

Rationalization and the Ross Paradox

Benj Hellie*

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1 Introduction

I *must* finish this chapter—and here I am, working away. To type this sentence, as I am doing, I *must* push the hyphen key three times—lo and behold, I just did it. Conversely, although I surely *should* donate a lot more to causes fighting the prison-industrial complex, and doubtless *oughta* do much more on behalf of ending fossil-fuel use, I will not do either (hey, nobody’s perfect!).

The link between ‘must’ and intentional action appears tight. Whatever one thinks one must do (whether important or trivial), one does. More cautiously—one might be stymied, after all—one acts with the *intention* of doing it.

Indeed, among the English modal auxiliary verbs I understand, the connection appears uniquely tight. What one thinks one *can*, *could*, *may*, *might*, *should*, or *would* do (whether important or trivial), one often enough does not even try to do (Sloman 1970, 391); and much of what one *will* do, one won’t do intentionally.¹ With the end of understanding relations among agency, language, and modality,

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¹What one *shall* do is a matter I fear I cannot make much sense of: ‘shall’ is not in good standing in my idiolect, and corpus uses are freighted with legalese and distorted by prescriptivism.

must stands out as central, the remaining modals retreat to the periphery.²

Other linguistic phenomena connected to intentional action are imperative sentences and the speech act of command. If Rance accepts Brent's command 'Pay the rent!', Rance forms an intention to pay the rent, thinks he must pay the rent, and endorses the sentence 'I must pay the rent.'. As for Brent, he issues the command endorsing 'Pay the rent!', in the belief that Rance must pay the rent, endorsing the sentence 'Rance must pay the rent.', and in order to put on display an intention for Rance to pay the rent.³ Running in the opposite direction, if Rance intends to pay the rent, he arguably does so thinking he must pay the rent and endorsing 'I must pay the rent.'⁴—and, with some plausibility, also endorsing the self-directed imperative 'Pay the rent!'.⁵

These various phenomena—individual and general-will intention, imperative sentences, the issuance and acceptance of commands, and the associated *obligative* readings of 'must'—I group together under the rubric of *obligation*.⁶ This chapter maps out the territory of obligation. At its center is the psychological state of *intention*. An intention for Rance to pay the rent has a content *R* of a kind I call a *procedure*—a structured, nonpropositional representation of a plan of action.⁷ The imperative sentence 'You pay the rent!', with Rance as the addressee, shares this procedural content *R*.⁸ Brent's command addressed to Rance using this sentence puts on display an intention—harbored by the 'general will'

²If 'ethics' is understood as the theory of *value*, ethics may be appropriately aligned with *should/ought*. But if 'ethics' is understood as the theory of *practical reason*, ethics is appropriately aligned with *must*.

³Castañeda (1960, 21) brings Kant and Hare to his side in maintaining that 'imperatives are at the center of the connection of 'ought' []-assertions with both action and rationality'. To my mind, this settles on the wrong modal.

⁴Contrast the view that the obligative meaning of *must*-sentences is ineliminably social (Ninan 2005, 5.1; Portner 2007, 364).

⁵Contrast the view that the obligative meaning of imperative sentences is ineliminably social (Lewis 1979a; Kaufmann 2011).

⁶The deontic logic tradition studies an operator \bigcirc . That operator is given the intuitive glosses 'ought' and 'it is obligatory that'. On my terminology, 'obligation' is expressed with *must* rather than *ought*. If ordinary 'obligation' conforms to my usage, \bigcirc is either underspecified or semantically defective.

⁷Contrast the view that intentions are a kind of belief (Setiya 2007, 48).

⁸A selection of opposing views of imperative contents: properties (Portner 2004, 239); propositions (Kaufmann 2011); satisfaction–violation pairs (Vranas 2008); 'actions' understood as 'relations over worlds' (Barker 2012).

of a community including Rance and Brent, voiced by Brent—with the content *R*. Accepting the command, Rance forms an intention with content *R*.⁹

My core thesis is that *a modal sentence given an obligative reading is a modalized imperative* (Castañeda 1975, Hellie 2015).¹⁰ Read obligatively (and as addressed to Rance), ‘You must pay the rent.’ operates syntactically on the imperative ‘You pay the rent!’ and semantically on its content *R*.¹¹ As is standard (Kaplan 1977), I analyze modals as operators on intensions, and assign intensions the job of representing entailment; but I analyze entailment as the preservation not of *truth*, but of *support* (an endorsement-like mental attitude toward a sentence: Hare 1952, 172; contrast Castañeda 1960).¹² In consequence, modals are

⁹For a nuance, see note 27.

Charlow (2014) salutarily links imperatival content to practical psychological states: for α an imperative, $\llbracket\alpha\rrbracket$ is a condition on ‘plans’, objects with at least the structure of a set of action-kinds (644); plans are (near enough, perhaps) contents of a practical psychological state of ‘planning to do’ (Charlow 2011, passim). The communicative meaning of an act of command using α is then (roughly) to constrain the content of the audience’s ‘planning to do’-state so that it satisfies $\llbracket\alpha\rrbracket$.

Charlow (2011, IV) advances this as improving upon a ‘Standard Account’ affirming the *Display Thesis*, on which the content of σ is the content p of a mental state conventionally put on display by speech acts with σ , and the orthodox *Propositional-Attitude Thesis* that all mental states are propositional attitudes. To my mind, Charlow makes a compelling case that the Standard View founders on practical language phenomena.

On the present approach, the Display Thesis serves as a fixed point, while linguistic and psychological content float in unison. I fear that without the Display Thesis, both linguistic and psychological theory would be underconstrained. And (as a matter of personal taste) I would find questions about linguistic meaning far less interesting if they did not shed light on psychology. I also wonder about the stability of Charlow’s case against the Display Thesis: if ‘plans’ are psychological contents but are not propositional, the Propositional-Attitude Thesis is already abandoned; but that makes room to preserve the Display Thesis.

¹⁰In this slogan, a *modal sentence* is a sentence with a modal as its highest operator, while a *modalized blah* is a modal sentence in which the modal taking widest scope operates on a sentence of type *blah*—for example, a modalized declarative and a modalized imperative are both modal sentences. I presume that every modal sentence is declarative, so that a modalized imperative is a declarative modal sentence.

According to Geach (1958, 51), ‘[in] a language that entirely lacked proper imperatives[,] we could just use the plain future tense to give orders. [For example], ‘The patient will go down to the operating theater.’’. But if all directive modal sentences are modalized imperatives, this is wrong: the operand of ‘will’ is the indispensable imperative ‘The patient go down to the operating theater!’.

¹¹Contrast the ‘orthodox’ view that sentential-scope modals operate semantically on propositions (Lewis 1973, 1975, 1980a, 1986, 1.2; Kratzer 1977, 1981).

¹²The conception of entailment as support-preservation has been revived in more recent litera-

not ‘metaphysical’ but rather ‘perspectival’ operators. Gathering these theses, the predicted meaning of ‘You must pay the rent.’ is to express one’s perspective as supporting ‘You pay the rent!’—namely, to express one’s intention with content *R*.

A logical peculiarity unifies the phenomena of obligation, and contrasts them with *belief* and its affiliated language of declarative sentences and epistemic/metaphysical modality: the literature on obligative language labels this contrast the *Ross Paradox* (Ross 1944; compare Williams 1963).¹³

For belief, conjunction both introduces and eliminates classically, and disjunction introduces classically. I believe that I both posted this letter and drank up your wine just if I believe that I posted this letter and I believe that I drank up your wine. And if I believe that I posted this letter, I believe that I either posted this letter or drank up your wine. Similarly for declarative sentences. ‘You posted this letter and drank up my wine.’ is equivalent to the pair ‘You posted this letter.’ and ‘You drank up my wine.’ (the conjunction entails each conjunct, the conjuncts together entail the conjunction). And ‘You posted this letter.’ entails ‘Either you posted this letter or you drank up my wine.’. Similarly for epistemic/metaphysical uses of *must*. ‘You must have posted this letter and drank up my wine.’ is equivalent to the pair ‘You must have posted this letter.’ and ‘You must have drank up my wine.’ And ‘You must have posted this letter.’ entails ‘You must have posted this letter or drank up my wine.’.

In contrast, for intention, conjunction both introduces and eliminates classically, but disjunction does not introduce classically.¹⁴ If I intend to both post this

ture: see Kamp 1973, Veltman 1996, Gillies 2001, Yalcin 2007, inter alia.

This literature appears to presuppose support-preservation to be incompatible with the intensional-operator conception of modals—to my mind, incorrectly (Yli-Vakkuri 2013; Hellie 2015).

¹³According to Kamp (2001, 281), ‘Ross’s paradox [is] not only a reason to question the adequacy of this or that deontic-logical rule or axiom, but provide sufficient reason to drop the deontic-logical approach altogether’ (quoted as translated in Hansen 2006, 221n2). If so, casualties include Geach 1958; Chellas 1971, 124; Lewis 1979b; Parsons 2013. For a more extensive survey of the logical and linguistic impact, respectively, see Hansen 2006 and Starr 2011.

The subsequent literature on the Ross Paradox is extensive. A selection of notable recentish work: Lascarides and Asher 2003; Aloni 2007; Vranas 2008; Willer 2010; Kaufmann 2011; Portner forthcoming, 4.1; Starr 2011, 4.5; Charlow 2011, 4.4.6; Cariani 2013.

¹⁴Contrast the view that the Ross Paradox is essentially linguistic (‘the idea here is to introduce something like *disjunction at the level of speech acts*’: Charlow 2011, 146, original emphasis).

letter and drink up your wine just if I both intend to post this letter and drink up your wine. But if I intend to post this letter, I may well yet not intend to either post this letter or drink up your wine. Similarly for imperative sentences. Proceeding on a loose and intuitive understanding of imperative entailment, ‘Post this letter and drink up my wine!’ appears to entail both ‘Post this letter!’ and ‘Drink up my wine!’, and to be entailed by the pair of them: imperative conjunction both introduces and eliminates like declarative conjunction. But ‘Post this letter!’ appears not to entail ‘Post this letter or drink up my wine!’. Similarly for obligative *must*: ‘You must post this letter and drink up my wine.’ appears to entail both ‘You must post this letter.’ and ‘You must drink up my wine.’, and to be entailed by the pair of them. But ‘You must post this letter.’ does not entail ‘You must post this letter or drink up my wine.’.¹⁵

This chapter is divided roughly 60/40 between psychology (sections 2–5) and language (sections 6–8). The psychological part advances a technical and conceptual explanation of the psychological Ross Paradox. On the technical side (sections 2–3), I propose that while belief-contents are propositional (‘fundamentally’ so, in a sense to be developed in section 5.1), the procedural contents of intentions determine a *granulated proposition*—in effect, a pair of a proposition and a partition. While the partition allows procedure-conjunction to introduce and eliminate classically, it sets up a barrier to classical introduction for procedure-disjunction.¹⁶

On the conceptual side, I trace this contrast to the opposing polarities of belief and intention in *direction of rationalization* (sections 4–5): for belief, rationalization points ‘inward’; for intention, rationalization points ‘outward’. A well-known, related hypothesis similarly concerns the ‘normative order’ of psychol-

¹⁵I am inclined to agree with Charlow (2009, 31) that the Ross Paradox for *oughta* is not quite as compelling: ‘I oughta post this letter, so I oughta either post this letter or drink up all your wine.’ is better than the *must*/bare imperative versions; the same goes for *should*. This suggests that *oughta* and *should* cannot operate on procedures, but only on propositions.

Schroeder (2011) and Chrisman (2012) agree *oughta* operates sometimes on propositions; Schroeder but not Chrisman thinks *oughta* operates also on ‘actions’. Charlow’s observation supports Chrisman on the question about *oughta*; though on the general question whether deontic modals can operate on things other than *declarative* sentence semantic values, I side with Schroeder; though on the nuanced question whether they operate on *imperative* semantic values, Chrisman (449) expresses sympathy: compare Chrisman forthcoming, ch. 5.

¹⁶But procedure-disjunction does eliminate classically. Procedure-negation is similarly classical in elimination but not in introduction.

ogy: a *direction of fit* (Austin 1952; Anscombe 1963, section 32; Searle 1979) runs from ‘world-to-mind’ for belief but from ‘mind-to-world’ for intention. But in contrast (section 4), where *fit* connects the world to the mind, *rationalization* is psychology-internal, keyed to structural constraints on intelligibility (Heal 2003, Hellie 2011).

Rationalization in belief (section 5.1; Hellie 2014) starts in evidence about local physiological very recent history and flows toward belief about the more abstract and remote, along channels of expectation. The origination in evidence makes information stored in belief, such as that Rance posted a certain letter, *transferrable*: Fred preserves information for his future reference, or transmits it to Brent. But if Fred wants the information later or Brent wants it now, neither typically cares about why Fred wanted initially to get it: their interest is limited to the raw propositional information. In that sense, belief is ‘fundamentally’ propositional.

By contrast, rationalization in intention (section 5.2) originates in abstract long-term aims and/or open-ended projects and flows toward intention for the local physiological very near future, along channels of instrumentality: to walk to work, I walk halfway then the rest of the way; to walk halfway, I walk a quarter of the way then the rest of the way; and so on (Anscombe 1963, Thompson 2008). Because the rationalization of intention traces back to aims for which it is instrumental, intention has a constrained ‘mandate’. An intention to post a certain letter in service of paying the rent is rationalized only by the end of paying the rent; the intention to post the letter has no independent power to rationalize anything orthogonal to the end of paying the rent, such as drinking up all Brent’s wine. That excludes assigning significance in the content of intention to raw propositional information.

The closing 40% or so of the chapter brings obligative language into contact with this psychological story. Assigning granulated proposition-determining procedures as their contents, an abstract form of Ross-Paradoxicality percolates up to imperative sentences (section 6).¹⁷ The alignment between linguistic and psychological content subserves a unification of intention with natural-language

¹⁷Boolean connective semantic values are classical Boolean operations, while composition adjusts: Booleanizing imperatives, the Boolean operation acts on the propositions while the partitions are multiplied.

imperative endorsement and command (section 7): a ‘statics’ for imperative sentences grounding their endorsement in intentions, together with an analysis of sentential intensions (and the affiliated notions of *validity* and *entailment*) as reflecting patterns of support (rather than of truth),¹⁸ predicts natural-language Ross-Paradoxicality; a ‘dynamics’ for command models the distribution of intentions using imperative sentences in their canonical speech act. Obligative readings of *must*-sentences are brought into the fold by a combination of means: a familiar statics for declarative sentences and dynamics for their assertion; a grammatical analysis of obligative *must*-sentences as modalized imperatives; and an analysis of modals as intensional operators (section 8).¹⁹ This package predicts that ‘You pay the rent!’ and ‘You must pay the rent.’ have the same support-condition and are therefore equivalent; and that an assertion using the latter can have the same communicative meaning as a command using the former.

The phenomenon of *permission* appears only in concluding remarks:²⁰ I consider why the dual of obligative *must* is in a sense ‘elusive’ in natural language; and I propose a solution to Lewis’s (1979b) ‘problem about permission’.

¹⁸I therefore agree with Charlow (this volume) about the desirability of a ‘reorientation of semantic theorizing away from the aim of delivering a sentence’s truth-condition, toward the aim of modeling the state of competently accepting a sentence’.

¹⁹Cariani (2013) analyzes $\bigcirc\varphi$ as true just if (i) of the best salient options partitioning modal space, all entail $\llbracket\varphi\rrbracket$, (ii) no option entailing $\llbracket\varphi\rrbracket$ is ‘undesirable’, and (iii) the options do not cross-cut $\llbracket\varphi\rrbracket$. Cariani aims to solve, inter alia, the Ross Paradox and the ‘Professor Procrastinate’ puzzle (it is not true that he oughta accept an assignment, because he presumably won’t carry it out, but he oughta accept the assignment and carry it out).

Some worries. First, perhaps there is no Ross Paradox for *oughta*: see note 15. Second, Cariani complains his opponent ‘must resort [] to completely different resistance strategies for’ the Ross Paradox and the Professor Procrastinate puzzle. He also ‘refrain[s] from drawing any conclusions about the imperative version of the [Ross Paradox]’ (554n3). But there is no imperative version of the Professor Procrastinate puzzle, so a unified treatment is not warranted. Third, if it is salient that I play with the dog all afternoon and not ‘undesirable’ that I play with the dog but do not post this letter, then $\bigcirc(\text{post this letter})$ entails $\bigcirc(\text{post this letter} \vee \text{play with the dog all afternoon})$. Fourth, if it is salient that I drink up all your wine, then $\bigcirc(\text{post this letter})$ entails $\bigcirc((\text{post this letter} \wedge \text{drink up all my wine}) \vee (\text{post this letter} \wedge \text{don’t drink up all my wine}))$.

²⁰On a contrasting approach originated by Åqvist (1965) (compare Aloni 2007; Veltman 2008/2011; and Charlow’s 2009 and 2011, 4.4.6), an imperative has both *permission*- and *obligation*-content. I am sympathetic to the spirit behind this account, and I conjecture that its apparatus affords the same predictive resources as mine. But *permission* is a ‘positive’ phenomenon only socially, and not psychologically: postulating a distinctive variety of content for permission is theoretically odd, somewhat akin to doing so for *epistemic possibility*.

2 Concern and indifference in belief

A contrast between *concern* with a certain issue and *indifference* to it will be central to my case against propositional content for intention. So I want to be perfectly clear about what I mean by that contrast and why it matters to questions about content. I set up for that in this section by moving at a deliberate pace through potentially familiar ground regarding belief.

On the familiar system of representation, when Fred is in a certain psychological state, that fact can be factored into *content* and *attitude* (Lewis 1979a). For example, if Fred believes that goats eat cans and that lambs eat ivy but does not believe that ibexes eat amphorae, Fred bears the attitude *belief* toward the propositional contents *that goats eat cans* and *that lambs eat ivy* but not toward the propositional content *that ibexes eat amphorae*; if Fred wonders whether horses eat hay but not whether ibexes eat amphorae, Fred bears the attitude *wondering* toward the content of *whether horses eat hay* but not that of *whether ibexes eat amphorae* (whatever the contents of *wondering* may be).

The identification of propositions with sets of possible worlds (aka subsets of *modal space*) is familiar (Stalnaker 1970; Lewis 1970), if not completely uncontroversial. The proposition *that goats eat cans* is identified with the set of just those worlds in which goats eat cans; the proposition *that lambs eat ivy* with the set of just those worlds in which lambs eat ivy. A proposition is true or false depending whether it contains the actual world: horses do eat hay, so the set of worlds where they do includes actuality, so the proposition that they do is true; goats do not eat cans, so the set of worlds where they do excludes actuality, so the proposition that they do is false.

The content of Fred's total belief state can be identified with a certain set of worlds, Fred's *doxastic possibilities* (Hintikka 1962; Stalnaker 1984; Lewis 1986): the set containing w just if Fred, so to speak, takes w seriously as a candidate for actuality. In believing that goats eat cans, Fred has ruled out of candidacy for actuality all worlds in which goats do not eat cans; in his uncertainty whether horses eat hay, Fred takes seriously at least some worlds in which they do and some in which they do not. More generally, Fred believes that p just if his doxastic possibilities are a subset of the p -worlds, disbelieves that p just if his doxastic possibilities are a subset of the not- p -worlds, and is otherwise uncertain.

Belief is said to ‘aim at the truth’ (compare Frege 1892/1960). If belief aims at the *whole* truth—aims to get doxastic possibilities down to just the actual world—belief will never hit this target, because we never uncover the whole truth. By contrast, missing the target is easy: in ruling out all worlds in which goats do not eat cans, Fred has ruled out the actual world, and (unless he changes his mind) he will never get it back in his sights. As far as the whole truth is concerned, the best we can hope for is have not yet missed it.

Aiming at the whole truth might seem pointless: we can never succeed. And it might seem a fool’s errand: we can be reasonably sure of failing right out of the gate. Why bother? Fortunately, individual truths are much bigger targets. If Fred aimed at the truth that lambs eat ivy, he hit it: his doxastic possibilities are a subset of that proposition, so (unless he changes his mind) Fred will never lose his grip on that truth.

Does belief aim at the whole truth, or just at truths? —Let us not yet legislate, but instead distinguish *protean* belief, which aims at the whole truth, from *granular* belief, which aims at particular truths.

When aiming at the truth, in which direction do we point? —If Fred knew to aim at the truth that lambs eat ivy, he would already have it in hand as a truth, and would not need to aim at it. That at which we aim need not be fully specified, but only as the truth about a certain issue, whatever it may be: of the worlds where lambs eat ivy and the worlds where they do not, among which of the two actuality is included. Aiming at an individual truth is setting up a question to be answered; letting the arrow fly is answering it, by settling belief into one of its answers.

Setting up a question to be answered is *wondering* about it.²¹ If one wonders, one wonders *whether* goats eat cans, or *who* shot JR, or *where* Waldo went. Papering over certain complexities, whatever one wonders sets up a range of answers on which one might settle. A range of answers can be pictured as a bunch of lines of subdivision cutting through modal space: what the wonderer aims to settle is which of the subdivided lots contains the actual world. Or, more formally, as *partition of modal space*: a set of propositions—the *cells* of the partition—which, pairwise, do not overlap one another and which, collectively, subsume the entirety of modal space. For a ‘polar’ (yes/no) question like *whether goats eat cans*, the

²¹I retell an orthodox story: classic texts are Belnap 1976 and Groenendijk and Stokhof 1982, 1996.

partition is the pair set of a proposition (like *that goats eat cans*) representing the affirmative answer and another proposition, its *complement*, including exactly all other possible worlds, representing the negative answer. When p is a proposition, I notate the question whether p with $?p := \{p, \bar{p}\}$.

If Fred wonders whether horses eat hay and whether chickens bathe in dust, he wonders which on which side of the *horses do/don't eat hay*-line actuality lies, and also on which side of the *chickens do/don't bathe in dust*-line actuality lies. Assuming Fred thinks the answers are independent, these two questions subdivide modal space into four lots. More formally, when π and π' are partitions, their *product*, $\pi \times \pi'$, is the set containing the intersection of C and C' , whenever C is a cell of π and C' is a cell of π' .

The notion of a *subpartition* will be useful: intuitively, a subpartition draws every line drawn by its superpartition, and maybe draws more. More formally, ρ is a subpartition of π ($\rho \leq \pi$) just when for every $C \in \rho$, there is some $C' \in \pi$ for which $C \subseteq C'$. It is straightforward to see that the product of two partitions is always a subpartition of each.

The totality of the questions Fred has asked establishes a partition that is the product of the partitions giving the content of any question Fred has asked. The totality of Fred's granular belief can be represented as a pair of the propositional content of Fred's protean belief and the partitional content of the totality of Fred's questions. Say that a certain proposition p is *discriminable within* π just if p is the union set of some subset of π — p does not distinguish any pair of worlds occupying the same cell of (and therefore undistinguished by) π . Assuming Fred cannot answer a question he has not asked, the content of his protean belief will never distinguish worlds that are both within a single subdivided lot, and will therefore be discriminable within the partition-content of his questions.

The logics of protean and granular belief-content differ: protean content is classical, granular content Ross-Paradoxical. The story for protean belief is familiar: Boolean operations on propositions are Boolean set-theoretic operations; entailment is subset. So $p \wedge q = p \cap q$ (intersection), $p \vee q = p \cup q$ (union), and $\neg p = \bar{p}$ (complementation). And, allowing \vDash to stand for the content-entailment relation (as contrasted with the sentential-entailment relation \vdash , to be discussed later) $q_1, \dots, q_n \vDash p$ just if $q_1 \cap \dots \cap q_n \subseteq p$. It is straightforwardly seen that, on this analysis, the propositional connectives obey their classical introduction and

elimination rules.²²

The story for granular belief lacks a comparably orthodox treatment, but the following seems natural. Let some premiss-partitions entail a conclusion-partition $(\psi_1, \dots, \psi_n \vDash \pi)$ just if the product of the premisses is a subpartition of the conclusion: $\psi_1 \times \dots \times \psi_n \leq \pi$ (Groenendijk and Stokhof 1996). Let a ‘granulated proposition’ be a pair $\langle \pi, p \rangle$ such that p is discriminable within π . Some notation will be convenient: where $\langle X, Y \rangle$ is a granulated proposition, let $\pi\langle X, Y \rangle = X$ (the *partition-component*) and $p\langle X, Y \rangle = Y$ (the *proposition-component*). Then a Boolean operation O on granulated propositions P and Q may be stipulated to yield that granulated proposition with, as its partition-component, the product of the partition-components of P and Q , and with, as its proposition-component, the action of O on the proposition-components of P and Q : $P \wedge Q = \langle \pi(P) \times \pi(Q), p(P) \wedge p(Q) \rangle$, $P \vee Q = \langle \pi(P) \times \pi(Q), p(P) \vee p(Q) \rangle$, and $\neg P = \langle \pi(P), \neg p(P) \rangle$.

Granulated-proposition entailment $(Q_1, \dots, Q_n \vDash P)$ can then be identified with the both of (i) entailment by the partition-components of the premisses of the partition-component of the conclusion $(\pi(Q_1), \dots, \pi(Q_n) \vDash \pi(P))$ and (ii) entailment by the proposition-components of the premisses of the proposition-component of the conclusion $(p(Q_1), \dots, p(Q_n) \vDash p(P))$. Conjunction introduces and eliminates classically, and disjunction and negation eliminate classically. But disjunction does not introduce classically: in order that $\langle \pi, p \rangle \vDash \langle \pi \times \psi, p \cup q \rangle$, it must be that $\pi \leq \pi \times \psi$; but that requires $\pi = \pi \times \psi$ and therefore $\pi \leq \psi$. For example, let g be the proposition that goats eat cans and h be the proposition that ibexes eat amphorae. Then the disjunction of their ‘polar granulations’ $G = \langle ?g, g \rangle$ and $H = \langle ?h, h \rangle$ is $G \vee H = \langle ?g \times ?h, g \cup h \rangle = \langle \{g \cap h, g \cap \bar{h}, \bar{g} \cap h, \bar{g} \cap \bar{h}\}, g \cup h \rangle$. The partition component of $G \vee H$ is a subpartition of $\pi(G) = \{g, \bar{g}\}$, while the proposition component is a superset of $p(G) = g$: accordingly, neither G nor $G \vee H$ entails the other. Arguing similarly, nor does negation introduce classically either: Let $P, Q \vDash R$ and $P, Q \vDash \neg R$: then $\pi(P) \times \pi(Q) \leq \pi(R)$ and $p(P) \cap p(Q) = \emptyset$; while it follows from the latter that $p(P) \vDash \neg p(Q)$, all this is compatible with $\pi(P) \not\leq \pi(Q)$.

The metaphor of aiming, hitting, and missing can be discharged using talk of norms and standards, of success and failure, correctness and incorrectness, and related notions (Wedgwood 2002). The standard for belief is the actual world, in

²²Namely, \wedge -intro: $p, q \vDash p \wedge q$; \wedge -elim: $p \wedge q \vDash p, q$; \vee -intro: $p \vDash p \vee q$; \vee -elim: if $p \vDash r$ and $q \vDash r$ then $p \vee q \vDash r$; \neg -intro: if $p, q \vDash r$ and $p, q \vDash \neg r$, then $p \vDash \neg q$; \neg -elim: $\neg \neg p \vDash p$.

the following sense. The actual world fixes which propositions are true and which false: false propositions are incorrect, true propositions not incorrect. If one's doxastic possibilities are incorrect, one has failed at protean belief; if not, one has not (yet) failed at protean belief.

Granular belief may yet be successful. Fred has asked whether goats eat cans and answered it incorrectly. But he has also asked whether little lambs eat ivy—and answered correctly. The partition content of Fred's questions discriminates each of the set of the *goats eat cans*-worlds and the set of the *lambs eat ivy*-worlds from its complement. The content of Fred's protean belief is within the incorrect cell of the former, but within the non-incorrect cell of the latter. That makes Fred's granular belief whether goats eat cans a failure, but his granular belief whether lambs eat ivy a success.

More generally, suppose Fred's questions determine the partition π , with the actual world in cell $C_{@}$; and let Fred's protean beliefs have the content b , for b discriminable within π . Then consider some proposition p that is also discriminable within π . If b is a subset of p , we may say that Fred has the granular belief that p . That proposition p is not incorrect just if $C_{@}$ is a subset of p . That is so just if Fred's granular belief that p is a success; otherwise it is a failure. If b is not a subset of p , then Fred does not believe p : perhaps b is a subset of the complement of p , so Fred disbelieves p . Otherwise, b leaves it open whether p , and Fred is uncertain whether p ; moreover, when one is uncertain about a proposition discriminable within the questions one has asked, say one is *indecisive* about that proposition. The success/failure conditions of disbelief are obviously the opposite of those of belief.

What of those propositions p which are not discriminable within π ? For example, I supposed earlier that Fred had not asked whether ibexes eat amphorae. Perhaps that proposition is independent of the totality of questions Fred has asked: any cell in π contains worlds where ibexes eat amphorae and cells where they do not. Then Fred can have no opinion on the matter: Fred does not believe that ibexes eat amphorae, nor does he disbelieve it.

In that case, Fred has no opinion on the matter because nothing he cares to find out could settle it. But other cases of propositions not discriminable within the totality of Fred's questions are not that way. Consider the proposition that either goats eat cans or ibexes eat amphorae. Fred believes the stronger proposition

that goats eat cans. Does he thereby believe the weaker proposition? It is not clear whether intuition pronounces on the matter; perhaps theoretical motivations may be found for saying one, perhaps for saying the other; perhaps for giving one answer for some cases, the other for others. Fortunately, we have apparatus enough to allow us to say either. Fred protean-believes it because all doxastic worlds are members of the disjunctive proposition. As for granular belief, we could group the threefold division of *belief*, *disbelief*, and *indecision* under a category of *concern*, and set that supercategory against an exclusive category of *indifference*: and we could say that Fred has concern whether p just if p is discriminable within the content of his questions, and is otherwise indifferent. If we decided to use the apparatus in that way, we would rule Fred as indifferent regarding the disjunctive proposition.

How does the normative status of indecision (recall, uncertainty regarding a question one has asked) compare to that of indifference? Having asked a question but not answered it, one's situation is clearly neither (yet) a success nor a failure by the standards of belief. Arguably, though, indecision has a negative normative status along a different dimension: questions are 'to-be-answered'; having asked one, we are in a situation that is in a sense 'unsettled' until we settle it. Indifference, by contrast, is none of the above: neither successful nor failed, neither settled nor unsettled.

3 Concern and indifference in intention

Belief can perhaps be sensibly spoken of either as protean or as granular; intention, by contrast, is always granular, never protean.

So I claim. Intention is far less well-understood than belief, so I do not hope to ride the winds of orthodoxy, but will have to argue the point. Unfortunately, establishing a stable foundation requires drilling down far to hit bedrock: familiar formally-driven stories lack phenomenological plausibility; familiar phenomenologically-driven stories lack formal rigor. So rather than with tools or doctrine, I begin by collecting data—in particular, about the bearing of various worldly circumstances on the success or failure of intention. I will then run these through the apparatus of the previous section to generate my thesis that the content of intention is always granular.

For data-gathering, I exploit Lewis's (1980b) device of a 'questionnaire'—to my mind, and I hope you will agree, that it shows we have some 'very firm and definite opinions' about the success and failure of intention.

First question. On Monday, you are strolling along on the way to work, preoccupied with thoughts about deontic modals. Your attention is briefly drawn to a stone skittering along the sidewalk, which you appear to have just kicked. You pay it little mind and continue on your way. Consider the process-type *kicking a stone*. Does your having just undergone a process of this type constitute your success or failure at anything?

Answer. Of course not. Whether you kick any stones is a matter of complete indifference to you. Had you not kicked any stones at all on the way to work, that would not have constituted your success or failure at anything either.

Second question. On Tuesday, it is as on Monday, except that you have set out with a mindfulness project requiring not kicking any stones—but, again preoccupied, you kick a stone. Does your having just undergone a process of the type *kicking a stone* constitute your success or failure at anything this time?

Answer. Yes. You have failed at your intention to avoid kicking any stones.

Third question. On Wednesday, vexed at yesterday's failure, you have set yourself the intention of giving a good hard kick to some stone. Spotting one, you kick it. Does your having just undergone a process of the type *kicking a stone* constitute your success or failure at anything this time?

Answer. Yes. You have succeeded at your intention to kick a stone.

Last question. On Thursday, when you arrive at work, you set yourself the intention of either reading the new article by N or the new article by M that day. You manage to steal enough time to read the new article by N over the course of the day. Does your having undergone a process of the type *reading the new article by N* constitute your success or failure at anything?

Answer. Yes. You have succeeded at your intention to read either the new article by N or the new article by M. You would have equally well succeeded

had you instead read the new article by M. And you would have succeeded at that intention had you been able to read both.

If some of this does not seem obvious, please reconsider; if it all does, splendid! —Let us use it as evidence for what follows.

It is not the answers to the questionnaire by themselves that establish the content of intention to be never protean. For that matter, they do not even establish the content of intention to be ever granular. The obvious proposal for protean content is this: the content of intention includes, on Monday, both worlds where one kicks stones and worlds where one does not; on Tuesday, only worlds where one does not kick stones; on Wednesday, only worlds where one does kick stones; and on Thursday, only worlds where one reads either the article by N or the article by M (or both).

But a case for granular content emerges in working through the following exercise:

Exercise. On Friday, you are strolling along on the way to work, self-possessed, confident in your way of being. You have recognized that true mindfulness consists in equanimity about the trivial. Regarding the vexatious kicking of stones, you have set yourself the intention to either kick (one or more) stones or not kick (any) stones. As it happens, you kick a stone. Does your having just undergone a process of the type *kicking a stone* constitute your success or failure at anything this time?

Answer. Yes. You have succeeded at your intention to either kick stones or not kick stones. But you would have equally well succeeded had you not kicked any stones.

After all, recall that on Thursday, one has resolved to do at least one of two things: read the article by N; read the article by M. So long as one does at least one of them, one will succeed, and will not fail. One will fail only if one does neither.

To generalize, whenever one intends to act in at least one of two ways, one fails only by doing neither, succeeds by doing either (or both). It should not matter whether the thing in question is ‘positive’ or ‘negative’. If one resolves either to read the article by N or drink five or more cups of coffee,

one fails only if one neither reads the article nor drinks five or more cups of coffee, otherwise succeeds. And if one resolves either to read the article by N or *not* drink five or more cups of coffee, one fails only if one does not read the article and does drink five or more cups of coffee, otherwise succeeds.

Now, when an intention is to satisfy at least one of two properties, *one of them the negation of the other*, it should still be that one fails only by doing neither, succeeds by doing either (or both). Why should the content make a difference? Normative psychology does not ordinarily involve that kind of exception, so best not to postulate it now, either. Now of course, when one intends to act at least by kicking a stone or not kicking a stone, one cannot do neither and cannot do both. Because the only remaining alternatives are *do the one*—kick a stone—or *do the other*—do not kick a stone—one must do exactly one of those. Either way, one succeeds.

So sometimes one's intentions are guaranteed to succeed. I am inclined to see this consequence as an occasion for rejoicing, rather than resistance.

Intention, like belief, can involve a distinction between concern with an issue—the prospect of success or failure, depending on how the world meshes with the content of one's attitude—and indifference, in which the answer to the question cannot imbue the relevant aspect of one's mental state with either success or failure.

Indeed, distinguishing concern and indifference is at least as warranted for intention as for belief. Contrast Monday and Friday. on Monday, whether one kicks a stone is a matter of no significance for the success or failure of any intention: one is *indifferent*. But on Friday, there is an intention such that, if one kicks a stone, it will succeed, and if one does not kick a stone, it will succeed: one's attitude is *equanimity*.

The distinction between indifference and equanimity, like the distinction in belief between indifference and indecision, cannot be represented with protean content. On the earlier attempt at a protean characterization, the Monday content contained *kick a stone*-worlds and *don't kick a stone*-worlds; the Tuesday content contained only *don't kick a stone*-worlds; the Wednesday content contained only *kick a stone*-worlds. The Friday content should not distinguish *kick a stone* and *don't kick a stone*-worlds; and we surely do not want it do do so by excluding all

worlds; so the Friday content is like the Monday content in containing both kinds of world.

Saying what the content is does not yet settle the bearing of this on the success/failure-status of one's intention. Presumably false content makes for failure. Perhaps true content makes for success (as with granular belief). Then one's intention succeeds no matter what on Friday, desirably—but then it also succeeds no matter what on Monday, undesirably. Or perhaps true content makes for mere non-failure (as with protean belief). Then one's intention neither fails nor succeeds no matter what on Monday, desirably—but then it also neither fails nor succeeds no matter what on Friday, undesirably.

So protean intention cannot make enough distinctions. By contrast, this is no problem for granular intention: Monday does not partition *kick a stone* from *don't kick a stone*-worlds, but Tuesday, Wednesday, and Friday do; Tuesday, in addition, designates the *don't kick a stone*-answer, by involving a proposition that is a subset of the *don't kick a stone*-cell; Wednesday designates the *kick a stone*-answer, by involving a proposition that is a subset of the *kick a stone*-cell; and Friday designates both answers, by involving a proposition overlapping both cells.²³

4 Direction of rationalization

Belief is arguably both sensibly spoken of as protean and as granular; intention is at least sensibly spoken of as granular. Can intention be sensibly spoken of as protean? No: this section and the next trace the contrast between intention and belief to their opposing polarity in what I call *direction of rationalization*.

I will sidle up on this notion by reflecting on the challenges faced by the venerable if obscure notion of *direction of fit*. In the tradition, belief and intention (often: 'desire') are said to display oppositely polarized directions of fit with regard to the world: in some sense duly capturing their fundamental normative contrast, belief is to fit the world, whereas the world is to fit intention—where 'A is to fit B' means B somehow exerts normative pressure on A to fit B. Alluring, to be sure. But what on earth could it mean?

²³I explicate *designation* in terms of what makes for *success*, in contrast with in terms of what is *permitted*. In that sense, my view represents disjunction as 'alternative-presenting' in contrast with 'choice-offering': Rescher and Robison 1964, 179; Åqvist 1965.

For ‘fit’, I imagine truth of content is just enough. Substituting, beliefs are often true thanks to the world’s exerting normative pressure on them, while intentions are often true thanks to their exerting normative pressure on the world. This is not so plausible: howso, the world is ‘normatively pressured’ to make intentions true (Humberstone 1992; Frost 2014)? If an errant bus undermines my intention to walk to work, that may be problematic in many ways—but I doubt one of them will be the world’s regret at violating its half of the bargain struck in my forming my intention.

A popular fix has been to exchange *normative* pressure for *causal* pressure (Smith 1987; Velleman 2000; Setiya 2007). Beliefs are often true thanks to the world’s exerting causal pressure on them, while intentions are often true thanks to their exerting causal pressure on the world. Of course the world impinges on our sensory organs, and in this, we acquire evidence, which then ramifies in further, more abstract beliefs; when these do not go too far from evidence, they are often true. And of course, we implement our more abstract intentions through intentional motions of our bodies, which exert gentle and local causal pressure on the world; when we planned well and matters beyond our locality do not impede us, our intentions often come true. But the body is where causality *ends*, for belief, and *starts*, for intention. Characterizing the post/pre-bodily, psychological part of the story as just more causation does not square comfortably with ordinary understanding of psychology (is ‘phenomenologically off-key’). But the point of discussing direction of fit is to shed light on psychology: so this fix changes the subject unhelpfully.

This suggests that the remedy is to constrain our ambition: we need to limit the world’s exposure to normativity. We can continue to distinguish belief and intention in direction of normative pressure if the norm of *truth* is exchanged for a more psychology-internal, structural normative constraint based in a notion of *rationalization* (Hellie 2011, 2014). If psychology and the world are isolated, this will overshoot, kicking the world out of the story. Fortunately, psychology and the world overlap at the body: for belief, it is where causality ends, but also where rationalization begins; for intention, it is where causality begins, but also where rationalization ends. I will add detail to this sketch after introducing ‘rationalization’.

The root structural psychology-internal norm is *intelligibility*.²⁴ For an example of its absence, suppose Fred asserts ‘it is false that either goats eat cans or horses eat hay’ and then asserts ‘goats eat cans’. How is Fred thinking of the world? Without further information, I am in the dark.

This case does not distinguish the internal norm of intelligibility from the external norm of truth: Fred appears to have contradicted himself, so his statements at face value cannot possibly be true. But intelligibility and truth can be distinguished by means of *Moore’s Paradox*. Suppose we ask Fred whether goats eat cans, and he emphatically replies ‘yes, goats do indeed eat cans’. One of us did not catch his answer, and follows up: ‘sorry, was it your opinion that goats eat cans?’ Fred answers ‘Now you are asking me my opinion—I thought this was supposed to be about goats. No, I am not of that opinion.’ Here too it is hard to make sense of what is going on with Fred. But this is not because it is impossible for goats to eat cans though Fred lacks any opinion on the matter. Instead, it is because there is no intelligible perspective which pictures goats as eating cans without lucid recognition under at least some mode of presentation that it does so.

Why think intelligibility is a ‘norm’? —Not because intelligibility is guaranteed to be beneficial (contrast Parfit 1984, section 5; Harman 1995, 1.2) or is connected to anything else of value or to any other sort of value. Rather, Fred’s intelligibility is a boundary condition on our using psychological categories to describe or explain him. No intelligibility, no location in the ‘space of reasons’ at all: intelligibility is a ‘necessary precondition on the possibility of normativity’. If that is not enough for counting as a norm, but only as a ‘norm*’, I will concede the point—but in the interest of saving ink and trees, I leave the ‘*’ implicit.

Intelligibility provides a basis for defining a variety of ‘normative pressure’ exerted by mental states on one another: this *rationalization*. To illustrate, Sam thinks her pet rabbit, Rupert, has escaped. Why? Sam replies ‘if Rupert is in the house, I always know which room he is in; and I don’t know which room Rupert is in’—modus tollens. The first explanans is the state supporting the conditional: a pattern of expectation that is notably ‘stable’ in Sam’s mental life. The second is the rejection of the consequent, a temporary condition of uncertainty in Rupert’s

²⁴Compare ‘If we say ‘it does not make sense for this man to say he did this for no particular reason’ we are not ‘excluding a form of words from the language’; we are saying ‘we cannot understand such a man’ ’ (Anscombe 1963, section 18).

whereabouts. Having accepted these explanantes, we would find Sam unintelligible if she did not think Rupert had escaped. Unintelligibility is a ‘boundary condition’ on psychological characterization; knowing all this, we therefore have no choice but to accept the explanandum (that Sam thinks Rupert has escaped). So what we learned explains the belief; the bankshot off of intelligibility is what makes the explanation a *rationalization*. In general, then, some states rationalize another when the former explain the latter by way of its being unintelligible to have the former without the latter. Because rationalization is an explanatory relation, it inherits all the directedness of explanation.

I propose that the opposite polarity of belief and intention pertains to the direction of rationalization. For belief, rationalization flows, in a sense, ‘inward’; for intention, it flows ‘outward’. But inward or outward toward what and from what?

To see the answer, suppose Brent is off to Lower Slobbovia. He thinks Sam knows how one will eat in Lower Slobbovia, and wants to know how he will eat, so he asks her. Sam replies ‘you will eat terribly in Lower Slobbovia’. So: should Brent *believe* he will eat terribly in Lower Slobbovia? And should he *intend* to eat terribly in Lower Slobbovia?

Very plausibly, learning Sam’s answer rationalizes Brent believing he will eat terribly in Lower Slobbovia. Brent thinks Sam knows how one will eat in Lower Slobbovia, and has learned that Sam believes he will eat terribly in Lower Slobbovia. I am inclined to say that if *A* thinks *B* knows the answer to a certain question, and *A* thinks *B*’s answer to the question is *P*, that *A* thinks *P* is true—on pain of unintelligibility (‘Rance knows whether goats eat cans and thinks they don’t—but I don’t know whether goats eat cans’). So it is unintelligible for Brent not to believe he will eat terribly in Lower Slobbovia.

But Sam’s answer does not rationalize Brent’s *intending* to eat terribly in Lower Slobbovia. Brent might very well, knowing this, strive with great energy—albeit pessimistically—to eat well in Lower Slobbovia. Perhaps intention in some sense requires belief. But only for the completely acquiescent/megalomaniacal fatalist is belief always accompanied by intention!

To explain the contrast, I will quickly rehearse a model of ‘rational architecture’ I have defended elsewhere (Hellie 2011, 1.1; 2014, sections 2–3). Belief begins in *evidence*: evidence must be true, must be believed, consists of facts

broadly about ‘perception’, and is given under the ‘mode of presentation’ of perception (Lewis 1972/1997). So, for example, if I see a dagger before me, my evidence is some true proposition about my ‘visual condition’—perhaps that I see a dagger before me, perhaps that my retina is stimulated daggerishly, perhaps that my brain is agitated daggerishly. Evidence must be true and must be believed, but there remains plenty of room for falsehood in belief. The perceptual ‘mode of presentation’ typically must be ‘recoded’ into something less evanescent and more ‘conceptualized’ (Hellie 2011, 3.2–3); even then, my evidence concerns just my perceived surroundings up to the present, so my full rich set of beliefs requires extensive ‘amplification’ by my pattern of expectations.²⁵

(If I lucidly dream of a dagger before me, my evidence is again some true proposition about my ‘visual condition’—perhaps that I am imagining a dagger, or that my visual cortex is agitated daggerishly; either way, it is certainly possible to have mistaken ‘background beliefs’ about my condition and therefore ‘recode’ the perceptual mode of presentation in a way that conflicts with my actual evidence: in that case, I wind up with inconsistent beliefs, and am not fully intelligible: Hellie 2011, section 4, Hellie 2014, 4.1.)

Because evidence must be believed, evidence pushes belief around. Our expectations extract a great deal more from evidence than the content of evidence strictly entails; still, this leveraging move is not without risk, in that evidence might always surprise us and wind up contradicting what we expected: when that happens, we lapse from full intelligibility until consistency is restored (making ‘belief willy nilly’ generally to be avoided). In this sense, rationalization in belief flows ‘from evidence’.

That evidence can only have true content is good fortune, given that it must be believed. But this also gives evidence a foot on both sides of the fence: evidence is ‘in the world’, and also ‘in the mind’. In that sense, when rationalization in belief flows ‘from evidence’, it flows ‘inward’; and that is the good sense to be made of the claim that belief ‘fits the world’.

One’s evidence at a time pertains to the stretch of history ending at the present moment. But one’s intentions pertain to the stretch of history beginning at the

²⁵A congenial theory of the mechanics of expectation exploits the sort of construction behind ‘presumably’ in Veltman 1996 and exploited by Yalcin (this volume) in the analysis of *should*; I conjecture that ‘ranking functions’ (Spohn 1988; Huber 2007) afford an interchangeable apparatus.

present moment. So intention cannot radiate from evidence: evidence constrains intention at best in partnership with expectation. So rationalization for intention does not (for that reason anyway) flow ‘inward’.

Indeed, it flows ‘outward’. The paradigmatic rationalization in intention is from a longer-term, more ambitious ‘governing intention’ to a structure of shorter-term, less ambitious ‘implementing intentions’ undertaken as instrumental to executing the governing intention (Anscombe 1963, Thompson 2008). Suppose I set myself to walk to the office. I have to implement this intention somehow: I can’t just blink my eyes and it happens. In particular, I have to walk the first half of the way, and then walk the second half of the way. But to do all this *intentionally*, I need intentions to do both of those things. And they too need to be implemented: to do the first, I have to walk the first quarter of the way, then the second quarter of the way. And so on. At some point I am implementing my intention to walk to the corner by walking halfway to the corner, implementing that by taking three steps, implementing that by taking one step, . . . implementing that by exerting all sorts of micro-forces throughout my body

More generally, long-term ambitious intentions rationalize shorter-term less ambitious intentions, and so on—eventually arriving at motor coordination in the here and now. Now, motor coordination in the here and now is at least somewhat like evidence: each pertains to the here and now, and to matters impinging pretty closely on physiology. If they differ, it is in that motor coordination is about moving, while evidence is about sensing. And maybe moving and sensing, at the level of physiology, are not really dissociable: perhaps all there is is the ‘sensorimotor’. So either way, rationalization of belief flows ‘inward’ from evidence about the sensory (or perhaps the sensorimotor), while rationalization of intention flows ‘outward’ toward intention for the motor (or perhaps the sensorimotor).

So, in sum, belief and intention are oppositely polarized in their direction of rationalization: rationalization in belief flows inward from the physiological here and now qua evidence; rationalization in intention flows outward toward the physiological here and now qua near-momentary intentional action.

5 Polarity and content

5.1 Inward-directedness and protean content

The job assigned to granular belief was giving a point to inquiry: without questions to answer, success could only amount to omniscience, which we will never reach, and failure would be absolute and permanent with the smallest initial mistake.

Why is that not a compelling reason to jettison protean belief entirely? Because belief is inward-directed, it has a kind of ‘objectivity’: my beliefs are not just for me, nor yours just for you; rather, mine can be for you and yours for me. The granulation Fred imposes on his beliefs, conversely, lacks ‘objectivity’: it is an idiosyncratic matter of how Fred packages his information for himself, indispensable to Fred but potentially of no interest to anyone else. Fancy words? This ‘objectivity’ is easy enough to operationalize: the most salient symptom is that accepting testimony is rationalizable.

I therefore argue as follows. First: because belief is inward-directed, accepting testimony is rationalizable. Second: because accepting testimony is rationalizable, belief has protean content.

I do not expect the first premiss to be controversial. If Brent thinks Fred’s beliefs about animal husbandry have lost their moorings in evidence, it would be unintelligible for Brent to accept anything Fred says on the matter. Conversely: even if Rance thinks Sam’s beliefs about the culinary amplify her evidence considerably, that is not in itself a reason not to accept what she says. After all, it may be that Rance’s own beliefs amplify evidence in more-or-less the same pattern as Sam’s. If so, Sam’s beliefs about the culinary can be expected to be roughly the same as Rance’s would be if he had her evidence. So if this is Rance’s view, Rance will expect Sam’s testimony to just put on display reasonable expectations on the basis of some (in absentia) evidence. If so, when Sam asserts that p , and Rance is concerned whether p , for Rance to remain indecisive whether p would be either just as hard to make sense of as ignoring evidence on a matter of concern or just as hard to make sense of as skepticism about his own reasonable expectations. So without inward-directedness, accepting testimony would not be rationalizable, while with inward-directedness, what is not rationalizable is indecisiveness re-

garding at least favorable cases of testimony over matters of concern. So in that sense, it is because of the inward-directedness of belief that accepting testimony is rationalizable.

Now to the second premiss. The idea here is that sometimes when accommodating your issues would be a barrier to my acquiring information from you, I can still do it; but then I can gather your information without accommodating your issues; and if so, I think of your information as protean belief.

Suppose each of Rance and Brent was indifferent whether Amber would be at the party when he arrived, and indifferent whether Bruno would be there. But while individually, neither Amber nor Bruno was a matter of concern to either Rance or Brent, Amber and Bruno collectively were a matter of concern to each—though in different ways. Rance was worried that both Amber and Bruno might be there, while Brent was worried that Amber might be there without Bruno. The issue of concern to each, then, was a polar question: for Rance, $?(a \cap b)$; for Brent, $?(a \cap \bar{b})$. Upon arrival, each was happy to answer his question in the negative—Rance believing $\overline{a \cap b}$, Brent $a \cap \bar{b}$ —and gave it no further thought.

Some time later, Sam, who does not know Bruno, is wondering whether Amber was at the party: of concern to Sam is $?a$, but not $?b$. Knowing Rance and Brent were there, she asks each whether Amber was there. Rance replies ‘I don’t know, but Amber wasn’t there with Bruno’; Brent replies ‘I don’t know, but Amber wasn’t there without Bruno’. Sam concludes that Amber was not at the party—believing \bar{a} and thereby settling her question in the negative.

Suppose now for reductio that belief is essentially granular. In that case, the content of Rance’s posterior belief entails neither $\langle ?a, W \rangle$ nor $\langle ?(a \cap \bar{b}), W \rangle$, the content of Brent’s posterior belief entails neither $\langle ?a, W \rangle$ nor $\langle ?(a \cap b), W \rangle$, and the content of Sam’s posterior belief entails neither $\langle ?(a \cap b), W \rangle$ nor $\langle ?(a \cap \bar{b}), W \rangle$.

Grant me that if an assertion communicates a content X , for the speaker’s posterior belief-content S and the audience’s posterior belief-content A , both $S \vDash X$ and $A \vDash X$. So whatever was communicated to Sam by either Rance or Brent entails neither $\langle ?a, W \rangle$, nor $\langle ?(a \cap b), W \rangle$, nor $\langle ?(a \cap \bar{b}), W \rangle$. But the only other granulated proposition even conceivably in the ballpark for what was communicated is the trivial granulated proposition $\mathbf{1} := \langle \{W\}, W \rangle$ —which could not have settled anything for Sam, as the communications evidently did.

So, I conclude, belief is not essentially granular. Rance's assertion communicated his protean belief that $\overline{a \cap b}$, and Brent's communicated $\overline{a \cap \overline{b}}$, with the consequence that Sam's posterior protean belief entails \overline{a} —and her question $?a$ is settled in the negative.

5.2 Outward-directedness and granular content

The direction of rationalization of intention is oppositely polarized, in the sense that what rationalizes a given intention is (ordinarily,²⁶ and in significant part) the 'governing' intention it implements.

For this reason, intentions are not in any reasonable sense transferrable. Suppose Rachel intends to hang a picture Joan thinks is ugly. Joan need not intend to hang that picture herself; or support Rachel in doing so; or even stay out of the way: Joan might quite reasonably try, perhaps, to *foil* Rachel. The intention itself does not transfer. Each of Joan and Rachel might be confident their side will prevail, with Rachel believing she will hang the picture and Joan believing Rachel will not hang the picture. Any belief associated with the intention of either need not transfer either. The reason for ascribing protean content to belief does not apply to intention.

Conversely, the business of intention—implementing its governing intention—requires its content to be granular. A given intention has a limited mandate: it is to carry out its role in the plan for implementing the governing intention, and then expire.

For example, Sam intends to do her job today, and to do that, she has to get to the office. So she sets out after breakfast with an intention to walk to work at the office. She plans, and therefore intends, to do this by first walking halfway and then the rest of the way; she plans to walk halfway by first walking the first quarter of the way and then walking the second quarter of the way; and so forth. The intention to walk the second quarter of the way is rationalized by the intention to walk halfway. Its mandate is just to do its part in making Sam walk halfway, and

²⁶The hedge is to accommodate the rationalization of 'highest' intention: it has to top out at some stage, so at that stage intention is either self-rationalizing or arational or rationalized by a governing non-intention. The status of the 'ultimate sources of motivation' is orthogonal to the planning relations that are the focus of my argument.

then expire. While exercising that mandate requires the creation of other intentions it in turn rationalizes, the intentions it creates are rationalized only through the contribution they make to attaining the goal of walking the second quarter of the way.

So in particular, the intention to walk the second quarter of the way could rationalize an intention to walk the third eighth of the way. But it could *not* rationalize a disjunctive intention to either walk the third eighth of the way or empty the bank account and go on a drinking spree after work. That disjunctive intention succeeds unless one neither walks the third eighth of the way or empties the bank account and goes on a drinking spree after work. In particular, it succeeds if one empties the bank account and goes on a drinking spree after work. An intention rationalizes whatever might be appropriate for realizing its success-condition; so the disjunctive intention rationalizes, if appropriate, emptying the bank account and going on a drinking spree. So if the intention to walk the second quarter of the way rationalizes the disjunctive intention, it can (derivatively) rationalize emptying the bank account and going on a drinking spree (just so long as it also rationalizes walking the third eighth of the way). But emptying the bank account and going on a drinking spree makes no contribution whatever to walking to work; so because the intention to walk to work is what governs the intention to walk the second quarter of the way, the rationalization bestowed on the latter is restricted to its contribution to the aim of walking to work. If it ‘goes rogue’, so to speak, it does so without any rationalization, and therefore without any power to rationalize the wacky splinter plan.

So a disjunctive intention is not in general rationalized by its disjunct-intentions. But a conjunctive intention does seem to rationalize its conjunct-intentions: if I intend to have breakfast and then go back to sleep, I intend to have breakfast and I intend to go back to sleep (after eating breakfast).

Disjunction-introduction, understood as a rule of rationalization for the contents of intentions, seems to be invalid; but conjunction-elimination is valid. That means that rationalization by the contents of intention sometimes traces decreasing strength of protean content, but not always. If not, the contents of intentions cannot be protean.

Whatever these contents may be—I call them *procedures*—it is plausible that they at least determine a partition over something. As above, entailment over gran-

ular propositions validates classical conjunction-introduction and -elimination, and disjunction- and negation-elimination, but neither disjunction- nor negation-introduction. Intentions, we have seen, validate conjunction-elimination but not disjunction-introduction: and they seem to conform to the remaining patterns as well. Conjunct-intentions do seem to rationalize their conjunctive intention: if I intend to work and I intend to whistle, I intend to whistle and work (conjunction-introduction). If I intend to not fail to have breakfast, I intend to have breakfast (negation-elimination). Something like disjunction-elimination seems valid: to get downtown, you gotta ride a bus; to get uptown, you gotta ride a bus; so to get uptown or downtown, you gotta ride a bus. But negation-introduction seems invalid. The case against disjunction-introduction works equally well: the intention to go to work is incompatible with the intention to neither go to work nor to have a drinking spree; but that is so without the intention to go to work requiring an intention to either go to work or have a drinking spree.

A range of further phenomena militate for ascribing still further varieties of structure to the procedural contents of intentions, which provide affordances for further intricacies of natural language.²⁷ I set these aside for treatment elsewhere.

6 Imperative semantics

The conclusion of the foregoing discussion is that belief and intention contrast in that belief is optionally (and perhaps fundamentally) protean, while intention is mandatorily granular. This mandates Ross-Paradoxical behavior for the content of intention while permitting classical behavior for the (perhaps fundamental) content of belief.

I now transition from the psychology to the language of obligation. In these concluding sections, I will describe a series of increasingly complex languages. Collectively, these (i) show how agents conforming to my psychological hypotheses might perform speech acts to put (individual) beliefs and (general-will) in-

²⁷Three examples. *Delayed triggering*: wait five minutes, then stir; remove on July 21; leave at sundown; smile when they walk in; fold in the whipped cream, then put it in the oven. *Condition-alization*: if it is *A*, do α ; if it is *B*, do β ; if you can't tell, do γ . *Delegation*: the contrast between Rance's intention to pay the rent, which is essentially 'de se' or subjectless, and our intention for Rance to pay the rent, which adds a subject.

tentions on display, in order to collect individual beliefs and parcel out collective intentions; and (ii) illustrate meanings within such a practice for expressions designed to resemble ordinary-English Boolean connectives and modal operators. These languages conform, in various nuanced ways, to the phenomena of obligation with which I opened the chapter: the connection between the language and psychology of obligation; certain commonalities and distinctions between command uses of imperatives and obligative uses of *must*, including the Ross Paradox; the elusiveness of their ‘duals’, permission and permissive *can/could*.

The initial language L_0 contains simple imperative sentences, with *procedure*-type semantic values, and classical Boolean connectives: with its natural entailment relation, the \models of section 2, L_0 is Ross-Paradoxical. Next, L_1 introduces subject-predicate structure to imperatives, allowing connectives to take scope: narrow-scope disjunction ‘softens’ Ross-Paradoxicality, as in English.

In section 7, L_2 wraps a ‘language game’ around L_1 : command uses of imperatives distribute an initially centralized stock of general-will intentions to appropriate participants in a conversation. With context now playing a significant role, L_2 modernizes entailment as the ‘diagonal’ notion of preservation of endorsement in all contexts.

In section 8, L_3 extends this language game, with declarative sentences used in assertion to centralize a distributed stock of beliefs. The final stage language L_4 adds modal operators: these are affiliated with the theory of entailment, and accordingly behave either *epistemically*, when operating on declaratives, or *obligatively*, when operating on imperatives.

Stipulations for L_0

In the elementary language L_0 , as well as in its extensions to follow, Boolean connectives are semantically ‘classical’ in that they have Boolean set-theoretic operators as their semantic values: as usual, ‘the semantic value of ξ ’ is abbreviated $\llbracket \xi \rrbracket$; ‘the semantic value of ξ in context c ’, as $\llbracket \xi \rrbracket^c$.²⁸ The non-classical Ross-Paradoxical behavior is generated by assigning procedures as imperative semantic values, in combination with a composition rule directing the action of a

²⁸The treatment of compositionality falls short of full rigor, but without I hope undermining my central aims.

connective to the proposition-components of its operands and composing their partition-components by multiplication.

Syntax *Lexicon*: (i) a stock of elemental imperative sentences A, B , etc.; (ii) Boolean connectives \wedge, \vee, \neg .

Composition: if $\alpha_1, \dots, \alpha_n$ are imperative sentences and C is an n -place Boolean connective, $C(\alpha_1, \dots, \alpha_n)$ is an imperative sentence.

Semantics *Ontology*: Let \mathcal{PR} be the set of all *procedures*; and for present purposes, let a procedure be a granulated proposition: recall, a partition paired with a proposition it discriminates (so $\mathcal{PR} := \{\langle \pi, p \rangle : \pi \text{ a partition of } W \wedge (\exists S \subseteq \pi)(p = \bigcup S)\}$); for $\Pi = \langle X, Y \rangle$, let $\pi(\Pi) = X$ and $p(\Pi) = Y$.

Lexicon: (i) When α is an elemental imperative, for some $p \subseteq W$, $\llbracket \alpha \rrbracket = \langle \{p, \bar{p}\}, p \rangle$; (ii) $\llbracket \wedge \rrbracket = \cap$, $\llbracket \vee \rrbracket = \cup$, $\llbracket \neg \rrbracket = \lambda x \bar{x}$.²⁹

Composition: $\llbracket C(\alpha_1, \dots, \alpha_n) \rrbracket = \langle \pi(\llbracket \alpha_1 \rrbracket) \times \dots \times \pi(\llbracket \alpha_n \rrbracket), \llbracket C \rrbracket(p(\llbracket \alpha_1 \rrbracket), \dots, p(\llbracket \alpha_n \rrbracket)) \rangle$.

Entailment If $\beta_1, \dots, \beta_n, \alpha$ are sentences of L_0 , then $\beta_1, \dots, \beta_n \vdash \alpha$ just if $\llbracket \beta_1 \rrbracket, \dots, \llbracket \beta_n \rrbracket \vDash \llbracket \alpha \rrbracket$.

Discussion

It is easy to see that classical conjunction-introduction and -elimination, and negation- and disjunction-elimination are valid, but negation- and disjunction-introduction are invalid: L_0 is, in effect, a projection into the syntactic of the procedure-contents of intention, with an entailment relation isomorphizing content-entailment. (Example: let $\llbracket A \rrbracket = \langle \{q, \bar{q}\}, q \rangle$ and $\llbracket B \rrbracket = \langle \{r, \bar{r}\}, r \rangle$; then $\llbracket A \vee B \rrbracket = \langle \{q, \bar{q}\} \times \{r, \bar{r}\}, q \cup r \rangle$; the partition component of $\llbracket A \vee B \rrbracket$ is then $\{q \cap r, q \cap \bar{r}, \bar{q} \cap r, \bar{q} \cap \bar{r}\} \leq \{q, \bar{q}\}$, while the proposition component is $q \cup r \supseteq q$, so whenever q and r are orthogonal, neither $A \vdash A \vee B$ nor $A \vee B \vdash A$.)³⁰

²⁹I am not actually so confident that negation can take wide scope over imperatives: ‘Fred don’t open the door!’/‘Fred open the door and Brent take out the trash!’/?‘Don’t (Fred open the door and Brent take out the trash)!’.

³⁰Starr’s otherwise congenial (if intricate) approach (Starr 2011, 4.5) unfortunately predicts that it is always preferable to carry out both disjuncts (when compatible) to just carrying out one (Charlow 2011, 143–6).

Stipulations for L_1

The imperatival language L_1 extends L_0 with subject–predicate structure and permits Boolean connectives to take either wide (sentential) or narrow (predicate) scope. This produces a ‘softer’ Ross-Paradoxical effect arising when disjunction takes narrow scope.

My intent with this is to model the datum that imperative disjunction-introduction can be coerced into sounding less objectionable. It takes some advance preparation. First reflect on the excellence of the conjunction-elimination argument. Then highlight that if something is a certain way, it is thereby any less specific way—so in particular, if someone undergoes a process of a certain type, they thereby undergo a process of any more inclusive type. Then think of a disjunctive action-characterization as just less specific than either disjunct action-characterization. And finally, coerce the disjunctiveness in the conclusion into the disjunctiveness of its action-characterization, as follows: ‘Fred post this letter!—So: Fred (either-post-this-letter-or-take-out-the-trash)!’. That sounds better than before, if not yet strikingly valid.³¹

The predicates of L_1 pertain to *action-kinds*. Actions are sustained processes; an action-kind groups together a bunch of processes under a certain respect of similarity. In the modal spirit, a property of *individuals* is a set of ‘centered worlds’ aka world–time–individual triples: *red* is the set containing $\langle w, t, j \rangle$ just if, in w , at t , j is red. A similarity-class of processes could then be thought to determine a set of ‘extended centered worlds’ aka world–*interval*–individual triples: *running around the block* is the set containing $\langle w, (t, t'), j \rangle$ just if, in w , at t , j commences a run around the block which concludes at t' . But for the purposes of the speech act theory to be introduced for language L_2 , it is only the *onset* of the action that will matter: the impact of our imperative language will paper over matters of practical reasoning and the structuring of actions for which this matters; the issue of interest will be setting each other to action through issuing commands, for which it is only the onset that matters.³²

³¹Starr (2011, 37–8) maintains that disjunction takes scope either over or under the imperatival mood operator $!$, and takes pains to secure the equivalence of the updates associated with both.

³²This actually raises an annoying side-issue: as the pragmatics for L_2 is set up, commands occur in a temporal sequence, and require immediate commencement. But they also put on display stable intentions that are not in a position to be acted upon until speech acts occur. In natural lan-

Syntax *Lexicon*: (i) a stock of elemental action-predicates F, G , etc.; (ii) a stock of names a, b , etc.; (iii) Boolean connectives \wedge, \vee, \neg .

Composition: (i) if $\Gamma_1, \dots, \Gamma_n$ are action-predicates and C is an n -place Boolean connective, $C(\Gamma_1, \dots, \Gamma_n)$ is an action-predicate; (ii) if ν is a name and Γ is an action-predicate, $\nu\Gamma$ is an elemental imperative sentence; (iii) if $\alpha_1, \dots, \alpha_n$ are imperative sentences and C is an n -place Boolean connective, $C(\alpha_1, \dots, \alpha_n)$ is an imperative sentence.³³

Semantics *Ontology*: Let W, T , and J be (respectively) the sets of worlds, times, and individuals. Let \mathcal{A} be the set of all action-kinds, where an action-kind determines a set of world–time–individual triples, with which I will identify it: where K is an action-kind, I will notate $\{w : \langle w, t^*, j^* \rangle \in K\}$ as $K(t^*, j^*)$.

Let a context c determine a moment of time t_c .

Lexicon: (i) For an elemental action-predicate Γ , $\llbracket \Gamma \rrbracket^c \in \mathcal{A}$; (ii) for a name ν , $\llbracket \nu \rrbracket^c \in J$; (iii) $\llbracket \wedge \rrbracket^c = \cap$, $\llbracket \vee \rrbracket^c = \cup$, $\llbracket \neg \rrbracket^c = \lambda x \bar{x}$.

Composition: (i) $\llbracket C(\Gamma_1, \dots, \Gamma_n) \rrbracket^c = \llbracket C \rrbracket^c(\llbracket \Gamma_1 \rrbracket^c, \dots, \llbracket \Gamma_n \rrbracket^c)$; (ii) $\llbracket \nu\Gamma \rrbracket^c = \langle ?(\llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)), \llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c) \rangle$; (iii) $\llbracket C(\alpha_1, \dots, \alpha_n) \rrbracket^c = \langle \pi(\llbracket \alpha_1 \rrbracket^c) \times \dots \times \pi(\llbracket \alpha_n \rrbracket^c), \llbracket C \rrbracket^c(p(\llbracket \alpha_1 \rrbracket^c), \dots, p(\llbracket \alpha_n \rrbracket^c)) \rangle$.

Entailment If $\beta_1, \dots, \beta_n, \alpha$ are sentences of L_1 , then $\beta_1, \dots, \beta_n \vdash \alpha$ just if, for all c , $\llbracket \beta_1 \rrbracket^c, \dots, \llbracket \beta_n \rrbracket^c \vDash \llbracket \alpha \rrbracket^c$.

Discussion

The scope of negation does not matter: $\llbracket \neg(\nu\Gamma) \rrbracket^c = \llbracket \nu(\neg\Gamma) \rrbracket^c$. After all, $\llbracket \neg(\nu\Gamma) \rrbracket^c = \langle ?(\llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)), \llbracket \neg \rrbracket^c(\llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)) \rangle = \langle ?(\llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)), \overline{\llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)} \rangle = \langle ?(\llbracket \neg \rrbracket^c \llbracket \Gamma \rrbracket^c(\llbracket \nu \rrbracket^c, t_c)), \llbracket \neg \rrbracket^c \llbracket \nu(\neg\Gamma) \rrbracket^c \rangle$.

But the scope of disjunction does matter. Let p be the proposition that Fred opens the door, q that he takes out the trash. The content of ‘Fred take out the

guage, that would be resolved by tacit or explicit temporal adverbialization; or perhaps by looseness about what counts as the onset of action (compare: ‘is there much to choose between ‘She is making tea’ and [] ‘She is going to make tea’? Obviously not’: [Anscombe 1963](#), section 23).

³³For further details of the syntactic and semantic composition of imperatives, see [Hellie 2015](#). In eschewing an imperatival operator, I side with [Portner 2004, 2007, forthcoming](#); and I oppose [Hare 1952, 2.2](#); [Kenny 1966](#); [Stenius 1967](#); [Chellas 1971](#); [Lewis 1979b](#); [Kaufmann 2011](#); [Charlow 2011, 2014](#); and [Starr 2011](#).

trash!’, of course, is $\langle ?q, q \rangle$. And the content of ‘Fred (open-the-door-or-take-out-the-trash)!’ is $\langle ?(p \cup q), p \cup q \rangle$, while that of ‘(Fred open the door) or (Fred take out the trash)!’ is $\langle ?p \times ?q, p \cup q \rangle$.

The sentential-scope disjunction raises more issues than the disjunct: it offers Fred the option of success by opening the door in addition to by taking out the trash. Consequently, the partition component is stronger than that of the disjunct. By contrast, the predicate-scope disjunction offers Fred only one option, that of taking the disjunctive action: the resulting partition is neither stronger nor weaker than that of the simple imperative.

Perhaps we reason about the relative strengths of natural-language imperatives α and β by comparing the relative strengths of their partitions and of their propositions. If so, we might perceive the predicate-scope Ross-inference as ‘more valid’ than the sentential-scope: with predicate scope, the premiss proposition is stronger than the conclusion, the partitions orthogonal; with sentential scope, the premiss proposition is stronger, the conclusion partition is stronger. With sentential scope, respects of strength are balanced between premiss and conclusion; with predicate scope, by either measure, whenever their strength is comparable, the premiss is stronger than the conclusion.

7 Command pragmatics

Next, the language L_2 assembles a command-‘language game’ around L_1 -syntax/semantics. Conversations in L_2 are centered on the intentions held in a ‘general will’ of the assembled conversationalists—which I understand as to be a group mental state in a fully robust sense (Rovane 2014, Hellie 2015), in many salient respects just like individual intention.³⁴ (Why? If canonical episodes of command require some sort of ‘group project’, it would be peculiar for non-Catholics to issue commands to the Pope (compare Ninan 2005, 5.3); involving a collective mental state permits reusing structures for individual practical rationality. A more Lewis (1979b)/Portner (2004)-like approach, on which commands merely *constrain* without *rationalizing*, must also find some alternative explanation of the imperative Ross Paradox.) Conversation begins in common ignorance of how

³⁴For differences, see fn. 27.

the general will intentions obligate the conversationalists, and moves toward total common recognition of the obligations of each; the acts of command in L_2 serve to distribute this common recognition (I set to the side the epistemological question of how anyone knows what the general will intends; note also that L_2 stripped of its speech act theory can represent individual practical calculation).

In mechanical terms, L_2 extends L_1 with a statics and a dynamics: the statics postulates *contexts* to record the general will intentions, the intentions of each conversationalist, and the extent of common knowledge of the distribution of general will intentions;³⁵ the dynamics sets the start- and end-states of the language game and the effect of speech-acts on the state of play.³⁶

In its postulation of psychologically-laden contexts, the statics of L_2 permits a definition of *support*: endorsement of a sentence by a mental state (individual or group), represented by positive evaluation at a context of the content of the sentence, interpreted at that context.³⁷ Armed with support, entailment can be defined as support-preservation: any context supporting all premisses supports the conclusion.³⁸ Despite this, the entailment-relation of L_2 does not differ extensionally from that of L_1 : that happens only in L_3 , with the introduction of intensional operators to shift the context at which a sentence is evaluated for support.

The objective is to illustrate these points: how the contents of commands and of intentions can be identified; how the meaning of a command can be obligative, in the sense of binding its recipient to the course of action commanded; and, more programmatically, what is required of the relations among content, mental states, speech act meaning, and entailment.

Stipulations for L_2

Statics *Ontology*: The central device of statics is the set C of all *contexts*. The approach uses a particular context $c \in C$ to represent a mental state relevant to

³⁵In this mentalistic conception of context, I side with Stalnaker (1970, 1998), Lewis (1979a), and Veltman (1996, 247), and oppose Kaplan (1977) and Lewis (1980a); compare Hellie 2015 and, to a considerable extent, Yalcin 2007.

³⁶My story here is a spin on the familiar Stalnakean pragmatics (Stalnaker 1970, 1978).

³⁷In this ‘diagonal’ conception of entailment, I side with Kaplan (1977) and against Stalnaker (1975), Veltman (1996), and Yalcin (2007).

³⁸In thinking of entailment as support-preservation, I side with Hare (1952), Veltman (1996), and Yalcin (2007), and against Stalnaker (1975) and Kaplan (1977).

language use. Those include the mental states of individual language-users, of course, but also collective mental states of groups of individuals gathered in conversation. A context representing an individual I call *solo*; a context representing a conversational group I call *social*. (I allow that contexts may represent not only ‘sincerely’ held mental states, but also other states ‘treated as’ held—as for hypothesis, fiction, contingency planning, and so forth: [Stalnaker 1978](#), 84. I henceforth pass over this qualification without acknowledgement.)

A mental state needs to know whose it is, and when—matters of particular significance for agency, because if I am uncertain whether I am BH in Toronto in as I write this or Julius Caesar just before his crossing the Rubicon, I will have no idea whether to type or command my army (and I certainly have no idea how to split the difference!).³⁹ Accordingly, I continue to let c determine its time t_c . I also require c to determine a nonempty set of individual conversationalists $V_c \subseteq J$: c is *solo* just if V_c is a singleton set, otherwise *social*. For solo c , V_c is the singleton of the agent whose state of mind is represented by c ; for social c , V_c represents the members assembled in conversation as represented by c .

In order to link imperative meaning to intentions, a context must bear a representation of the content of someone’s or some group’s intention. Accordingly, let c determine a procedure $\Pi_c \in \mathcal{PR}$: for solo c with $V_c = \{j\}$, Π_c is the procedural content of the intentions of j at t_c . For social c , Π_c represents the obligations imposed on one another by the ‘general will’ of the assembled members of the conversation ([Castañeda 1960](#), 44–5).

I next insist that, for social c , for any $j \in V_c$, there is a solo context c^j such that $V_{c^j} = \{j\}$. After all, j can only converse in an imperative language-game if it can place itself under obligations (by having intentions, and thereby being representable by a device c^j representing j as itself and the content of its intentions as Π_{c^j}); I impose the requirement that c^j be solo to exclude the bizarre prospect that a conversation is a conversationalist in another conversation. And next, I require that $t_c = t_{c^j}$: at any time, it is the same time for everyone.

The purpose of assembling in L_2 -conversation is to parcel out the general will. If that is what we are doing, we need a sense for how much of it we have parceled out. To this end, I require a social context c to determine a *to-do list* function td_c

³⁹In requiring this ‘immunity to error through misidentification’, I side with [Stalnaker \(2008\)](#) and against [Kaplan \(1977\)](#) and [Lewis \(1979a\)](#).

assigning a procedure to each member of the conversation ($td_c : V_c \rightarrow \mathcal{PR}$):⁴⁰ td_c represents the common recognition among the conversationalists of how much of the general will has been parceled out, and to whom. In order for td_c to parcel out *the general will*, I require that it assign no procedure to anyone unless the general will intends that procedure to be carried out by them (for $j \in V_c$, $\Pi_c \vDash td_c(j)$). And in order for a to-do list to reflect the *parceling out* the general will—for it to be commonly recognized who is under an obligation to do what—it needs to be that it accurately reflects what the intentions of the assembled conversationalists actually are ($\Pi_{c_j} \vDash td_c(j)$).

Entailment Sentential-entailment in L_2 is grounded in its statics: the relation at the definitional basis L_2 -entailment, (*content*)-*support*, holds between a context c' (a creature of the statics) and a content Π , in virtue of the content-entailment of Π by a parameter of c' (namely $\Pi_{c'}$). In formal terms: $c' \Vdash \Pi := \Pi_{c'} \vDash \Pi$. Because contexts represent mental states, this makes support psychological; because support is fundamental to entailment, it in turn makes entailment fundamentally psychological, and excises the linguistic (including speech-acts) from the basis of entailment.⁴¹

Sentential entailment, of course, is a relation between sentences. L_2 -entailment insists on the fundamental role of the psychological by bringing language in as a product of content-support and semantic-valuation. In formal terms: $c' \text{ } c\text{-supports } \alpha$ ($c' \Vdash^c \alpha$) := $c' \Vdash \llbracket \alpha \rrbracket^c$.

This relation, c -support, is a context-relative binary relation between a context and a sentence. Contexts, therefore, do two jobs when c' c -supports α : there is the job of c , which I call *interpretation*; and there is the job of c' , which I call *evaluation*. The interpreting context c is in charge of saturating context-dependent parameters in the semantic-valuation clause for α (for example, c turns the sentence ‘You pay the rent!’ to the procedure committing the addressee of c to set

⁴⁰I borrow the ‘to-do list’ terminology from Portner (2004, 2007, forthcoming) (my conception of these objects adjusts Portner’s to meet with my apparatus and doctrine, as needed, and without comment). Portner acknowledges Han (1998) and Potts (2008/2011) as precursors; and of course (Charlow 2011, 2.1–2) the to-do list is a generalization of Lewis’s single-subject ‘sphere of permissibility’ (Lewis 1979b).

⁴¹In this latter respect I side with Yalcin (2007) and against ‘dynamists’ (Veltman 1996, Starr 2011).

about paying the rent at the moment of c). The evaluating context is in charge of assessing whether that interpreted semantic value is content-supported (for example, whether it is intended by the general will that, at the moment of c , the addressee of c set about paying the rent).

This dual role of context sets up a ‘two-dimensional intension’ for a sentence (the horizontal is the evaluating axis, the vertical the interpreting axis, the ‘extensions’ are support- rather than truth-values): I follow the two-dimensionalist tradition (Kaplan 1977; Chalmers 2005) in affiliating rational power with the diagonal. The diagonalization of c -support, *support (simpliciter)*, is c -support by c : $c \Vdash \alpha$ just if $c \Vdash^c \alpha$.

Finally, (sentential-)entailment is *support-preservation*: universal supportedness of the conclusion conditional on supportedness of all premisses. In formal terms: $\beta_1, \dots, \beta_n \vdash \alpha$ just if whenever $c \Vdash \beta_1, \dots, c \Vdash \beta_n$, then $c \Vdash \alpha$. Collecting together: just if whenever the general will-intention is sufficient to support the premisses, it is sufficient to support the conclusion.

Dynamics *Ontology*: The central device of dynamics is a set of events \mathcal{E} , the set of all *speech acts*. I have troubled to tease apart the speech acts from the contexts—the former are linguistic, the latter mental—in order to emphasize the purification of sentential-entailment of any linguistic componentry. In this respect, the approach contrasts with ‘dynamic semantics’.

The dynamics pictures a temporally-ordered sequence of speech acts—a *conversation*, $conv = \langle e^0, \dots, e^\omega \rangle$ —with each $e^i \in \mathcal{E}$. A speech act e determines a moment of time $t(e)$ at which it occurs: the temporal sequencing of $conv$ means that $t(e^i) < t(e^{i+1})$.

A speech act determines an individual $s(e)$, its *speaker*, by whom it is performed. In L_2 , a speech act also determines a distinct *addressee*, $a(e) \neq s(e)$, at whom it is directed. A speech act of L_2 can only occur inside of circumstances in which, at its time, its speaker and addressee are assembled for conversation. That requires a social context in which speaker and addressee are conversants. Accordingly, e determines a social context $c(e)$: the time of the context and speech act are the same ($t(e) = t_{c(e)}$); and the addressee and speaker are conversationalists in $c(e)$ ($\{s(e), a(e)\} \subseteq V_{c(e)}$).⁴² I assume that conversationalists do not come and go:

⁴²Speech acts therefore *require* contexts, and not the other way around. In this sense, con-

$V(conv) := V(c_{e^i}) = V(c_{e^{i+1}})$.

In a speech act, a sentence is spoken: accordingly, e determines a sentence $\sigma(e)$. The purpose of the speech act is to adjust its context $c(e)$ —the *prior* context—into a *posterior* context $cc(e)$. In L_2 , perhaps sanding down certain intricacies of natural language, the adjustment occurs in line with the conventional force of $\sigma(e)$ and with the content of $\sigma(e)$ interpreted against the prior context. (I assume that it is not the role of speech acts to adjust the general will: $\Pi_{c(e)} = \Pi_{cc(e)}$.)

The aim of a L_2 -speech-act is to adjust the to-do list in a way that furthers the ‘aim’ of the conversation. The aim of the conversation is characterized by its commencement- and conclusion-states, as reflected in its initial prior and final posterior contexts. Conversation commences absent any common recognition of the obligations placed by the general will on any conversationalist: the to-do list for each member according to the prior context of the initial speech act e^0 assigns the trivial procedure $\mathbf{1}$ (more formally, for each $j \in V(conv)$, $td_{c(e^0)}(j) = \mathbf{1}$). Conversation concludes in a state in which there is nothing in the general will remaining to be parcelled out to some appropriate conversationalist (more formally, $\bigwedge_{j \in V(conv)} td_{cc(e^\omega)}(j) = \Pi_{cc(e^\omega)}$). In that sense, the aim of the conversation as a whole is to spread the whole general will over the to-do list; an individual speech act contributes to that aim by moving a bit of the general will to the to-do list. That makes these speech acts *commands*.⁴³

In its mechanics, a command is subject to the constraints of *relevance* and *legitimacy*. A command is relevant just if its content pertains to its addressee (so for any e , if $\sigma(e) = v\Gamma$, $\llbracket v \rrbracket^{c(e)} = a(e)$: compare [Portner 2004](#), 237); a command is legitimate just if supported by the general will (so for any e , $c(e) \Vdash \sigma(e)$: compare [Castañeda 1975](#); contrast [Portner 2004](#), 4.1). The effect of e is then to determine a posterior context $cc(e)$ adjusting the prior context $c(e)$ just by adding the content of $\sigma(e)$ to the to-do list of the addressee ($td_{cc(e)}(a(e)) = td_{c(e)} \wedge \llbracket \sigma(e) \rrbracket^{c(e)}$):

texts are essentially psychological, and when associated with speech acts, are only accidentally so: compare [Chalmers 2005](#) against ‘contextual two-dimensionalism’; contrast [Stalnaker 1999](#), passim; [Kaplan 1977](#); and [Lewis 1980a](#). This wiggle-room introduces the prospect that meanings of personal pronouns are grounded in dynamic strategies for resolution of anaphora rather than static rules of interpretation.

⁴³Imperatives can be used for a laundry list of speech acts other than commands ([Kaufmann 2011](#); compare [Portner 2004](#), [2007](#), [Starr 2011](#), 5.1, [Charlow 2011](#), 2.4). Because of space, I set this aside.

compare Portner 2004, 237—but Portner (2004, 240) then goes on (to my mind, incongruously) to explain this force as an effect of ‘pragmatic reasoning’.⁴⁴

If contexts evolve between speech acts, they do so in the direction of strength: $c(e^{i+1}) \vDash cc(e^i)$ —the posterior context of the leading speech act is entailed by the prior context of the trailing speech act. What is entailment between contexts? The objective is to ensure that what speech acts do does not get undone: if my command or assertion is accepted, behind-the-scenes manoeuvring won’t later ‘de-accept’ it (Charlow 2011, 148). So in particular, whenever x_c is a parameter of context adjustable by speech acts, $c^1 \Vdash c^0$ only if $x_{c^1} \Vdash x_{c^0}$.

This requirement is weak in two ways. First, it says nothing about parameters that cannot be adjusted by speech acts. Second, it does not require identity between speech-act parameters of leading posterior and trailing prior contexts. The latter weakness leaves room for the important phenomenon of *accommodation* (Lewis 1979c): roughly, the natural-language practice of ‘working outside of the rules’ to permit speech acts to succeed. For example, ‘Pick up Sam’s sister at the airport!’ *presupposes* that Sam has a sister (compare Stalnaker 1998, III, IV): a command using that sentence fails unless that presupposition is common belief in the prior context of its command. But if the prior-context common beliefs need not just be the leading posterior-context common beliefs, the conversationalists may strengthen the latter with the presupposition so as to ready the former for updating with the command.⁴⁵

A final point: *truth* is nowhere to be seen in L_2 .⁴⁶ Imperative contents cannot be truth-conditions: they are too finely-grained for that. So truth is nowhere in the semantics. The entailment-relation on L_2 is, as I have argued, fundamentally

⁴⁴See footnote 32 for a complication.

⁴⁵Another use for this looseness is negotiation of the ‘general will’ (compare Lewis (1979c, example 8) on the collective negotiation of a plan): the requirement that $\Pi_{c(e)} = \Pi_{cc(e)}$ requires merely that there is no speech act *essentially* for adjusting the general will. That is compatible with the prospect of adjusting the general will ‘between speech acts’.

⁴⁶Do I thereby abandon the ‘utility of the truth-conditional paradigm’ (Charlow 2014, 620)—namely, the ‘standard’ notion of entailment as incompatibility of the truth-condition of the negation of the conclusion with the truth-conditions of the premisses, and the ‘familiar’ conception of compositionality as requiring the contribution of the truth-condition of an embedded sentence to the meaning of its embedding sentence? I do not feel discomfitingly anxious. What is actually the ‘standard’ (Kaplan 1977) notion of entailment is truth of the conclusion at the index of the context whenever the premisses are true at the index of the context; as noted above, support does as well in this role as truth. And needless to say, objects other than truth-conditions can embed.

and ineliminably psychological. While entailment is preservation of something-or-other, it is not truth that is preserved, but the psychological relation of support. So truth is nowhere in the statics. Finally, the aim and practice of command is of distributing the general will by putting it on display. There is nothing like a ‘truth-norm’ for command—nor any recasting of a truth-norm in other terms, such as a ‘goodness-norm’ (the good may suffuse *should* and *oughta*, but imperatives and intentions are rather less permeable: contrast Castañeda 1960). So truth is nowhere in the dynamics.⁴⁷

8 Obligative modals

Extending L_2 with modals requires not just new semantic clauses for modals, but an extension of the pragmatics. Modal sentences are declarative; accordingly, sticking with the support-preservation conception of entailment requires a declarative statics; moreover, the use of obligative ‘must’-claims to command is among the explananda, requiring a declarative dynamics. The language L_3 extends L_2 with a declarative pragmatics; L_4 , at last, adds modals.

The move to L_3 is easy, because the general form of a declarative pragmatics is relatively clear: each asserts propositions they believe to enrich common belief. Support is belief; the ‘essential effect’ of assertion intersects the proposition asserted with the ‘context set’. While the rigors of segregating statics and dynamics call for a few subtle adjustments, the pragmatics is in the main orthodox.

The semantics for modals presented in L_4 is less so. The founding datum of this chapter, recall, is that one thinks one must do something just if one acts with the intention of doing it. Or, expressed in the jargon, a solo context (sentence-)supports ‘I must pay the rent.’ just if it (content-)supports the procedure *pay the rent*.⁴⁸

The orthodoxy says that modals are intensional operators, comparing their operanda to aggregations of those things at which intensions are evaluated, what-

⁴⁷Might truth still be lurking at the bottom, in the contents of intentions, by (as suggested in the early sections) grounding success of intentions in truth of their contents? —But with intelligibility in hand as a norm, I suspect truth can be stripped even of that job, and content can be given a purely phenomenological reduction. Further discussion must await another forum.

⁴⁸See fn. 27.

ever they may be. It says also that modals quantify over worlds, comparing their operands to aggregations of worlds. The orthodoxy can say both things because it thinks intensions are evaluated at worlds. But I think entailment is support-preservation, intensions are evaluated at representations of mental states—what I have been calling *contexts*—so I have to choose. (Revanchists attempting to avoid this choice by retreating from support-preservation must find some alternative approach to the imperative Ross Paradox.)

One option is to break the contact between modals and intensions but preserve contact between modals and worlds:⁴⁹ on this approach, ‘I must pay the rent.’ is sensitive to whether, in all worlds that have been somehow selected by the context, I pay the rent. Selected how? Perhaps they are those where I do as I intend. Suppose I intend to pay the rent; then in all worlds where I do as I intend, I pay the rent; then in all worlds selected by the context, I pay the rent. And that is what it takes for ‘I must pay the rent.’ to be ‘good’ (true, or necessary, or acceptable, or what have you). Unfortunately, in all worlds where I pay the rent, I either pay the rent or drink up all your wine. So if I intend to pay the rent, ‘I must either pay the rent or drink up all your wine.’ is also ‘good’. So if I intend to pay the rent, I think I must either pay the rent or drink up all your wine. But I don’t! The Ross Paradox works for obligative modals, too.

The other option holds out more promise of a treatment of the Ross Paradox for *must*: break the contact between modals and worlds but preserve contact between modals and intensions. On this option, modals are intensional operators, comparing their operands to aggregations of contexts, looking for patterns of support. What makes a reading of ‘must’ obligative is its use to compare its procedural operand to the intentions of the context at which it is interpreted; with the consequence that a solo context supports obligative ‘I must pay the rent.’ just if it supports *pay the rent* and ‘I must pay the rent or go drinking.’ just if it supports *pay the rent or go drinking*.

⁴⁹Compare Yalcin 2007: intensions are evaluated at information states: ‘ φ_c is *accepted* in information state s iff for all worlds w in s , $\llbracket \varphi \rrbracket^{c,s,w}$ is true’; ‘ $\Gamma \vDash_i \varphi$ just in case for every context c and body of information s , if every member of Γ_c is accepted in s , then φ is accepted in s ’ (1004). But modals quantify over worlds: ‘ $\llbracket \Box \varphi \rrbracket^{c,s,w}$ is true iff $\forall w' \in s : \llbracket \varphi \rrbracket^{c,s,w'}$ is true’ (1014).

Stipulations for L_3

Syntax *Lexicon*: A stock of elemental declarative sentences P, Q , and so forth.

Composition: If $\varphi_1, \dots, \varphi_n$ are declarative sentences and \mathbf{C} is an n -place Boolean connective, $\mathbf{C}(\varphi_1, \dots, \varphi_n)$ is a declarative sentence.

Semantics *Lexicon*: For an elemental declarative sentence φ , $\llbracket \varphi \rrbracket^c \subseteq W$.

Composition: $\llbracket \mathbf{C}(\varphi_1, \dots, \varphi_n) \rrbracket^c = \llbracket \mathbf{C} \rrbracket^c(\llbracket \varphi_1 \rrbracket^c, \dots, \llbracket \varphi_n \rrbracket^c)$.

Statics *Ontology*: Let a context $c \in \mathcal{C}$ determine an *information state*, a nonempty set $i_c \subseteq W$ (compare [Stalnaker 1978](#), 84–5; also (inter alia) [Veltman 1996](#), [Yalcin 2007](#)). For solo c , i_c represents the content of the beliefs of the subject c represents; for social c , i_c represents the *common beliefs* of the participants to the conversation c represents (the strongest proposition each believes each believes . . . each believes). The nonemptiness requirement on i_c stipulates that no (nondefective) belief state can be inconsistent. A *union-set requirement* on social contexts stipulates that nothing can count as common belief unless it is severally believed: if c is social, then $\bigcup_{j \in V_c} i_{c^j} \subseteq i_c$.

Entailment To bring in declarative entailment, it suffices to add a ‘module’ for content-support of declarative content: namely, contextual support of a proposition is believing it ($c' \Vdash p := i_{c'} \vDash p$).

Dynamics *Ontology*: As with L_2 , a conversation in L_3 is a temporally ordered sequence of speech acts $conv = \langle e_0, \dots, e_\omega \rangle$. Conversation progresses from complete common ignorance to complete common belief in that which is not controversial (more formally: $i_{c(e^0)} = W$ and $\Pi_{c(e^0)} = \mathbf{1}$; $i_{cc(e^\omega)} = \bigcup_{j \in V(conv)} i_{c^j}$ and $\Pi_{cc(e^\omega)} = \bigwedge_{j \in V(conv)} td_{cc(e^\omega)}(j)$). By contrast with command, which aims at parceling out content which is already common to us, assertion aims at gathering up content which had been scattered among us. This opposite polarization of assertion and command recapitulates the opposite polarization of belief and intention: command is polarized from the social to the solo, assertion is polarized from the solo to the social.

As with L_2 , an L_3 -speech act $e \in \mathcal{E}$ determines a sentence $\sigma(e)$. If $\sigma(e)$ is an imperative sentence α , e is a *command*; if $\sigma(e)$ is a declarative sentence φ , e is an *assertion*. But the opposing polarities of command and assertion make for opposing polarities in the appropriateness-conditions of the speech-act types. Commanding α is appropriate when the content of α is intended by the general will, where this is represented as support by the social context. But when an assertion contains only information already included in common belief, it is redundant, and will typically be undesirable: asserting φ is generally inappropriate when the social context supports φ , appropriate when the speaker's context supports φ .

Accommodating this requires an extension of the apparatus. In addition to its prior and posterior contexts $c(e)$ and $cc(e)$, e determines also an *imputed context* $mc(e)$: the imputed context of a speech act is, in effect, the context 'authorizing' the speech act, or 'in the voice of which' the speech act is advanced. When e is a command, $mc(e)$ is typically $c(e)$, the social context of the conversation; when e is an assertion, $mc(e)$ is typically $c^{s(e)}$, the solo context of the speaker. Either way, it is required that the imputed context does indeed support the sentence advanced in the speech act ($mc(e) \Vdash \sigma(e)$).

Finally, the update rule for assertion is the familiar intersective 'essential effect': $i_{cc(e)} = i_{c(e)} \cap \llbracket \varphi \rrbracket^{mc(e)}$.

Stipulations for L_4

Statics *Ontology*: If modals are intensional operators, they 'quantify' over whatever intensions are evaluated at. If entailment is support-preservation, intensions are evaluated at contexts. So modals quantify over contexts. As on the orthodoxy, the quantification is typically restricted. I represent the restriction with a set of contexts C_c determined by c . As on the orthodoxy, restriction represents a limitation in which entities are 'taken seriously' in c ; those not meeting the restriction are 'ignored'.

The orthodoxy typically imposes a requirement of *realism* (Kratzer 1991, 646): actuality is never ignored, always taken seriously. I impose a similar restriction: a context never ignores itself, always takes itself seriously. Formally: $c \in C_c$.

Syntax *Lexicon*: Modal operators \Box, \Diamond .

Composition: If σ is a sentence and M is a modal operator, $M\sigma$ is a declarative sentence.

Semantics *Lexical*: $\llbracket \Box \rrbracket^c = \lambda F \{w : (\forall c' \in C_c)(c' \Vdash F(c))\}$, $\llbracket \Diamond \rrbracket^c = \lambda F \{w : (\forall c' \in C_c)(c' \Vdash \overline{F(c)})\}$.

These clauses represent a modal as an intensional operator, comparing the C_c -restricted set of contexts to the content of its operand: for \Box , whether *all* members of C_c *support* that content; for \Diamond , whether *not all* members of C_c *antisupport* the content. I return to ‘antisupport’ in the wrap-up discussion. What it is for a modal sentence to be ‘good’ is for it to be supported, as interpreted, however evaluated: as a declarative sentence, that is for it to be noncontingent—trivial if true, absurd if false.⁵⁰

Composition: $\llbracket M\sigma \rrbracket^c = \llbracket M \rrbracket^c(\lambda c' \llbracket \sigma \rrbracket^{c'})$.

Obligative necessity

Every context takes itself seriously: $c \in C_c$. But under *reflexive restriction*, a context takes only itself seriously: $C_c = \{c\}$. Let \boxplus and \diamond be unofficial symbols for \Box and \Diamond under reflexive restriction: then $\llbracket \boxplus \rrbracket^c = \lambda F \{w : (\forall c' \in C_c = \{c\})(c' \Vdash F(c))\} = \lambda F \{w : c \Vdash F(c)\}$; similarly, $\llbracket \diamond \rrbracket^c = \lambda F \{w : c \Vdash \overline{F(c)}\}$.

Accordingly, $\llbracket \boxplus \sigma \rrbracket^c = \{w : c \Vdash \llbracket \sigma \rrbracket^c\} = \{w : c \Vdash \sigma\} = W$ just if $c \Vdash \sigma$, otherwise \emptyset : from which it follows that $c \Vdash \boxplus \sigma$ just if $c \Vdash \sigma$, from which it follows that $\sigma \vdash \boxplus \sigma$ and $\sigma \vdash \boxminus \sigma$: any sentence is equivalent to its reflexively-restricted necessitation (compare [Yalcin 2007](#), 1014).

And accordingly, $\llbracket \diamond \sigma \rrbracket^c = \{w : c \Vdash \overline{\llbracket \sigma \rrbracket^c}\}$. Now, as it happens, for imperative $\sigma = \alpha$, $\llbracket \alpha \rrbracket^c$ is undefined, and with it $\llbracket \diamond \alpha \rrbracket^c$: I return to this point in the concluding remarks. But for declarative $\sigma = \varphi$, $\llbracket \diamond \varphi \rrbracket^c = \{w : c \Vdash \neg \varphi\}$. It follows that $c \Vdash \neg \diamond \varphi$ just if $c \Vdash \neg \varphi$, from which it follows that $\neg \varphi \vdash \neg \diamond \varphi$ and $\neg \diamond \varphi \vdash \neg \varphi$: the negation of any declarative sentence is equivalent to the negation of its reflexively-restricted possibilization (compare [Yalcin 2007](#), 994, 1005).

⁵⁰Because the completely ignorant belief state must still support a ‘good’ modal sentence, the content of that sentence can be no more specific than the trivial proposition.

Because modal sentences are declaratives, they conform to disjunction-introduction.⁵¹ In particular, $\Box\alpha \vdash \Box\alpha \vee \Box\beta$. But because $\Box\sigma$ and σ are equivalent, the Ross-Paradoxical nonentailment $\alpha \not\vdash \alpha \vee \beta$ results in a comparable nonentailment $\Box\alpha \not\vdash \Box(\alpha \vee \beta)$. (Compare: I must post this letter, so either I must post this letter or I must drink up all your wine—sounds fine to me.) Of course $\Box\varphi \vdash \Box(\varphi \vee \psi)$.

An assertion e of $\Box\alpha$ can have a force resembling that of a command e' of α . How so? Let the imputed context $mc(e) = c(e)$, the social context of the speech act (while assertion is *ordinarily* imputed to the speaker, that is not a hard-and-fast rule: in general, epistemic uses of ‘they might have already left’ to probe for compatibility with common belief imputes to the social context). Then, by stipulation, $mc(e) = c(e) \Vdash \Box\alpha$. That is so just if $c(e) \Vdash \alpha$. That is so just if $\Pi_{c(e)} \models \llbracket\alpha\rrbracket^c$.

Now, by stipulation, $c(e') \Vdash \sigma(e') = \alpha$ as well. But command has the conventional force of introducing $\llbracket\alpha\rrbracket^c$ to the to-do-list of its addressee, while assertion has no such conventional force. To bridge the gap, observe with [Stalnaker \(1998, 100–2\)](#) that common belief includes commonly salient information, in addition to the content so far asserted. Common extraction of the content of a speech act requires common recognition of the act’s occurrence as an act with a certain conventional meaning. So typically this common recognition regarding a speech act will be incorporated into its prior context (recall that a trailing prior context need only be *at least as strong* as its leading posterior context).

So if the assertion e of $\Box\alpha$ has been accepted, it is commonly recognized that the social context has accepted a speech act requiring $\Pi_{c(e)} \models \llbracket\alpha\rrbracket^c$ and in that sense commonly recognized that the general will supports α . But that has the same effect as putting this support on display, the conventional force of command. The objective of the conversation *conv* is to parcel out the content of $\Pi(\text{conv})$ among the conversationalists by putting the general will on display; they should not care whether that is done by a speech act with that conventional force or via this more scenic route.

⁵¹Not so disjunction-elimination. Read epistemically, \Box gives this counterexample: $\varphi \vdash \Box\varphi$, $\neg\varphi \vdash \Box\neg\varphi$, $\vdash \varphi \vee \neg\varphi$, but $\not\vdash \Box\varphi \vee \Box\neg\varphi$.

Obligative possibility?

I will close by reflecting on the dual of obligative necessity, namely obligative possibility. I consider an explanation for the difficulty in perceiving obligative readings of natural language strong possibility modals. And I propose a resolution of David Lewis's 'problem about permission' (Lewis 1979b).

On its elusiveness Suppose we let go this paperweight here. Fred says it must fall, Brent says it can/could fail to fall. It is easy to access a reading on which they disagree, hard to access readings on which they do not; it would be unnatural for Brent to rejoin %'There are a lot of things this object can/could fail to do that it must do—so what?'. Suppose we toss this chemical into water. Fred says it can/could explode, Brent says it must fail to explode. Again, it is easy to access a reading on which they disagree, hard to access readings on which they do not; it would again be unnatural for Brent to rejoin %'There are a lot of things this chemical can/could do that it must fail to do—so what?'. Giving them 'natural capacity' readings, it is not easy to avoid coordinating *must* and *can/could* so that they are dual to one another.

What to do about the rent? Brent says Rance must pay the rent, Fred says Rance can/could fail to pay the rent. It is easy to access readings on which they do not disagree; indeed, Brent quite sensibly rejoins 'There are a lot of things Rance can/could fail to do that he must do—so what?'. What to do about Brent's wine? Brent says Rance must not drink it all up, Fred says Rance can/could drink it all up. Again, it is easy to access readings on which they do not disagree; again, Brent sensibly rejoins 'There are a lot of things Rance can/could do that he must not do—so what?'. Giving *must* its obligative reading, it is easy to avoid coordinating *can/could* so that they are dual to one another.⁵²

Granting my core thesis, the obligative readings of *can/could* Fred hopes to access would be reflexively-restricted possibility-modalized imperatives: Brent

⁵²Our parents would irritate us by answering 'Can I be excused from the table?' with 'You can if you may.'. Our parents were correct that the issue was compatibility with a gradated standard of good manners. We did not embrace those standards as our own, but instead felt them to have the force of natural bonds—they imprisoned us at the table. Our irritation was at being not only imprisoned, but compelled to ritually legitimize the imprisoning standards as a condition of freedom.

says $\Box\beta$, Fred says $\Diamond\neg\beta$; Brent says $\Box\neg\alpha$, Fred says $\Diamond\alpha$. In L_4 , $\llbracket\Diamond\neg\beta\rrbracket^c = \{w : c \Vdash \overline{\llbracket\neg\beta\rrbracket^c}\}$; $\llbracket\Diamond\alpha\rrbracket^c = \{w : c \Vdash \overline{\llbracket\alpha\rrbracket^c}\}$. What is the problem?

The problem is that, for a granulated proposition P , it is not straightforward to define its complement \overline{P} . Now, for a (protean) proposition p , its complement \overline{p} is the unique proposition for which the \subseteq -maximally weak ‘trivial’ proposition \top is the strongest proposition weaker than both p and \overline{p} and the \subseteq -maximally strong ‘absurd’ proposition \perp is the weakest proposition stronger than both p and \overline{p} .

Is there a comparable definition of partition-complementation? In the set of partitions of W , the \leq -maximally weak partition $\mathbf{1}$ is the one-cell partition $\{W\}$, while the \leq -maximally strong partition $\mathbf{0}$ is the partition into singletons $\{\{w\} : w \in W\}$. But a partition does not generally have a unique complement—namely, given π , there is sometimes more than one π' for which $\mathbf{1}$ is the strongest partition weaker than both and $\mathbf{0}$ is the weakest partition stronger than both. Example: over the set $\{A1, B1, C1, A2, B2, C2\}$ let π be the ‘column’ partition $\{\{A1, A2\}, \{B1, B2\}, \{C1, C2\}\}$; let ρ be the ‘row’ partition $\{\{A1, B1, C1\}, \{A2, B2, C2\}\}$; and let ρ' be the ‘skew’ partition $\{\{A1, C2\}, \{B1, C1\}, \{A2, B2\}\}$. Then each of ρ and ρ' meets the conditions required of a complement for π : for both of the pairs $(\pi$ and $\rho)$ and $(\pi$ and $\rho')$, $\mathbf{1}$ is the strongest partition weaker than both and $\mathbf{0}$ the weakest partition stronger than both.

If partitions cannot be complemented, what sense can be made of the complement of a granulated proposition? We manage to determine $\llbracket\neg\alpha\rrbracket^{c53}$ by complementing the proposition-component of $\llbracket\alpha\rrbracket^c$. So perhaps while $\llbracket\Diamond\alpha\rrbracket^c = \{w : c \Vdash \overline{\llbracket\alpha\rrbracket^c}\}$ is incomprehensible as $\{w : c \Vdash \langle\overline{\pi(\llbracket\alpha\rrbracket^c)}, p(\llbracket\alpha\rrbracket^c)\rangle\}$, we can make sense of it as $\{w : c \Vdash \langle\pi(\llbracket\alpha\rrbracket^c), p(\llbracket\alpha\rrbracket^c)\rangle\}$. Still, the latter competes with the ‘natural capacity’ reading of ‘Rance can drink up Brent’s wine.’ as the modalized declarative $\Diamond(\text{Rance drinks up Brent’s wine})$. Because the proposition *Rance drinks up Brent’s wine* can be complemented without digging under the hood, we ceteris paribus prefer the natural capacity reading to the obligative reading. That is why it is easy to avoid coordinating the obligative readings of *must* and *can/could*.

On Lewis’s problem about permission Lewis’s ‘problem about permission’ (Lewis 1979b) is this. Allow (against the view of this chapter) that obligation and

⁵³If we do: see note 29.

permission are dual necessity and possibility modals over propositions: p is obligatory just when, for a ‘sphere of permissibility’ s , $s \subseteq p$; p is permitted just when $s \not\subseteq \bar{p}$. Suppose s obligates p , then the obligation is lifted, resulting in s' . What is s' ? If $s' = s \cup \bar{p}$, then everything compatible with the once forbidden is now permitted—so retracting an obligation to carry rocks all day would issue a permission to drink up all my wine. Canvassing various alternatives, Lewis concluded none worked.

But if obligation is a mandate to settle the question $?p$ in favor of p , permission—the retraction of obligation—is just the retraction of the mandate to settle the question. The obligation had surely made the not- p worlds ‘prohibited’. But retracting the obligation does not make the p worlds ‘permitted’—it just makes the not- p worlds ‘unprohibited’. The retraction just makes it no longer a matter of significance whether p . And then the fact that in a possibility, not- p , does not make that possibility in itself either prohibited or required, nor permitted. Instead, that fact is restored to its initial evaluative neutrality—for a time a matter of concern, it subsides once again back into indifference.

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