available to any counterexample, however, because that requires \(\emptyset = c^* \cap [\rho]^c \not\subseteq [\varphi]^c\)—an impossibility.

Option (B) requires \(\rho\) to be a test, with \([\rho]^c = W\) and \([\rho]^c = \emptyset\). Because \([\rho]^c = W\), \(r = c^* \cap [\rho]^c = c^* \cap W = c^*\). But then it is not possible that \([\varphi]^c = W\) and \([\varphi]^c = \emptyset\).

So there can be no counterexample.

42. For some \(c, c \vdash \rho \rightarrow \varphi\) only if \(\rho \vdash \varphi\)

   • In (a) above, note that to trigger lemma i, the full assumption of theoremhood is not required—only support at some context or other.

While the equivalence between the theoremhood of a conditional and the entailment by its antecedent of its consequent is in place for \(\rightarrow\) and \(\rightarrow\), either can be sometimes supported without itself being a theorem.

Ramsey Test  In its validation of conditional proof, the rigidifying conditional \(\rightarrow\) is a pure ‘Ramsey Test’ conditional. For standard-independent consequents, it is entailed by the material conditional: let \(c \subseteq [\neg P \vee Q]^c = [P]^c \cup [Q]^c\); then \(c \cap [P]^c \subseteq ([P]^c \cup [Q]^c) \cap [P]^c = [Q]^c = [Q]^c \cap [P]^c\); in which case \(c \subseteq [P \rightarrow Q]^c\).

Stalnaker-Lewis  Our borrowing of Lewis’s famous box-arrow sign (Lewis 1973) for the Stalnaker-Lewis conditional reflects some degree of affinity to our vertical conditional. Our stipulation for the latter, recall, is this: \([\rho \rightarrow \varphi]^c = T[(\forall c')(c' \cap [\rho]^c) \subseteq
texts, once updated with ρ as we understand it, then support ϕ as we understand it. By distinguishing the contexts of evaluation and interpretation and saturating the latter with the context of utterance, our conditional (very roughly) treats the context of evaluation ‘as counterfactual’. In that way, our conditional retains that centeredness on the context of utterance characteristic of the Stalnaker-Lewis conditional.

It does not, however, put the context of utterance to the job of limiting the ‘strictness’ of the conditional. In particular, it does not do so by restricting the quantifier to contexts of evaluation pegged in some way to the context of utterance; let alone (in a Ludovician spirit) to those contexts of evaluation which, updated with ρ interpreted in line with the context of utterance, adopt a form maximally similar to that of the context of utterance.

Conditionals are affiliated with modals, and are therefore amenable to pragmatic restriction of contexts of evaluation. Presumably this restriction is frequently enough pegged to the context of utterance. But it is exceedingly hard to see how or why we would ‘reverse-engineer’ this restriction to force a certain result following on the update with ρ. Otherwise we would be stacking the deck against anything interesting ever showing up under a hypothesis! It would make much more sense to require similarity of some desired sort prior to the update.

Unfortunately, that path is available only to the standard-set approach. For there is no making sense of updating a world: a world is what it is. Nor would it make sense to select out the ‘pre-update’ worlds most similar to some other world, if those worlds are to have ‘gaps’ into which the update pours: a world is complete in all detail.

### 2.4.4 Contingent apriority

We arrive at long last at the ‘contingent apriority’ of ‘P just if ▷ P’. We interpret the contingency of this biconditional to involve the non-theoremmhood of either the vertical conditional or its negation in either direction; we interpret its apriority to involve the theoremhood of the diagonal conditional in each direction:

43. Let \([P]^c\) = p for every c′, with \(\emptyset \subseteq p \subseteq W\)

(a) Contingency

i. A. \(\varphi \quad \Box \quad \Diamond \varphi\)
   - Let c \(\notin p\) and c \(\notin \overline{p}\); then \(\Box \varphi\) = \(\emptyset\).
     In that case, \([P \Box \Diamond \varphi]\) = \(T[(\forall c')(c' \cap [P]^c \subseteq \Diamond \varphi)]\); but because \(\emptyset \subseteq p\), it is not generally the case that \(c' \cap p \subseteq \emptyset\). Accordingly, \(T[(\forall c')(c' \cap p \subseteq \emptyset)] = \emptyset\), so that \(c \not\in P \Box \Diamond \varphi\).

B. \(\varphi \quad \neg(P \Box \Diamond \varphi)\)
   - Let c \(\subseteq p\); then \(\Box \varphi\) = \(W\).
     In that case, \([P \Box \Diamond \varphi]\) = \(T[(\forall c')(c' \cap [P]^c \subseteq \Diamond \varphi)]\) = \(T[(\forall c')(c' \cap p \subseteq W)]\) = \(W\), so \(\neg(P \Box \Diamond \varphi)\) = \(\emptyset\). But in that case \(c \not\in \neg(P \Box \Diamond \varphi)\), so that \(c \not\in \neg(P \Box \Diamond \varphi)\).

ii. A. \(\varphi \quad \Diamond \varphi \quad \Box \quad P\)
   - Let c \(\subseteq p\); then \(\Diamond \varphi\) = \(W\).
     In that case, \([\Diamond \varphi \Box P]\) = \(T[(\forall c')(c' \cap [\Diamond \varphi]^c \subseteq P)]\) = \(T[(\forall c')(c' \cap W \subseteq p)]\) = \(T[(\forall c')(c' \subseteq p)]\); because p \(\subseteq W\), this value is \(\emptyset\); but in that case c \(\not\in \Diamond \varphi \Box P\).
B. $\neg(\bigvee P \square \rightarrow P)$

- Let $c \not\subseteq p$; then $\llbracket \bigvee P \rrbracket^c = \emptyset$.

  In that case,
  \[
  \llbracket \neg(\bigvee P \square \rightarrow P) \rrbracket^c
  = \overline{\text{T}(\forall c')(c' \cap \llbracket \bigvee P \rrbracket^c \subseteq \llbracket P \rrbracket^c)}
  = \overline{\text{T}(\forall c')(c' \cap \emptyset \subseteq p)}
  = \overline{\text{T}(\forall c')(\emptyset \subseteq p)}
  = \overline{W} = \emptyset;
  
  But in that case $c \not\Rightarrow \neg(\bigvee P \square \rightarrow P)$.

(b) Apriority

i. $\vdash P \rightarrow \bigvee P$

- (A) $\llbracket P \rightarrow \bigvee P \rrbracket^c$
  \[
  = \text{T}(\forall c'(c' \cap \llbracket P \rrbracket^c \subseteq \llbracket \bigvee P \rrbracket^c)).
  
  (B) $\llbracket \bigvee P \rrbracket^c \cap \llbracket P \rrbracket^c = \text{T}(c' \cap \llbracket P \rrbracket^c \subseteq \llbracket \bigvee P \rrbracket^c)'
  = \text{T}(c' \cap p \subseteq p)
  = W.$

  By (B), $\llbracket P \rightarrow \bigvee P \rrbracket^c = \text{T}(\forall c'(c' \cap p \subseteq W)) = W$; in which case $c \not\vdash P \rightarrow \bigvee P$. But $c$ was chosen arbitrarily, so $\vdash P \rightarrow \bigvee P$.

ii. $\vdash \bigvee P \rightarrow P$

- $\llbracket \bigvee P \rightarrow P \rrbracket^c$
  \[
  = \text{T}(\forall c'(c' \cap \llbracket \bigvee P \rrbracket^c \subseteq \llbracket P \rrbracket^c \cap \llbracket \bigvee P \rrbracket^c)).
  
  Our search for counterexamples to the universal quantification splits into two cases.

  Case 1: $\llbracket \bigvee P \rrbracket^c = W$. Then $c' \cap \llbracket \bigvee P \rrbracket^c \subseteq \llbracket P \rrbracket^c \cap \llbracket \bigvee P \rrbracket^c$ just if $c' \cap W \subseteq \llbracket P \rrbracket^c \cap W$ just if $c' \subseteq \llbracket P \rrbracket^c$. But that is just what is required by the assumption of the case, so such a context cannot be a counterexample.

  Case 2: $\llbracket \bigvee P \rrbracket^c = \emptyset$. But then $c' \cap \llbracket \bigvee P \rrbracket^c \subseteq \llbracket P \rrbracket^c \cap \llbracket \bigvee P \rrbracket^c$ just if $c' \cap \emptyset \subseteq \llbracket P \rrbracket^c \cap \emptyset$; and that is so no matter what $\llbracket P \rrbracket^c$ may be.

  There are no counterexamples, so $\llbracket \bigvee P \rightarrow P \rrbracket^c = W$; as above, then, $\vdash \bigvee P \rightarrow P$.

3 Booleans and rigidification

The classical inference rules for the Boolean connectives include the suppositional rules of reductio and dilemma. The former is the classical introduction rule for negation: granting that $\psi$ entails $\varphi$, reductio extracts $\neg\psi$ from $\neg\varphi$. The latter is the classical elimination rule for disjunction: granting moreover that $\psi'$ entails $\varphi'$, dilemma extracts $\varphi \lor \varphi'$ from $\psi \lor \psi'$.24

These suppositional rules are straightforward projections of elementary properties of Boolean algebras: if $p$ precedes $q$, the complement of $q$ precedes the complement of $p$; if $p'$ moreover precedes $q'$, the supremum of $p$ and $p'$ precedes the supremum of $q$ and $q'$. As we will discuss, Boolean algebra structure is plausibly the structure of descriptive content. So unless the entailment relationships governing the Boolean connectives in a language are isomorphic to precedence relationships governing Boolean algebraic operations, the language does more than encode descriptive information.

The suppositional rules, familiarly (see the works cited in footnote 2), fail for standard-set languages containing rigidifiers: while $\varphi \vdash \bigvee \varphi$, $\neg \neg \varphi \not\vdash \neg \varphi$ (contra reductio); and moreover $\varphi \lor \psi \not\vdash \bigvee \varphi \lor \bigvee \psi$ (contra dilemma). This section first gets under the hood to pin down exactly why that is. After that we present several historical, empirical, and philosophical reflections on the failure of the suppositional rules.

24The rules are ‘suppositional’ in the sense that they appeal not just to individual sentences but to relationships between sentences; in Fitch-type systems, these relationships are established in subproofs, which in turn are supposed to represent reasoning under the guidance of a supposition.
3.1 Rigidifiers, the fundamental semantic relation, and suppositional rules

This subsection begins by asking what it would take for a language to invalidate reductio: the fundamental semantic relation $F$ has to be $\subseteq$; and the language has to contain standard-dependent sentences. Various plausible assumptions tightly constrain the range of possible counterexamples. We then argue that reductio and dilemma are dual to one another, so that either is valid just if the other is. We note finally that the material conditional, as a disjunction scoping over a negation, is poorly suited to expressing ‘contingent apriority’ effects.

3.1.1 Reductio

According to the classical negation-introduction rule reductio,

44. If $\psi \vdash \varphi$, then $\neg \varphi \vdash \neg \psi$

Why believe it?

Analyzed in accord with our modular characterization of entailment, that comes down to the following:

45. If, for all $c$,

(a) $\gfrak F(c, \llbracket \psi \rrbracket^c) \Rightarrow \gfrak F(c, \llbracket \varphi \rrbracket^c)$,

then, for all $c$,

(b) $\gfrak F(c, \llbracket \neg \varphi \rrbracket^c) \Rightarrow \gfrak F(c, \llbracket \neg \psi \rrbracket^c)$

The following is certainly true:

45. For all $c$, (a), just if, for all $c$,

(c) not-$\gfrak F(c, \llbracket \varphi \rrbracket^c) \Rightarrow$ not-$\gfrak F(c, \llbracket \psi \rrbracket^c)$

Assume $\gfrak F$ to be consistent, in the sense that it is never the case that $\gfrak F(c, p)$ and $\gfrak F(c, \overline{p})$. If so,

45. For all $c$,

(d) $\gfrak F(c, \llbracket \neg \varphi \rrbracket^c) \Rightarrow$ not-$\gfrak F(c, \llbracket \varphi \rrbracket^c)$

And (c) and (d) together yield

45. If, for all $c$, (a), then, for all $c$,

(e) $\gfrak F(c, \llbracket \neg \varphi \rrbracket^c) \Rightarrow$ not-$\gfrak F(c, \llbracket \psi \rrbracket^c)$

Now, to get from (e) to (b), we would need the following:

45. For all $c$,

(f) not-$\gfrak F(c, \llbracket \psi \rrbracket^c) \Rightarrow \gfrak F(c, \llbracket \neg \psi \rrbracket^c)$

Which would follow if $\gfrak F$ is decisive, in the sense that it is never the case that neither $\gfrak F(c, p)$ nor $\gfrak F(c, \overline{p})$.

Consistency and decisiveness follow from self-duality. Accordingly, if $\gfrak F$ is $\in$, (f) is an attribute of all $c$, and reductio is valid. But if $\gfrak F$ is instead $\subseteq$, $\gfrak F$ is not decisive. If (f) is an attribute of all $c$ and reductio is valid, it must be for some other reason.

For the purposes of assessing what a minimal counterexample to reductio would be, then, let us suppose that $\gfrak F$ is $\subseteq$. A minimal counterexample would then be a pair of sentences $\psi$ and $\varphi$ and a context $c^*$ for which:

46. (a) For all $c$, $c \subseteq \llbracket \psi \rrbracket^c \Rightarrow c \subseteq \llbracket \varphi \rrbracket^c$

(b) $c^* \subseteq \llbracket \varphi \rrbracket^c$

(c) $c^* \perp \llbracket \psi \rrbracket^c$
Where \( a \perp b := a \not\in b \) and \( a \not\in b \). (After all, without (a), the case does not fit the presupposition \((45a)\) of reductio; without (b), the case does not fit the premiss (the antecedent of \((45b)\)); and regarding (c), \( c^* \subseteq \lbrack \psi \rbrack^c \) is incompatible with (a), while \( c^* \subseteq \lbrack \psi \rbrack^c \) would mean the case conforms to reductio.)

In particular, this requires either \( \psi \) or \( \varphi \) to be standard-dependent. For suppose \( \psi \) is standard-independent. Then if \( c^* \perp \lbrack \psi \rbrack^c := q \), then for some \( c', c' \subseteq q = \lbrack \psi \rbrack^c \), in which case, by (a), \( c' \subseteq \lbrack \varphi \rbrack^c =: p \); so if \( \varphi \) is also standard-independent, then \( q \subseteq p \); but then (b) and (c) are incompatible. In addition to standard-dependence, several ‘non-triviality’ conditions are required: the range of contexts accessible to the speakers of the language must not be restricted to atomic contexts; the language must have the expressive capacity to generate a sentence \( \psi \) for which (c) is an option.

Note, however, that if all standard-dependent sentences are tests, the existence of a counterexample requires \( \psi \) to be standard-independent. For otherwise the condition \((46c)\) cannot be met: that is possible only when \( \lbrack \psi \rbrack^c \) is nonextremal (strictly between \( \emptyset \) and \( W \)).

One way to generate cases of the sort in \((46)\) is via an acceptance-tracking operator, one with the following properties:

\[ 47. \]

(a) If \( \not\mathcal{H}(c, \lbrack \psi \rbrack^c) \), \( c \subseteq \lbrack O(\psi) \rbrack^c \)

(b) If not-\( \mathcal{H}(c, \lbrack \psi \rbrack^c) \), \( \lbrack O(\psi) \rbrack^c \subseteq c \)

The ‘positive’ case \((47a)\) underwrites \((46a)\); and unless \( \mathcal{H} \) is self-dual, the ‘negative’ case \((47b)\) underwrites the consequence of \((46b)\) on \((46c)\).

One (not fully general) way to define acceptance-tracking operators is this:

\[ 48. \]

Where, for all \( c, c \subseteq f(c) \)

\[ \bullet \lbrack O(\psi) \rbrack^c = T[\mathcal{H}(c, \lbrack \psi \rbrack^c)] \otimes f(c) \]

After all, when \( \mathcal{H}(c, \lbrack \psi \rbrack^c) \), this value is just \( f(c) \supseteq c \); and when not-\( \mathcal{H}(c, \lbrack \psi \rbrack^c) \), this value is just \( f(c) \subseteq c \)—in accord with

And as should be evident, our rigidifier \( \triangledown \) is an instance of this form, for the constant function \( f(c) = W \).

Let us now peer through the opposite end of the telescope. A job description for a rigidifier is an ‘object-language projection of acceptance’:

\[ 49. \]

(a) \( \mathcal{H}(c, \lbrack R \varphi \rbrack^c) \Leftrightarrow \mathcal{H}(c, \lbrack \varphi \rbrack^c) \)

(b) \( \mathcal{H}(c, \lbrack R \varphi \rbrack^c) \Leftrightarrow \not\mathcal{H}(c, \lbrack \varphi \rbrack^c) \)

From these stipulations, it follows that for all \( c \), either \( \mathcal{H}(c, \lbrack R \varphi \rbrack^c) \) or \( \mathcal{H}(c, \lbrack \varphi \rbrack^c) \). In particular, either \( \mathcal{H}(c, \lbrack R P \rbrack^c) \) or \( \mathcal{H}(c, \lbrack R P \rbrack^c) \); and yet \( \not\mathcal{H}(c, \lbrack R P \rbrack^c) \Leftrightarrow \mathcal{H}(c, \lbrack P \rbrack^c) \). But unless \( \mathcal{H} \) is self-dual, it is false that for all \( c \), either \( \mathcal{H}(c, \lbrack P \rbrack^c) \) or \( \mathcal{H}(c, \lbrack P \rbrack^c) \); and, as a result, there are \( c \) for which \( \mathcal{H}(c, \lbrack R P \rbrack^c) \) while yet not-\( \mathcal{H}(c, \lbrack P \rbrack^c) \). But this together with stipulation (a) establishes that there are counterexamples to reductio. The equivalence \( \mathcal{H}(c, \lbrack R P \rbrack^c) \Leftrightarrow \mathcal{H}(c, \lbrack P \rbrack^c) \) moreover requires that if \( R P \) is standard-independent, its content is equivalent to that of \( P \). But in that case, there cannot be cases in which \( \mathcal{H}(c, \lbrack R P \rbrack^c) \) but not \( \mathcal{H}(c, \lbrack P \rbrack^c) \).

Either way, we observe that if reductio is valid for a nontrivial language, the language is either a standard-point language or lacks any acceptance-tracking operator, (in particular, any rigidifier); and, in light of the great simplicity of rigidification as a variety of standard-dependence, presumably is bereft of standard-dependence altogether.

In light of the widespread endorsement of rigidification, most philosophers will therefore see the issue of whether reductio is valid for natural language as pertaining to whether the fundamental semantic relation for natural languages is \( \in \) or \( \subseteq \). This is, as
we will discuss in following sections, tied up with the most profound questions about the nature of meaning. In deriding the test of context as a mere ‘technical trick’, Schroeder seems to have overlooked these connections.

3.1.2 Dilemma

Reductio (assisted by conjunction-elimination and -introduction) is dual to the classical disjunction-elimination rule, dilemma:

50. If $\psi \vdash \varphi$ and $\psi' \vdash \varphi'$, then $\psi \lor \psi' \vdash \varphi \lor \varphi'$

- Dilemma derivable as reductio of conjunction-introduction of reductio of conjunction-elimination
  - DeMorganizing the argument in dilemma yields $\neg(\neg\psi \land \neg\psi') \vdash \neg(\neg\varphi \land \neg\varphi')$
  - Applying conjunction-elimination twice, we extract from $\neg\varphi \land \neg\varphi'$ the premises $\neg\varphi$ and $\neg\varphi'$
  - Applying reductio twice, $\neg\varphi \vdash \neg\psi$ and $\neg\varphi' \vdash \neg\psi'$
  - Applying conjunction-introduction to the conclusions of the reductios, $\neg\varphi \land \neg\varphi' \vdash \neg\psi \land \neg\psi'$
  - And applying reductio, we recover the DeMorganization of dilemma

The conjunction rules are trivial, so reductio is valid just if dilemma is.

3.1.3 The material conditional

The material conditional is a disjunction scoping over a negation: $\psi \supset \varphi := \neg\psi \lor \varphi$. So obviously within standard-set semantics the material conditional will exhibit nonclassical behavior. In particular contrast with $\langle P \equiv AP \rangle = W$, while $[\supset \varphi P \equiv P] = C$, $[P \supset \varphi P] \subseteq C$:

51. (a) $[\neg \psi \lor \varphi] = [\neg \psi \lor \varphi] \cup [P] = [\psi] \lor [P] = T[c \subseteq p] \cup \varphi$; when $c \subseteq p$, this is equal to $\varnothing \cup \varphi$, so $c \vdash \psi \lor \varphi$; when $c \nsubseteq p$, it is equal to $W$, and so too $c \vdash \psi \lor \varphi$.

(b) $[\neg P \lor \psi] = [\neg P] \lor [\psi] = [P] \lor [\psi] = \neg P \lor \varphi \lor \varphi \lor \varphi \lor \varphi$. This is because the condition (46c) cannot be met: because $[\varphi] \supset \psi$ is extremal, there is no $c$ such that $c \nsubseteq \varphi$.

3.2 Reflections

After advancing some historical speculation about the allure of reductio and dilemma, I consider various candidates for natural language rigidifiers, and then draw a connection to expressivism.

3.2.1 Surprised?

We are all informed very early on in our philosophical education that reductio and dilemma are valid. Many of us later learn that various nonclassical weakenings of the Boolean connectives invalidate the suppositional rules; but it is a widely shared view that, without teasing the Boolean connectives away from the Boolean operations, we must learn to live with these rules. And yet, I will
argue, the collapse of these suppositional rules is implicit in the
collected foundational philosophical texts in natural language se-
mandics. If this collapse is nevertheless a surprise, that is because
it has been camouflaged by a disunity of approach.

What is a context? According to Stalnaker (1975), ‘The most
important element of a context, I suggest, is the common knowl-
edge, or presumed common knowledge and common assumption
of the participants in the discourse’ (273), where ‘in the possi-
ble worlds framework, we can represent this background informa-
tion by a set of possible worlds’ (274). Stalnaker’s common-
assumption contexts are sets, mentalistically interpreted.

According to Kaplan (1977/1989), by contrast, a context is a
‘possible occasion of use’ (494), where (at least to judge by those
aspects of contexts deemed worth mentioning) an occasion of use
determines an agent, a time, a position, and a world (543). Ka-
plan’s use-occasion contexts are points, locationally interpreted.

What is entailment? According to Stalnaker, the ‘basic notion’
(285) of entailment is a relation between propositions: p basically-
entails q just if p ⊆ q. Stalnaker recognizes also a ‘logical con-
cept of entailment’ (285): ψ logically-entails φ just if at all c, p.
Both of Stalnaker’s notions are relations of descriptive strength: p basically-entails q just if p is at least as strong descriptively as q; ψ logically-entails φ just if whatever descriptive power either may have, the power of the former is at least as great as that of the latter.

According to Kaplan, by contrast, ‘from the perspective of LD,
validity is truth in every possible context’ (549); correlative, ψ
LD-entails φ just if whenever ψ is true in c, φ is true in c; alter-
atively, just if whenever c ∈ [ψ], c ∈ [φ]—namely, just if
[ψ] ⊆ [φ]. Kaplan’s LD-entailment is acceptance-preservation.

Are the suppositional rules valid? If entailment is one of Stal-
aker’s relations of descriptive strength, then even if contexts
are Stalnaker’s sets, the suppositional rules remain valid. Basic-
entailment is straightforwardly in conformity with (analogues of)
reductio and dilemma: if p ⊆ p’ and q ⊆ q’, then p’ ⊆ p and
p ∪ q ⊆ p’ ∪ q’. Because logical-entailment just generalizes over
basic-entailment, logical-entailment is also in conformity with re-
ductio and dilemma.

If contexts are Kaplan’s points, then even if entailment is Ka-
plan’s relation of acceptance-preservation, then, as just argued, the
suppositional rules remain valid.

However, by combining Stalnaker’s sets with Kaplan’s
acceptance-preservation, standard-set semantics invalidates the
suppositional rules.25

I disagree, therefore, with Lewis’s (1980) famous assessment
that nothing of substance hangs on whether we go with Stalnaker’s
approach or with Kaplan’s. As ‘package deals’, their differences
cancel one another out. But recombining the parts of the packages
makes a dramatic difference.

3.2.2 Natural language rigidifiers

Does any operator in natural language have logical properties like
those of ∨?

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25Stalnaker comes very close to advancing just this combination: his notion of ‘reasonable inference’ (286), notated here ⇒, is this: ⟨ψ₁,…,ψₙ⟩ ⇒ φ := (∀c)(c ∈ [ψ₁] ∧ … ∧ [ψₙ] ∧ … ⊆ [φ]). Reasonable inference is ‘dynamic’: while ⟨ψ, ¬ψ⟩ ⇒ ⊥, ⟨¬ψ, ψ⟩ ⇒ ⊥—in that respect it differs from standard-set entail-
ment. However, I conjecture that a proof akin to that in (41) would establish the equivalence of single-premiss reasonable inference with single-premiss support-preservation. Stalnaker’s discussion of fatalism (his section V) exploits the dilemma-invalidating power of reasonable inference without (so far as I can tell) essentially incorporating its
dynamism.
‘Certainly’ One candidate is the sentential adverb ‘certainly’ (with ‘perhaps’ being a candidate for the logical properties of $\Delta$):\(^{26}\)

52. (a) ‘Certainly’ is ‘transparent’: anyone endorsing $\phi$ endorses ‘certainly, $\phi$’, and vice versa; compare (37d)
(b) ‘Certainly’ violates the suppositional rules
   i. Anyone endorseing ‘it is not the case that $\phi$’ endorses ‘it is not the case that, certainly, $\phi$’, but not vice versa; contra reductio
   ii. Anyone endorseing ‘either certainly $\phi$ or certainly $\psi$’ thereby endorses ‘$\phi$ or $\psi$’, but not vice versa; contra dilemma
(c) Diverging from this paradigm while hopefully preserving meaning, something like iteration of ‘certainly’ does not violate the suppositional rules
   i. Anyone endorseing ‘it is certainly not so that certainly, $\phi$’ endorses ‘it is not so that certainly, $\phi$’, and vice versa
   ii. Anyone endorseing ‘it is certain that either certainly $\phi$ or certainly $\psi$’ thereby endorses ‘$\phi$ or $\psi$’ and/or ‘it is certain that certainly, $\phi$ or it is certain that certainly, $\psi$', and vice versa

‘It is like this’ Less cautiously, another candidate may be ‘it is like this’, a sentential operator with the meaning assumed in the literature on avowals of consciousness:

53. (a) ‘It is like this’ is ‘transparent’: anyone endorseing ‘$\phi$’ endorses ‘it is like this: $\phi$’, and vice versa; compare (37d)
(b) ‘It is like this’ violates the suppositional rules
   i. Anyone endorseing ‘it is not the case that $\phi$’ endorses ‘it is not like this: $\phi$’, but not vice versa; contra reductio
   ii. Anyone endorseing ‘either it is like this: $\phi$ or it is like this: $\psi$’ thereby endorses ‘$\phi$ or $\psi$’, but not vice versa; contra dilemma
(c) Iteration of ‘it is like this’ does not violate the suppositional rules
   i. Anyone endorseing ‘it is not like this: it is like this: $\phi$’ endorses ‘it is not like this: $\phi$', and vice versa
   ii. Anyone endorseing ‘either it is like this: it is like this: $\phi$ or it is like this: it is like this: $\psi$’ thereby endorses ‘either it is like this: $\phi$ or it is like this $\psi$', and vice versa

Epistemic uses of ‘must’ As noted at the outset, the test of context was developed initially as a representation of ‘epistemic’ uses of modals: cases like (checking the watch) ‘they must be here by now’ or (assessing whether to bring an umbrella) ‘it might rain’. Epistemic ‘must’ certainly does behave in many ways like a rigidifier:

54. (a) Epistemic ‘must’ is ‘transparent’: anyone endorseing ‘$\phi$’ endorses ‘must $\phi$’, and vice versa; compare (37d)
(b) Epistemic ‘must’ violates the suppositional rules
   i. Anyone endorseing ‘it is not the case that $\phi$’ endorses ‘it is not the case that must $\phi$’, but not vice versa; contra reductio

\(^{26}\)I leave belief avowals off this list because the explicit subject-position leads to an ease of breaking the context of interpretation and the context of utterance that softens up the data.
ii. Anyone endorsing ‘either must \( \varphi \) or must \( \psi \)’ thereby endorses ‘\( \varphi \) or \( \psi \)’, but not vice versa; contra dilemma

(c) Modal auxiliaries do not iterate; still, ‘certainly’ applies to modalized sentences with rigidifier-like results

i. Anyone endorsing ‘certainly, must not \( \varphi \)’ endorses ‘must not \( \varphi \)’, and vice versa

ii. Anyone endorsing ‘either certainly, must \( \varphi \); or certainly, must \( \psi \)’ endorses ‘either must \( \varphi \) or must \( \psi \)’, and vice versa

And yet as a modal, epistemic ‘must’ is not a rigidifier.

Still, this rigidifier-like behavior plausibly results from reflexive restriction on the range of interesting contexts: restriction just to the context of utterance. For with the universal quantifier restricted to \( \{c\} \), \[\llbracket \Box \varphi \rrbracket^c = T[(\forall c' \in \{c\})(c' \subseteq \llbracket \varphi \rrbracket^c)] = T[c \subseteq \llbracket \varphi \rrbracket] = \llbracket \Box \varphi \rrbracket^c = T[(\forall c' \in \{c\})(c' \subseteq \llbracket \varphi \rrbracket^c)] = \llbracket \Box \varphi \rrbracket^c.\]

### 3.2.3 Expressivism

The entailment relationships in a language reflect ‘language-internal’ aspects of the meanings of its sentences. According to expressivism, the language-internal component of meaning is rich; according to descriptivism, it is poor. The way this debate shakes out has profound philosophical consequences. If language-internal meaning is impoverished, then our discourse imposes heavy burdens on the world; conversely, the burdens on the world diminish reciprocally with enrichment of the language-internal side of meaning.

If all classical rules for Boolean connectives are valid, the entailment relationships in a language never contravene the relationships of descriptive strength recorded in Stalnaker’s basic-entailment. Their very validity is itself language-internal. But all that means is that language is ‘apt’ to bear information, by containing structures reflective of fundamental structures within information.

To fail to contravene the classical rules is not to do nothing at all, of course. The Kaplanean package, familiarly, validates sentences like ‘I am here now’ (Kaplan 1977/1989, remark 3, 547). This reflects, perhaps, something like the recognition that episodes of language-use are acts—particular occurrences performed by agents with distinctive locations in a broad spatiotemporal manifold. That does, salutarily, take the burden of supplying ‘perspectival facts’ (Fine 2005b, Hare 2009) off of the world. But while enriching the language-internal with just the barest requirement on the possibility of language-use frees the world only from the burden of accommodating language-use itself, it allows that what goes on once language-use is up and running places all further burdens on the world.

But invalidating the suppositional rules in the manner of \( \downslice \) imposes a cross-cutting system of language-internal meaning on top of the fundamental Boolean structures. The source of the equivalence of \( \downslice \varphi \) and \( \varphi \)—the reason why \( \varphi \vdash \downslice \varphi \) and \( \downslice \varphi \vdash \varphi \)—after all, is not that \( \downslice \varphi \) has exactly the same meaning as \( \varphi \). It cannot be, inasmuch as \( \neg \downslice \varphi \not\vdash \neg \varphi \). But if not that, then the distinctive contribution of \( \downslice \) cannot consist in its transforming the descriptive power of \( \varphi \) to some other form of descriptive power.

A test case will make these perhaps gnomically abstract remarks more vivid. Consider the following case against the coherence of metaphysical indeterminacy (Williamson 1994, Barnett 2009). Let \( F\varphi \) mean ‘it is fixed whether \( \varphi \)’, where metaphysical indeterminacy corresponds to nonfixity. Plausibly \( \varphi \vdash F\varphi \) and \( \neg \varphi \vdash F\varphi \); after all, once the sea-battle starts (fails to start), nothing more can or needs to be done to make it have determinately started (failed to start). But then, by reductio, \( \neg F\varphi \vdash \neg \varphi \) and \( \neg F\varphi \vdash \neg \neg \varphi \); so \( \neg F\varphi \vdash (\neg \varphi \land \neg \neg \varphi) \); so \( \vdash F\varphi \).
But the appeal to reductio presupposes that the meaning of \( F \) consists in its transforming the descriptive power of \( \varphi \) to some other form of descriptive power. If so, this is the power to convert any sentence to a triviality: after all, if \( \varphi \vdash \varphi' \) and \( \neg \varphi \vdash \varphi' \), then if the meaning of \( \varphi \) and \( \varphi' \) is exhausted by their descriptive power, \( \varphi' \) is that sentence exceeded in descriptive power by both \( \varphi \) and \( \neg \varphi \); which means \( \varphi' \) has trivial descriptive power. Then of course \( \neg \varphi' \) has the limitless descriptive power of the contradiction.

If ‘metaphysical indeterminacy’ requires of the world that it supply indeterminacy for us—that the meaning of \( \neg F \varphi \) is exhausted by its descriptive power—then this would in my view count as a good argument against metaphysical indeterminacy. But there is a further possibility: the meaning of \( F \) is primarily language-internal. What that might be is not a task for this article.\(^{27}\) The point here is rather that it is possible to think that it is unfixed whether there will be a sea-battle tomorrow without requiring by this anything distinctive of the world: all there is to this thought is the bearing of a certain attitude toward the question whether there will be a sea-battle.

### 4 Meaning and support, propositions and answers

This section discusses the ‘problem of the declarative sentence’: what is it that makes propositions, with their overall Boolean structure, so attractive as semantic values for declarative sentences? On the interpretation I advance, this stems from the Boolean structure of \textit{pictures of the world}; where a picture of the world, in turn, is a positions that can be occupied in the course of inquiry into a range of intertwined yes–no questions. It is the intertwining of such questions that engenders Boolean structure. This is pillar (C) of the framework, the conception of \textit{propositions as epistemic}.

What is meaningful in the first instance is then the \textit{endorsement} of a declarative sentence. Doing so marks something about one’s picture of the world, where exactly what is so marked is a matter of the distinctive conventional meaning of the sentence. Propositional semantic values distinguish among the conventional meanings of declarative sentences, but are abstractions from those meanings. We advance a ‘lifted’ version of Kaplan’s identification of meaning with character; on the standard-set approach, the analogous doctrine is that the primary meaning of a sentence is its \textit{support-condition}. This is pillar (D) of the framework, the conception of \textit{meanings as endorsement-conditions}.

But that allows us to see descriptively rich propositional content as an optional aspect of declarative meaning. Support is already a mentalistic notion, and therefore no further mentalization need be undertaken to introduce sentences with purely ‘internal’ or ‘regulative’ or ‘expressive’ roles. If they are to be declarative, they must conform to the Boolean demands of that class, but that can be fulfilled perfunctorily, with what is distinctive to their meaning located beyond the bare nod to Booleanism. Thus rigidifiers; thus, moreover, their \textit{flaunting} of their nondescriptive meaning as observed in the previous section.

In our discussion of \textit{pictures of the world}, we will take note of a complexity in our attitudes toward \textit{opposition} (section 4.1): toward one person answering a question in the affirmative and another answering the same question in the negative. It is never OK for a single person to do that at a single time; but sometimes it is also not OK for a single person to do that over time, or for distinct people to do that at one time (section 4.1.2). Sometimes, indeed, we treat those who oppose us even as \textit{unintelligible} (section 4.1.1): we call questions for which that is so \textit{insubstantial}. It is to re-connect the meaning of questions for which collective op-

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\(^{27}\)One proposal is that \( \neg F \varphi \) rules out asking whether \( \varphi \).
position is not OK (global questions) to those for which collective opposition is OK (local questions) that we postulate contexts of interpretation: for there are indeed tight connections between their meanings (section 4.2.1); the shape of their connection is just that a local question plus a context of interpretation determines a global question (section 4.2.2). Propositions are apt to measure any state of progress in answering any global questions (or interpretation-relative local questions) anyone has put themself (section 4.3.1); one’s picture of the world is just the progress one has made on one’s own questions (section 4.3.2); and, finally, the problem of the declarative sentence is resolved by distinguishing meaning as support-condition from propositional content (section 4.3.3).

In the final section, we gather this interpretation with our formal apparatus for rigidifiers, modals, and conditionals on behalf of resolving the Dorr-Lee objection and the Ramsey–Moore–God paradox.

4.1 Questions and opposition

Consider a polar question—one with a yes–no answer, like those in (55):

55. (a) Do goats eat cans?
   (b) Are my shoes untied?
   (c) Are Fred’s shoes untied at t∗?
   (d) Is Julius Whitcomb Judson?
   (e) Did Julius invent the zip?
   (f) Is 7 + 5 = 14?

But contrasting with (56):

56. Who shot JR?

If I answer a polar question yes while Fred answers it no, or if Fred answers yes and I answer no, then Fred and I oppose one another on that polar question. The range of attitudes toward this opposition available to me is nuanced.

4.1.1 Your intelligibility through opposition

At one extreme, I might think Fred’s opposing position makes him unintelligible—or, at least, not prima facie intelligible. If I answer (55f), ‘is 7 + 5 = 14?’, in the negative but Fred answers it in the affirmative, I can’t make sense of what things are like for Fred such that he gives that answer. Under such circumstances, I must hunt around for special reasons Fred has for giving this answer: imperfect understanding; bad calculation; a brain glitch.28 (The same goes if I answer the question in the affirmative and Fred in the negative. Given how I in the real world, as the author answer the question, it would then be me who is unintelligible. But one never seems unintelligible to oneself.)

If I think of opposition on a certain question as unintelligible, I also think of uncertainty as unintelligible. If I can’t make sense of what it is like for Fred answering (55f) in the affirmative, I can’t make sense of some third party, Sam, making sense of what it is like for Fred or someone like him in giving that answer. But I can only make sense of indecision regarding an option if I can make sense of making sense of the option. Because uncertainty is indecision between the affirmative and negative answers, I therefore can’t make sense of Sam’s uncertainty.

28 ‘So what if one is unintelligible? Maybe that meets some pragmatic requirement. If so, what is it that is somehow bad about the situation?’ —I see no reason to accept the demand for a further norm to motivate conformity to rational psychology. All norms are internal to rational psychology. If one doesn’t conform, then we just can’t bring the rational-psychological strategy for understanding to assessing one, any more than chemistry doesn’t work at the heart of the sun. What we choose to make of either situation is up to us; the importance we place in making sense of one another makes us likely to attempt to browbeat one another back to intelligibility, but that is a norm within rational psychology rather than any external ground.
At the other extreme, I might think there is no problem at all with this opposition. If I answer (55b), ‘are my shoes untied?’, in the affirmative while Fred answers it in the negative, that need not raise any problem at all. The same is true if I answer the question in the affirmative at one time but in the negative at another time; or if Fred does so. It is an extreme problem, however, if Fred answers the question simultaneously in the affirmative and the negative. Then Fred is in opposition to himself: an absolute prohibition in the question-and-answer game. In violating this prohibition, Fred makes himself in that way unintelligible.

4.1.2 Our intelligibility through opposition

In an intermediate position, I might think there is a problem with our opposing one another, though the problem is not Fred’s unintelligibility. If I answer (55a), ‘do goats eat cans?’, in the negative while Fred answers it in the affirmative, then I simply think he is wrong: the way he pictures the world in giving his answer to (55a) is incompatible with the way I picture the world in giving mine. But in order for that to be so, there must be an intelligible way Fred pictures the world in giving that answer.

The problem is rather with us. Calling into being a collective inquirer with opinions given as the aggregate of my opinion and Fred’s, that collective inquirer answers (55a) simultaneously in the affirmative and in the negative. That is an absolute prohibition in the question-and-answer game. Consequently, if the collective inquirer is playing the question-and-answer game, it is in violation; so unless there is some violation, Fred and I are not aggregating our opinions into an aggregate. Perhaps it is in violation of some broad norm of inquiry for a society to fall short of fully collective inquiry.

And the same goes if it is not Fred but some distant temporal stage of myself who is in opposition to me as I am now. Then the aggregate of my opinion over time does not fold into an intelligible total picture. Perhaps it is in violation of some broad norm of inquiry for a person to lack an aggregable progression of opinions.

4.2 Global and local questions and interpretation

Let us call questions for which we are—the collective inquirer is—unintelligible in opposition global, the rest local. Then (55f), ‘is 7 + 5 = 12?’, and (55a), ‘do goats eat cans?’, are global questions; while (55b), ‘are my shoes untied?’, is a local question. Let us moreover call questions for which one regards those opposing as unintelligible insubstantial. Then (55a) and (55b) are substantial, (55f) insubstantial.

4.2.1 The connection between global and local questions

Global and local questions are connected in their meanings. Grant that (55c), ‘are Fred’s shoes untied at t∗?’, is a global question. If Fred endorses a negative answer to (55b), ‘are my shoes untied?’, but an affirmative answer to (55c), while also endorsing bridging identities ‘I am Fred’ and ‘it is now t∗’, he is unintelligible here just by virtue of the logic of identity. But, arguably, there are modes of presentation under which (55c) retains its globality, (55b) retains its locality, and Fred is unintelligible even if he fails to endorse the bridging identities. For suppose Fred at t∗ asks (55b), ‘are my shoes untied?’—a public speech act. The only means anyone has of answering Fred’s speech act is to answer it in accord with (55c). If I do not know who was asking, or what time they were asking, I cannot answer. Or if Mo is under the misimpression it was Ro asking; or if Rance is under the misimpression it was Brent asking: then they will answer irrelevantly. If this is not the reaction Fred intends to provoke in performing the speech act, then we have no idea what his intention was. But that can only be the reaction he
intends to provoke if he endorses ‘I am Fred’ and ‘it is now $t^*$. More generally, if at $t$, $x$ endorses opposing answers to ‘am I $F$?’ and ‘is $x$ $F$ at $t$?’, $x$ is thereby in violation of the question-and-answer game.

Still more perplexingly, two distinct local questions might not accommodate opposing answers, if the times at which or agents by which they are addressed are aligned just right (or just wrong). For example, when ‘were his shoes untied then?’ is intended in such a way that ‘he’ concerns Fred and ‘then’ concerns $t^*$, it cannot be given an opposing answer to (55b) as asked by Fred then.

### 4.2.2 Contexts of interpretation

We can accommodate this connection in the meanings of global and local questions with our notion of the context of interpretation. For any local question and any context of interpretation, there is some global question such that if the local question is asked by someone whose circumstances we associate with the context of interpretation, the answers they give to the local and the global question cannot oppose. Under these circumstances, say that the global question underlies the local question against that context of interpretation.

What determines whether opposition is a violation of the question-and-answer game, given this apparatus, is the underlying global question. If a local question $?\ell$ is underlain by a global question $?\gamma$ against a context of interpretation $c$, while a local question $?\ell'$ is underlain by a global question $?\gamma'$ against a context of interpretation $c'$, then opposition between answers to $?\ell$ against $c$ and $?\ell'$ against $c'$ is legitimate just if opposition between answers to $?\gamma$ and $?\gamma'$ is.

Doubtless, moreover, there is some sense in which it is possible to adjust one’s context of interpretation so that it does not just align directly with one’s present circumstances—via flights of the imagination; in tracking temporally complex narrative; in understanding the context-sensitive utterances of others. If so, then if Fred and I adjust our contexts of interpretation so that they are unified, our attitudes toward any given question would determine our attitude toward its underlying global question against the context of interpretation to which we have adjusted. Under such a condition of cointerpretation, it would be impermissible for us to oppose one another on the local question.

### 4.3 Propositions and meaning

It is positions in the search for answers to global polar questions (and the local polar questions they underlie) that we use propositions to represent. To foreshadow a bit: propositions measure how much information someone has. To assign a proposition $p$ as the semantic value of a sentence is to say that the sentence is endorsed by anyone with at least as much information as the amount $p$ measures. So if $p = W$, the sentence is endorsed by everyone, while if $p = W$, the sentence is endorsed by no one. The same goes when the assignment is relative to a context of interpretation: in particular, $\forall P$, with its test-semantics, is represented as endorsed by exactly those who interpret it against a $P$-supporting context.

### 4.3.1 Where propositions come from

Let $?\gamma$ and $?\eta$ be global polar questions. Then if each is substantial, anyone who has made it her business to answer $?\gamma$ thereby projects a Boolean algebra of information states she might occupy in the course of the investigation: uncertainty; affirmation; negation; and inconsistency (this last failure state must be present in the system if it is obligatorily be avoided, after all); and the same goes for anyone who has made it her business to answer $?\eta$. Affirmation and negation are the success states, when the question has
been answered; uncertainty, though not a failure state, is also not a success state. If \( \leq \) is the ordering of the algebra, then inconsistency is the \( \leq \)-minimal state \( \bot \), while uncertainty is the \( \leq \)-maximal state \( \top \); affirmation and negation are intermediate and mutually incomparable. The success states are the atoms of the Boolean algebra, so the number of success states equals the dimension \( D \) of the algebra: two success states makes for a two-dimensional Boolean algebra; the cardinality of the algebra is \( 2^D \)—in the present case, four, with the two success states accompanied by uncertainty and inconsistency. We could then think of the ‘objective’ one sets in asking a global polar question as being to go as far down in the Boolean algebra as one can get without bottoming out in inconsistency.

Say \( ?\gamma \) and \( ?\eta \) are independent just if each pairing of a success state for one and a success state for the other is an intelligible view. Example: ‘do goats eat cans?’ and ‘do horses eat hay?’ are independent (one might intelligibly think any of: both do, neither do, just the former do, just the latter do); ‘do goats eat cans?’ and ‘do either (goats eat cans) or (horses eat hay)?’ are not independent, because an affirmative answer to the former and a negative answer to the latter add up to an unintelligible view. When one entertains a pair of independent questions, the number of success states is the product of the number of success states to each in the pair: in the present case, four. Correspondingly, the Boolean algebra of informational positions on the two questions consists of all points corresponding to success states still left open: in other words, all disjunctions of success states, together with inconsistency. In the present case, the dimensionality of the algebra is 4, so its cardinality is \( 2^4 = 16 \).

If we consider the largest mutually independent subset of the set of all possible substantial global polar questions, the cardinality of that set is some vast number, \( Q \). Let the total inquiry be the product of all those questions. The cardinality of the set of success states for the total inquiry is \( 2^Q \). And the cardinality of the algebra projected by the total inquiry is then \( 2^{2^Q} \). We identify the propositions as the members of that algebra. Because every Boolean algebra is isomorphic to a field of sets (the power set of some set, ordered by \( \subseteq \)), there is some set the field of which is isomorphic to the algebra of propositions. We call that set modal space, notate it \( W \), and identify its members as the members of the singletons that are the atoms of the field of sets isomorphic to the algebra of propositions. We may then finally reconstruct the semantics of and pragmatics of questions using a familiar apparatus of partitions of modal space, with a polar question partitioning modal space into just a proposition representing its affirmative answer and a proposition representing its negative answer, and a substantial polar question requiring each of those sets to be nonvacuous.

### 4.3.2 Propositions, pictures of the world, and meaning

What we mean, therefore, in representing Fred with a context \( c^{\text{fred}} \) the standard of which is \( s(c^{\text{fred}}) \), a (subset of modal space standing in for a) certain member of the algebra of propositions, is this. Considering the total inquiry, \( s(c^{\text{fred}}) \) represents the position of leaving open just a certain range of success states. We then mean that Fred, in his inquiries, has attained a position leaving open just that range of success states for the total inquiry. Let an individual’s picture of the world be the range of success states for the total inquiry they leave open.

What we mean, moreover, in identifying Fred’s implicit endorsement of \( \varphi \) (namely, \( c^{\text{fred}} \vdash \varphi \)) with \( s(c^{\text{fred}}) \subseteq \llbracket \varphi \rrbracket^{\text{fred}} \) is that to answer \( ?\varphi \) in the affirmative—with \( \varphi \) interpreted against the circumstances in which Fred finds himself—it suffices to picture the world as Fred does. The formal artifact of core significance here, however, is support (as a representation of an attitude toward an
interpreted sentence) rather than *semantic-valuation* (as a representation of an abstract representational power of an interpreted sentence). The light this sheds on the meaning of \( \varphi \) is just that it is such as to be accepted by a context of interpretation/evaluation when, so interpreted, \( \varphi \) with its conventional meaning is thereby implicitly evaluated as accepted.

The point of assigning context-relative propositional semantic values to declarative sentences is then to depict their aptness as vehicles for recording positions in a question-and-answer game. Propositional semantic values are not themselves meanings, even relative to contexts. The best they can do is distinguish the meanings of declarative sentences from one another. They cannot by themselves shed light on what it is to be a declarative sentence, on the nature of declarative meaning itself—in contrast, say, with imperative or interrogative meaning. In order to understand that, we need to know what it is to *accept* a declarative sentence.

### 4.3.3 Boolean connectives and meaning

Boolean connectives apply to declarative sentences because multiple interrelated questions project a Boolean structure, and because declarative sentences answer questions. It is for that reason that Boolean connectives are represented with Boolean operators on semantic values: for instance, \( \llbracket \varphi \lor \psi \rrbracket^c = \llbracket \varphi \rrbracket^c \cup \llbracket \psi \rrbracket^c \). Namely, the affirmative answer to \(?(\varphi \lor \psi)\) is exactly as strong as the strongest position weaker than both the affirmative answer to \(?\varphi\) and the affirmative answer to \(?\psi\). Relatedly, the affirmative answer to \(?(\neg \varphi)\) and the negative answer to \(?(\varphi)\) are one and the same, so that, in particular, one can no more answer both \(?(\neg \varphi)\) and \(?\varphi\) in the affirmative than one can answer \(?\varphi\) in both the affirmative and the negative: arguably the fundamental fact about the meaning of negation.

But Boolean connectives are *not* representable by Boolean operators on support-conditions. To answer \(?(\varphi \lor \psi)\) in the affirmative is not to have either answered \(?\varphi\) in the affirmative or answered \(?\psi\) in the affirmative. The information that \(\varphi \lor \psi\) is *emergent*: it is available only when one considers \(?\varphi\) and \(?\psi\) as potentially bearing on one another. So long as one treats the questions in isolation, one will never be in a position to attain the disjunctive information state. Similarly, to have failed to have answered \(?\varphi\) in the affirmative is not the same as having answered \(?(\neg \varphi)\) in the affirmative—for otherwise the former would be the same as having answered \(?\varphi\) in the negative, which would make uncertainty and therefore the very practice of question-and-answer games impossible. Uncertainty is a feature of us, not of the world into which we inquire; identifying the support condition of \(\varphi\) and the ‘antisupport’ condition of \(\neg \varphi\) would collapse the distinction between self and world.

### 5 Modality and perspective, propositions and answers

We have embraced the identification of modals with quantifiers over contexts (36), hypothesizing that the semantic valuation clauses for vertical and diagonal necessity modals are these:

\[
\begin{align*}
(36a) \quad \llbracket \Box c \rrbracket^c &= T[(\forall c')(c' \subseteq \llbracket \varphi \rrbracket^c)] \\
- \quad c \vdash \Box \varphi \quad \iff s(c) \subseteq T[(\forall c')(s(c') \subseteq \llbracket \varphi \rrbracket^c)] \\
& \quad \iff (\forall c')(s(c') \subseteq \llbracket \varphi \rrbracket^c)
\end{align*}
\]

\[
\begin{align*}
(36b) \quad \llbracket \Box c \rrbracket^c &= T[(\forall c')(s(c') \subseteq \llbracket \varphi \rrbracket^c)] \\
- \quad c \vdash \Box \varphi \quad \iff s(c) \subseteq T[(\forall c')(s(c') \subseteq \llbracket \varphi \rrbracket^c)] \\
& \quad \iff (\forall c')(s(c') \subseteq \llbracket \varphi \rrbracket^c)
\end{align*}
\]
The discussion of the previous section feeds into the interpretation of this apparatus as follows.

A polar question embedding a rigidified sentence—whether $\nvdash \varphi$—is both insubstantial and local (section 5.1): local, in that in the absence of cointerpretation, opposition between you and me is no problem; insubstantial, in that given cointerpretation, I find your opposition unintelligible.

In section 5.2, we discuss the Dorr-Lee objection: the prediction $\Box \nvdash \varphi \lor \Box \neg \nvdash \varphi$ (37c) makes it ‘metaphysically noncontingent’ what I believe, absurdly. But in light of the perspectival conception of modality, what that means is that anyone disagreeing with me about my own mental state is unintelligible (section 5.2.1): a claim of first-person authority to which mindset semantics already commits in making a representation of one’s mental state the standard for sentential acceptance (24a). The source of the allure of the assumption that it is contingent what I believe is not far off (section 5.2.3): because of this first-person authority, the value to me of Fred’s belief-avowal is in the descriptive information I can glean from it via reasoning backward from whatever means by which I get from descriptive information to my ‘sense’ for Fred’s mental state. Because a vertical modal brings in an indefinite range of contexts of evaluation distinct from one’s own context of interpretation, there is a natural slide into thinking of that descriptive information rather than the meaning of the prejacent when evaluating the modalized sentence. We also discuss (section 5.2.2) whether we should in fact think of answers to insubstantial local questions as, in fact, a posteriori: we argue instead that it occupies a third category—the classical dichotomous classification another casualty of the loss of self-duality of the fundamental semantic relation.

In section 5.3, we discuss the Ramsey–Moore–God paradox: the allegedly paradoxical conclusion being the theoremhood of ‘$P$ just if I believe that $P$’, with this theoremhood alleged to be paradoxical because it represents us as having the ‘epistemic powers of a god’. I argue that distinguishing contexts of interpretation and evaluation undermines the sense that this is what is represented: a god seems godlike not just reflexively but to everyone. At the same time, it is true that we can’t distinguish ourselves from gods without getting outside our own heads: the coindexation of the evaluation and interpretation contexts for the cases when the biconditional is a theorem represents that we have not gotten outside our own heads; so if there is this prediction, that is a feature rather than a bug.

5.1 Local insubstantial questions

The value of the metalinguistic test operator for us, on display here, is its ability to ‘disquote’: because $s(c) \subseteq T[\sigma]$ exactly when $T[\sigma] = W$ exactly when $\sigma, s(c) \subseteq T[\sigma] \Leftrightarrow \sigma$.

What we mean, then, by ascribing the semantic valuation clauses in (36) is then to depict someone’s endorsement of a modalized sentence as (and pending further standard-dependence of the prejacent $\varphi$) independent of their picture of the world. What is relevant is instead the scope of the set of contexts over which the quantifier ranges, and whether its members support $\varphi$ as contexts of evaluation, given an appropriately indexed context of interpretation. In particular, Fred endorses $\Box \varphi$ just if all contexts relevant to Fred picture the world in accord with $\varphi$ as interpreted by Fred; and Fred endorses $\nabla \varphi$ just if all the relevant contexts support $\varphi$.

5.1.1 Opposition and vertical modality

These modalized sentences are associated with distinctive attitudes toward opposition. If I endorse $\Box \varphi$ and my context is ‘sincere’—it is relevant to itself—then I endorse $\varphi$; same for $\neg \varphi$. I then think of an opposing view whether $\varphi$ as intelligible only if differently
interpreted. This attitude is familiar from discussion of examples like (55d), ‘is Julius Whitcomb Judson?’. Recall that ‘Julius’ is stipulated to refer to the inventor of the zip, whoever he may be; use of the name with this stipulation, let us suppose, is part of its conventional meaning. In my opinion, Whitcomb Judson invented the zip; so, as I interpret it, one should have the same attitude toward (55d) and the trivial question ‘is Whitcomb Judson Whitcomb Judson?’. It is unintelligible to me how the latter could get a negative answer; and so it is unintelligible to me how, given how I interpret it, (55d) could get a negative answer. And yet at the same time I do think opposition over the question is permissible! For it is not part of the conventional meaning of ‘Julius’ that it refers to Whitcomb Judson: that is just my opinion about who invented the zip, where the conventional meaning is exhausted by the latter. If Fred mistakenly thinks that Tenzing Norgay invented the zip, he could perfectly well answer (55d) in the negative—without either his lapsing into unintelligibility or our collectively violating the norms of question-and-answer games.

This attitude is distinctively different from that I bear to substantial questions whether local or global. Unlike with global questions, opposition is compatible with collective intelligibility. Unlike with local questions, opposition together with counterinterpretation is incompatible with your intelligibility.

5.1.2 Opposition and diagonal modality

Another attitude still is associated with □, which we attach only to theorems. If I think every (relevant) context supports ϕ, then when I am sincere, I think my context supports ϕ; and when you are relevant, I think your context supports ϕ. Because of the first, I endorse ϕ; so if your answer to whether ϕ opposes mine, you reject ϕ. Because of the second, I view you as rejecting ϕ even though your context supports it; namely, in my view, your total picture of the world is not coherent, with your implicit picture confirming ϕ and your explicit picture confirming ¬ϕ. This is our attitude toward (55f): opposition over ‘is 7 + 5 = 14?’ is, as we announced at the outset, a strong leading indicator of accusations of unintelligibility.

And yet examples such as (55e) suggest that even certain theorems allow for opposition, if contexts of interpretation and evaluation are dissociated. For we perhaps reject □(Julius invented the zip). If my view sets the context of interpretation, the question ‘did Julius invent the zip?’ is equivalent to ‘did Whitcomb Judson invent the zip?’; and that question is (tense aside) an ordinary global question, admitting opposition without accusations of unintelligibility. If ‘did Julius invent the zip?’ is interpreted with me but evaluated with someone else, opposition is no problem.

5.2 Modals scoping over rigidifiers

Let us throw our range of relevant contexts wide open and turn to the case of modals scoping over rigidifiers. Let g := ‘goats eat cans’, where [g]c = G (for any c). Applying our clauses for vertical and diagonal necessity to ▽g generates the following:

57. [□▽g]c = T[(∀c′)(s(c′) ⊆ [▽g]c′)]

- c ⊨ □▽g
    ⇔ s(c) ⊆ T[(∀c′)(s(c′) ⊆ [▽g]c′)]
    ⇔ (∀c′)(s(c′) ⊆ [▽g]c′)
    ⇔ (∀c′)(s(c′) ⊆ T[s(c′) ⊆ [g]c′])
    ⇔ (∀c′)(s(c′) ⊆ T[s(c′) ⊆ G])
    ⇔ (∀c′)(s(c′) ⊆ G)

So c ✯ □▽g

58. [□▽g]c = T[(∀c′)(s(c′) ⊆ [▽g]c′)]
Fred’s does. So for Fred, positive evaluation of \( \varphi \) just if Fred’s accepts \( \varphi \). As my context interprets it, inquiry, though opposition over the former does not. But it also draws \( \varphi \) apart from claims like ‘my shoes are untied’: cointerpreting the latter, opposition is a failure but still intelligible; cointerpreting the former, I regard anyone in opposition as thereby unintelligible.

What does this all mean? Suppose I endorse \( \varphi \) but Rance, purporting to have taken up my mental state as a context of interpretation—at pretending to be me—rejects \( \varphi \) (within the pretense). I then think Rance is not intelligible, prima facie. The natural fallback interpretation for me is just that Rance has not done what he has purported: he has, in fact, failed to take up my mental state. That assessment of the situation is, according to the framework, rationally mandatory: unless I am (unbeknownst to myself) somehow lapsed into unintelligibility, no one can know me better than I know myself (and therefore, in light of my perpetual self-intelligibility, so it will always seem, at least in the moment). Adequate interpretation of the question is all it takes to know its answer. In particular, opposition under such circumstances does not admit of explanation in terms of divergent pictures of the world: there is nothing relevant to the answering of substantial global polar questions which could conceivably be relevant to answering questions about my state of mind as posed using rigidifiers.

Is this a result we should welcome? I think it is. According to the mindset approach, the constitutive norm governing the meaning of declarative sentences relates one’s picture of the world to the conventional meaning of a sentence as one interprets it. If one’s picture of the world suffices for endorsing a sentence but one does not endorse it, one is thereby not prima facie intelligible. If endorsing sentences is not done ‘blindly’, but with full knowledge of what one is doing and why, this requires knowledge of one’s picture of the world. The sort of condition of ‘revelation’

\[ c \vdash \square \neg \varphi \]

\[ \iff s(c) \subseteq T[(\forall c')(s(c') \subseteq [\varphi])'] \]

\[ \iff (\forall c')(s(c') \subseteq [\varphi])' \]

\[ \iff (\forall c')(s(c') \subseteq T[s(c) \subseteq [\varphi]])' \]

\[ \iff (\forall c')(s(c') \subseteq T[s(c) \subseteq G])' \]

\[ \iff (\forall c')(s(c') \subseteq G) \]

\[ \iff s(c) \subseteq G \]

So \( c \vdash \square \neg \varphi \) just if \( c \vdash \varphi \)

The diagonally-modalized sentence is supported by a context of utterance (indeed, at any context) just if every context positively-evaluates \( \varphi \) as it interprets it. To do that is just to positively-evaluate \( \varphi \). And not every context does that; so no context supports \( \square \neg \varphi \).

We return to whether this makes \( \varphi \) somehow a posteriori.

### 5.2.1 Rigidification, metaphysical necessity, and first-person authority

The vertically-modalized sentence (58) is supported by a context of utterance—mine, or Fred’s, for example—just if every context positively-evaluates \( \varphi \) as my context, or as Fred’s, interprets it. As my context interprets it, \( \varphi \) is to-be-positively-evaluated just if my context accepts \( \varphi \). As Fred’s context interprets it, \( \varphi \) is to be-positively-evaluated just if Fred’s accepts \( \varphi \). Mine doesn’t, Fred’s does. So for Fred, positive evaluation of \( \varphi \) as he interprets it is requisite on pain of unintelligibility, while for me, negative evaluation of \( \varphi \) as I interpret it is requisite on pain of unintelligibility. So Fred must endorse \( \square \neg \varphi \) while I must reject it.

This draws the status of \( \varphi \) apart from that of claims like ‘goats eat cans’: opposition over the latter reflects a failure of collective inquiry, though opposition over the former does not. But it also

\[29\text{It predicts, moreover, that mistaken endorsement of rigidified sentences will count as departure from intelligibility. That is an attractive feature of the approach, because to endorse a false rigidified sentence is to conditionalize on the impossible proposition, and thereby enter a defective state. The corresponding feature of the classical}\]
predicted is in the blueprint of the mindset approach. So while it is a prediction of the system that $\forall \varphi$ is ‘metaphysically noncontingent’, this is actually in line with the broad programmatic aims of the system.

5.2.2 Knowledge a praesente

Classically, the nontheoremhood of $\Box \forall g$ is interpreted as the non-apriority of $\forall g$. That is fine. But non-apriority is also, in the interpretation of the classical framework, elided with aposteriority. That is not a welcome label. Paradigms of the a posteriori are substantial questions like ‘do goats eat cans?’ and ‘are my shoes untied?’. Even once commonality of interpretation has been secured, Fred’s opposition does not warrant my accusing him of unintelligibility. But as we have seen, if Fred opposes me over $\forall g$, that is a sure sign of a distinct context of interpretation.

The trouble stems from our use of contexts as the entities over which modals quantify. As noted in section 1.2.3, the classical diagonal remains a proposition. But on our approach, the diagonal of a sentence exhibits variation across contexts of interpretation in the difficulty of establishing the sentence. One way to avoid theoremhood is genuine aposteriority: all along the diagonal are nonextremal propositions. But another, observed here, is for the diagonal to be populated inconstantly by extremal propositions: on certain interpretations, the sentence is irrefutable; on the rest, ludicrous.

That distinction in the non-theorems—between the never trivial and the always trivial—is one that cannot be drawn classically: points, individual worlds, the constituents of the classical diagonal, lack sufficient internal structure. For this reason, self-knowledge of the sort expressed with rigidifying tests does not fit comfortably in the binary scheme of apriority versus aposteriority. It might be better to multiply category labels, calling the knowledge in question a praesente.

5.2.3 Other minds and descriptive information

And yet there is an obvious sense in which it is ‘metaphysically contingent’ what I come to believe: thus the Dorr-Lee objection. There is surely no change of subject between rigidification and ordinary belief-reports; accordingly, the ordinary belief predicate should behave like $\forall$ amended with various bells and whistles. Without taking a position on how that should go, our aim is to sketch an affiliation between a sort of straightforwardly metaphysically contingent descriptive sentence and $\forall \varphi$.

Even if belief-avowals are nondescriptive, their audiences surely acquire descriptive information about the avower. This acquisition may link the avowal and the corresponding descriptive information sufficiently tightly that we do not customarily distinguish between these in discussion of the ‘meaning’ of a belief-avowal. The path to generating some sense in which $\forall \varphi$ is metaphysically contingent is straight, broad, and well-demarcated: what is metaphysically contingent is the linked descriptive information, rather than the literal meaning of the avowal.

All that remains is to fill in the details. The heart of the approach would be some function $M : \mathcal{P}(W) \times T \times J \rightarrow \mathcal{P}(C)$: $M$ (‘Mu’) captures the range of contexts individual at a time isn’t certainly not in given a picture of the world. (Perhaps $M$ captures the content of empathy.) Then if I avow belief that $\varphi$ at $t$, a certain range $R$ of propositions is such that, for $P \in R$, for all $c \in M(t, BH, P)$, $c \in \lfloor \varphi \rfloor$. $R$ is something like a ‘diagonal supercontent’ of the avowal. What the audience to my avowal takes

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approach is correspondingly unattractive: $\varphi$ and $A \varphi$ are equivalent, so presumably one should endorse the one just if the other; but if $\varphi$ is false, $A \varphi$ expresses the impossible proposition; so the classical approach magnifies mistake into incoherence.
away is some proposition in \( R \): perhaps the disjunction of all of them; perhaps something more specific and fixed by other knowledge.

The allure of the protest that it is metaphysically contingent what I believe, despite its literal falsehood, is amplified by the quantification over contexts involved in \( \boxvert \). I know well that others are in a bad position to ascertain exactly which context I inhabit. Ordinarily, my belief-avowals are made with the expectation that my audience will take away some proposition in their diagonal supercontent. Accordingly, when I bring in an indefinite range of other contexts via the invocation of \( \boxvert \), my natural tendency is to switch from literal interpretation of my statement to the more audience-friendly diagonal supercontent.

More efficiently: what is metaphysically contingent is whatever descriptive fact about this organism obtains such that, in its obtaining, I rightly avow belief that \( \varphi \). If we adopt an appropriately hygienic separation of matters that can be described from matters that can only be expressed, we will not be tempted to think we have any purchase on how to do the latter which is continuous across a wide range of hypothetical circumstances. Change the facts about the organism and the context vanishes. Despite its evanescent character, the context is ineliminable from semantical discussion: there is no ‘semantics of the organism’. More explicitly: we grasp discussion of psychological matters, including meaning, only through being in mental states: a zombie could not understand discussion of psychology. It is to keep this fact in the picture that we do not dispense with contexts. That is what the ‘hard problem of consciousness’ looks like to the semanticist.

### 5.3 Conditionals scoping over rigidifiers

Bearing the distinction between the context of interpretation and of evaluation firmly in mind helps defuse the Ramsey–Moore–God paradox as an objection to the present framework. Recall (43b): \( \vdash P \rightarrow \lozenge P \) and \( \vdash \lozenge P \rightarrow P \). Do these *diagonal conditionals* not state it to be a theorem that if \( P \), I believe it; and if I believe that \( P \), \( P \)?Does that not represent it to be theorems that I am omniscient and infallible—that I have the ‘epistemic powers of a god’? (The same goes for the *rigidifying conditionals* \( P \supset \lozenge P \) and \( \lozenge P \supset P \), which are consequences of the diagonal conditional.)

Note first that the corresponding *vertical conditionals* are not theorems (43a): \( \nvdash P \square \rightarrow \lozenge P \) and \( \nvdash \lozenge P \square \rightarrow P \). On the semantics we have discussed, the former says that any context of evaluation updated with \( P \) accepts \( \lozenge P \) as my context interprets it while the latter says that any context of evaluation updated with \( \lozenge P \) as my context interprets it accepts \( P \).

Whether I represent myself as having the epistemic powers of a god would seem to concern my attitude toward the *vertical* conditionals rather than that toward the *diagonal* or rigidifying conditionals. The latter pertain to whether, without in some way alienating myself from my own mental state, I can discriminate it from the world. And of course I can’t: belief is transparent, so I find nothing believed to doubt and no fact not yet believed. What the diagonal and rigidifying conditionals mean is that the same goes for hypothesis: anyone hypothesizes \( P \) just if they hypothesize it as accepted.

The vertical conditionals, by contrast, pertain to the splitting of evaluation and interpretation: of considering my own point of view (qua interpreter) as from some alien point of view (qua evaluator). This splitting is intimately connected with our idea of ‘objectivity’, as reflected in the distinction between global and local questions. Opposition over local questions is rendered unproblematic by multiplying contexts of interpretation, and noting that what is oppositely-evaluated by the other is not the question as I interpret it but as the other interprets it.

A god is presumably objectively so: would be recognized as
omniscient and infallible from all contexts of evaluation. (The discussion of the previous subsection suggests that we think of ourselves as having the epistemic powers of gods over our own mental states.) As we multiply contexts, we proportionately thin down our own egos. Someone might well represent themself as having the epistemic powers of a god because they neglect mental states other than their own. But I find that my own existence with a mental state is enough to refute them—not to mention the broad range of mental states I am perfectly comfortable recognizing.
References


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