Regarding a question as determinately answered

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What's with that old suit?

Splain me something

- A question:
  - What does ‘it is indeterminate whether φ’ mean?
  - Some intuitive constraints: ...
    - It negates ‘it is determinate whether φ’
    - It is in some sense ‘at odds with’ both φ and $\neg\varphi$
  - How so?
- A problem: these can’t be respected while acknowledging any scope for indeterminacy
  - Williamson roughs out the argument in *Vagueness*, Barnett cleans it up in ‘Is vagueness *sui generis*?’, Hellie simmers it to a rich demiglace

Williamson's proposed analyses

1. Neither φ nor $\neg\varphi$: $\neg(\varphi \vee \neg\varphi)$
2. It is neither true that φ nor true that $\neg\varphi$: $\neg(T\varphi \vee T\neg\varphi)$

Problem with (1)

- $\neg(\varphi \vee \neg\varphi)$ DeMorganizes to

(*) $\neg\varphi \wedge \neg\neg\varphi$

- And that is a contradiction
- So if it is indeterminate whether φ, things are contradictory—so, presumably, it is determinate whether φ
  - $\neg$-E not needed
  - DeMorganization uncontroversially valid
  - The open future picture is one of gaps getting filled rather than gluts getting tidied up—more facts rather than less—so reconciling to gluttony is at least revisionary in this sense

Problem with (2)

- $\neg(T\varphi \vee T\neg\varphi)$ DeMorganizes to

(**) $\neg T\varphi \wedge \neg T\neg\varphi$.

- Now consider the 'intro' direction of the T-schema:

  T-I $\qquad \varphi \vdash T\varphi$

- This is valid;
- As, presumably, is its contraposed version:

Cp-T-I $\qquad \neg T\varphi \vdash \neg\varphi$

- But, assuming (**), Cp-T-I (and $\wedge$-I and -E) allow us to prove the inconsistent

(*) $\neg\varphi \wedge \neg\neg\varphi$:
No help here.

A third option and its discontents

- Barnett proposes a third analysis:
  3. It is neither *metaphysically fixed* that φ nor *metaphysically fixed* that $\neg\varphi$: $\neg(\mathrm{M}\varphi \vee \mathrm{M}\neg\varphi)$

  Well, (3) DeMorganizes to

(***) $\neg\mathrm{M}\varphi \wedge \neg\mathrm{M}\neg\varphi$.

- So consider the 'intro' direction of the M inference-schema:

  M-I $\quad$ $\mathrm{T}\varphi \vdash \mathrm{M}\varphi$

- This seems valid:
  - After all, its being true that φ surely suffices to fix in reality that φ;
- If so, then so, presumably, is the contraposed version:

Cp-M-I $\quad$ $\neg\mathrm{M}\varphi \vdash \neg\mathrm{T}\varphi$

- But, assuming (***), Cp-M-I (and $\wedge$-I and -E) allow us to prove (**), and onward to the inconsistent (*)—no help here either.

Some notation

- ω is a schematic letter ranging over *embedded questions*. Substitution instances: 'who shot JR'; 'what the meaning of life is' 'whether that is really your hair'
- $?\varphi$ abbreviates 'whether φ'
- $\mathrm{D}\omega$ abbreviates 'it is determinate ω'
- $\mathrm{D}!?\varphi$ abbreviates 'it is determinate that φ'
- $\neg\mathrm{D}\omega$ represents the logical form of 'it is indeterminate ω'

The short version

- These inference-schemata are intuitively valid:

D^{+}-I	$\varphi \vdash D?\varphi$
$D^{\neg}$-I	$\neg\varphi \vdash D?\varphi$

- If so, then so, presumably, are their contraposed versions:

Cp-D^{+}-I	$\neg D?\varphi \vdash \neg\varphi$
Cp-$D^{\neg}$-I	$\neg D?\varphi \vdash \neg\neg\varphi$

- So, assuming $\neg D?\varphi$, these rules (with $\wedge$-I) allow us to prove the contradiction (*) immediately;
- So (via $\neg$-I and -E), $D?\varphi$.

The essence of W-B-H

A. D?φ is weaker than both φ and $\neg\varphi$
  - Because it analyzes as a disjunction of operations on φ and $\neg\varphi$, where operator-intro is valid (Williamson, Barnett)
  - Brutely (Hellie)

B. The counterposed version of any valid rule is valid, so

- $\neg$D?φ is stronger than both φ and $\neg\varphi$.

Strategies of reply

A. D?φ is not weaker than both φ and $\neg\varphi$—because it is *epistemic* ('knowable whether φ': Williamson); or *sui generis cognitive* ('rough whether φ': Barnett); or *sui generis metaphysical* ('the world hasn't settled': Barnes and collaborators)

B. Contraposition is invalid; we thought otherwise because philosophers are mistaken about the subject-matter of logic (Hellie)
   - Confession:
     - Williamson and Barnett are really on thin ice—if we can come up with a sensible interpretation of not-merely-cognitive indeterminacy that will tie a nice bow on what everyone already knew, namely that metaphysical indeterminacy is important and we implicitly understand it
     - In abandoning (A) as part of our notion of metaphysical indeterminacy, Barnes and collaborators end up leaving me thinking that once we have some fact, it takes something else to make it determinate. This seems to change the subject: perhaps by D!?φ they mean 'God smiles when she entertains the fact that φ'

Expanding and digging

- Other rules bring similar worries
- A Moorean understanding of *belief* faces similar worries
- A commonality: inference involving supposition
- The root of the problem: information-sensitive content

From dilemma to determinacy

Contraposition isn't the only worry for friends of D^{+}-I and $D^{\neg}$-I. If dilemma is valid things also get freaky:

1	$\varphi \vee \neg\varphi$	Classical Tautology
2	$\quad\vert\ \varphi$	
3	$\quad\vert\ D?\varphi$	D^{+}-I: 2
4	$\quad\vert\ \neg\varphi$	
5	$\quad\vert\ D?\varphi$	$D^{\neg}$-I: 4
6	$D?\varphi$	$\vee$-E: 1, 3, 5

Moral: granting assumption (A), either everything is determinate or $\vee$-E is invalid

From conditional proof to determinacy

Same for conditional proof:

1	φ	
2	$D?\varphi$	D^+-I: 1
3	if φ, $D?\varphi$	if-I: 1, 2

1	$\neg\varphi$	
2	$D?\varphi$	$D^\neg$-I: 1
3	if $\neg\varphi$, $D?\varphi$	if-I: 1, 2

Those schematic conditionals look like the following should be permissible substitution-instances:

- If there will be a sea battle tomorrow then it is determinate whether there will be a sea battle tomorrow
- If there won't be a sea battle tomorrow then it is determinate whether there will be a sea battle tomorrow

Continuing

Those in turn seem to entail these:

- If there will be a sea battle tomorrow then it is determinate that there will be a sea battle tomorrow
- If there won't be a sea battle tomorrow then it is determinate that there won't be a sea battle tomorrow
  - Plausibly D!?φ shares semantic content with D?φ and presupposes φ

Those skate pretty close to asserting determinacy, if you ask me

Moore on the first-person

Fortunately, friends of indeterminacy can make common cause with other philosophers bedeviled by similar perplexities

- Let $B\varphi$ abbreviate 'I believe that φ'.
- Then famously Moore observed that one who accepts φ had better accept $B\varphi$ on pain of incoherence;
  - Conversely: one who accepts $B\varphi$ had better accept φ on pain of incoherence
- Somewhat plausibly, 'had better accept on pain of incoherence' is a kind of 'entailment'. If so, the following is 'valid' in that sense:

  B-I $\quad \varphi \vdash B\varphi$

  - Conversely:

    B-E $\quad \varphi \dashv B\varphi$

From contraposition to opinionation

- Granting the validity of contraposition, Mooreans accept the validity of this:

Cp-B-I $\quad \neg B\varphi \vdash \neg\varphi$

- Then:

1	$\quad\mid\quad \neg B\varphi \wedge \neg B\neg\varphi$	
2	$\quad\mid\quad \neg B\varphi$	$\wedge$-E: 1
3	$\quad\mid\quad \neg\varphi$	Cp-B-I: 2
4	$\quad\mid\quad \neg B\neg\varphi$	$\wedge$-E: 1
5	$\quad\mid\quad \neg\neg\varphi$	Cp-B-I: 4
6	$B\varphi \vee B\neg\varphi$	$\neg$-I: 1, 3, 5; DeMorgan

Moral: granting contraposition, Mooreans predict complete opinionation.

From dilemma to opinionation

Mooreans also have a problem with reasoning by dilemma:

1	$\varphi \vee \neg\varphi$	Classical Tautology
2	$\quad \varphi$	
3	$\quad B\varphi$	B-I: 2
4	$\quad B\varphi \vee B\neg\varphi$	$\vee$-I: 3
5	$\quad \neg\varphi$	
6	$\quad B\neg\varphi$	B-I: 5
7	$\quad B\varphi \vee B\neg\varphi$	$\vee$-I: 6
8	$B\varphi \vee B\neg\varphi$	$\vee$-E: 1, 4, 7

Moral: granting the validity of dilemma, Mooreans predict complete opinionation.

From conditional proof to ‘omniscience’?

Here’s half of the famous Ramsey-Moore-God paradox:

1	φ	
2	$B\varphi$	B-I: 1
3	if φ, $B\varphi$	if-I: 1, 2

That conditional sort of looks like it says ‘I’m omniscient’.

- Use the elimination rule to get the converse, which sort of looks like it says ‘I’m infallible’—Chalmers and Hajek: ‘we have the epistemic powers of a God’.
  - (Indeed they do!)

Why contraposition?

- Ordinarily it is regarded as a derived (meta-)rule.
- Suppose we have some rule

  P2R $\quad \pi \vdash \rho$

  where P2R is valid over a certain range of substitution-instances $\{\langle \pi_i, \rho_i \rangle\}_i$.
- Then we can argue for the validity of

Cp-P2R $\quad \neg\rho \vdash \neg\pi$

over the same range of substitution-instances as follows:

1	$\neg\rho$	
2	$\quad \pi$	
3	$\quad \rho$	P2R: 2
4	$\quad \neg\rho$	Reit: 1
5	$\neg\pi$	¬-I: 2, 3, 4

Morals

- Granting validity of ¬-I, Reit, and P2R we can prove the validity of Cp-P2R;
- More generally, granting validity of ¬-I and Reit we can prove the validity of the meta-rule Contraposition.
- ¬-I is well-motivated: granting that φ is the case, we argue to a contradiction; so to the extent that we are certain that the world isn't contradictory, φ had better not be the case. Boom! Done.
  - Probably not: aren't there people here who know Restall from Adam? I'm not one of them, regrettably
- What about Reit?
  - . . . and is there any connection to our perplexity with ∨-E and if-I?

When dreams take wing

Note that both rules involve interplay between the ‘fantasy’ of supposition and the (relative) ‘reality’ of the higher argument. Classical rules let pegasuses into the wild (or vice versa) to different extents and in different ways:

1. Some rules stay at a level:
   - $\wedge$-I
   - $\wedge$-E
   - $\vee$-I
   - $\neg$-E
   - if-E
2. Other rules mix the levels:

   2.1 These rules let you bring home to reality some things that became apparent in a supposition:
   - if-I
   - $\neg$-I

   2.2 This rule lets you bring some things from reality into the supposition:
   - Reit

   2.3 This rule lets you mush together something from reality and some stuff you learned in a supposition to say something about reality:
   - $\vee$-E

Ah, about that old horse on the front lawn . . .

- The problem here is that playtime has gotta end at some point; when it does that beautiful pegasus will return to being smelly, flea-bitten, swayback Old Paint.
- Less metaphorically, our classical rules were developed to describe the behavior of the classical connectives when the sentences on which they operate have a very important property:
  - **They don't shift meaning in the course of the argument.**
- After all, the following argument doesn't look so great:

1	$\neg\xi$	
2	$\quad \pi$	
3	$\quad \rho$	P2R: 2
4	$\quad \neg\rho$	Schmeit: 1
5	$\neg\pi$	$\neg$-I: 2, 3, 4

- But if the sentence that gets reiterated means something different once inside the supposition, this is the situation.

Why this might happen

- Context sensitivity has been driven like Old Paint the whole time since I was a kid.
- But a really important kind of context-sensitivity has been relatively neglected:
  - **Sensitivity to the information present in the context;**
    - Or to some other semantic parameter more generally.
- What if:
  - introducing a supposition manipulates parameters of that sort; and
  - the meaning of D?φ and Bφ is sensitive to the values of parameters of that sort?
- Then we could declare our problematic inference rules invalid.

A semantic fix

- Will use the apparatus of ‘test semantics’ to get this job done
  - General strategy from Veltman’s ‘Defaults in update semantics’
  - **NB: for the cognoscenti, my approach is propositional throughout—meanings aren’t ‘context change potentials’**
- Will treat ‘determinate’ as an operator on *questions*
- Will present an appropriate notion of validity

Taking stuff for granted

- At any stage in a process of inquiry, a certain amount is *taken for granted*:
  - In collective inquiry, this is the ‘common ground’, what is ‘jointly presupposed’ among all the interlocutors: perhaps the ‘beliefs’ of a ‘collective agent’ standing in for what everyone takes everyone to take everyone . . . to be presupposing
  - In solipsistic inquiry, this is the picture of the world one is working with
- The ‘taking for granted’ doesn’t need to be *serious* or *fully committed*: it might instead be *temporarily supposed*
- We can think of this as a single proposition, the conjunction of everything that is taken for granted

The context set

- We will use sets of worlds to represent propositions
  - Let W be modal space: then a proposition is an element of 2^W, the set of all sets of worlds
- So in particular, what is taken for granted at a stage is represented by the set of exactly the worlds compatible with what is taken for granted; the worlds that might be actual assuming what is taken for granted
  - We will call that the **context set** for that stage of inquiry
- We will represent the semantic state of play at a stage of inquiry by assigning it a *context*: a sequence of parametric values representing the stands taken in the stage on everything of semantic significance
- In particular, for the time being, we will assume $c = \langle i_c, \ldots \rangle$:
  - Where c is the context of a certain state of inquiry only if i_c is the context set for that state of inquiry

Semantic values

- We will use the following notation to abbreviate 'the semantic value of φ relative to the context c':

$$\lceil \varphi \rceil^c$$

- We will assume that, when it is defined, $\lceil \varphi \rceil^c \in 2^W$—that the semantic value of a sentence is a proposition

Assertion

- According to Stalnaker, conversation updates *intersectively*:
  - The 'essential effect' of the acceptance of an assertion at a stage of inquiry is this:
  - The inquiry updates to a new stage where the context set is the intersection of the old context set with the proposition expressed by the sentence asserted relative to that context
    - Note the connection to Bayesian learning theory: viewing the acquisition of evidence as updating the state of play of a solipsistic process of inquiry by coming to accept a sentence whose semantic value is that evidence, learning is also intersective in this way
- More formally:

$$i_{c \dotplus \varphi} = i_c \cap [\varphi]^c;$$

  where $c \dotplus \varphi$ is the context resulting from c by (directly) accepting an assertion of φ
- And in general:

$$i_{c'} = i_{c \dotplus \varphi};$$

  where c' is the context of the 'next' stage of inquiry

Accommodation

- 'Directly?' 'In general?'
- An absolute rule of inquiry:

$$i_c \neq \varnothing;$$

  'I will never play The Dane': one's ambition ceases
- Lewis: 'Make the message make sense'—if someone says something that seems nonsensical but which would be sensible if something additional were presupposed, we should within limits *accommodate*.
- More formally (and somewhat restrictedly):
  - Suppose the following:
    - $i_{c+\varphi} = \varnothing$
    - ψ is the weakest sentence such that $i_{c+\psi+\varphi} \neq \varnothing$
  - Then typically if φ is asserted against c, $i_{c'} = i_{c+\psi+\varphi}$

  Note that this sort of 'purely intersective accommodation' will only be an option when φ is information-sensitive!

The Veltman Diamond

- Veltman offers an semantics for 'epistemic *might*' which can be cast in our terms as follows:
  - $\lceil \Diamond\varphi \rceil^c = \ldots$
    - W iff $i_c \cap \lceil \varphi \rceil^c \neq \varnothing$
    - $\varnothing$ otherwise
  - $\Diamond\varphi$ has an **information-sensitive semantics:** what is presupposed influences which proposition its assertion expresses
- People put this by saying that $\Diamond\varphi$ 'tests the context set' for whether what φ expresses remains a live option:
  - If it does, $\Diamond\varphi$ gives a thumbs up;
  - Otherwise, a thumbs down
- Why?
  - Well suppose we haven't ruled out what φ expresses: then its semantic value as asserted will be trivially true, so we can accept the assertion in the normal way, intersectively
  - But if we have ruled it out, its semantic value as asserted will be trivially false, so we can't accept the assertion in the normal way: to do so would 'crash the context'
    - Indeed, it's not even possible to accept the assertion via purely intersective accommodation
    - So we will (barring some fancy footwork) reject the assertion.

The dual of the Veltman Diamond . . .

- Works like this:
  - $\lceil \Box\varphi \rceil^c = \ldots$
    - W iff $i_c \subseteq \lceil\varphi\rceil^c$
    - $\varnothing$ otherwise
  - That tests for whether what φ expresses is taken for granted
  - We could pronounce that . . .
    - 'it must be so that φ'
    - 'we are (I am) taking it for granted that φ'
    - 'we (I) believe that φ'—assuming no suppositions are turned on
- . . . is the Moorean B?

Test semantics for B?

- If so, then Bφ is information-sensitive
- That's something we want
- And it does kinda test for whether we believe what φ expresses
- Once we have a notion of validity on the table, we will also see that it validates B-I and B-E
- And invalidates our world-hopping rules

Test semantics for D?

Yes, but first we need to talk about questions

Semantic values of questions

- The semantic value of ω relative to c—$|\omega|^c$—is a partition of modal space (of the subregion of modal space where all presuppositions of ω relative to c are met)
  - A partition of S is a set of subsets of S such that no two of them contain the same member and any member of S is in one of them—they are *mutually exclusive and jointly exhaustive*
    - (This is only so for 'informational' questions—different for practical questions, questions about conditionals, and explanatory questions)
- We will write $\mathfrak{Q}(W)$ for the set of partitions of subregions of modal space
  - So $|\omega|^c \in \mathfrak{Q}(W)$

Examples

- $|$Who shot JR$|^c$ is the set of sets of worlds (at which JR was shot by a person) such that:
  - If at w, Suellen shot JR, while at w', Kristin shot JR, w and w' are in different cells;
  - If at both w and w', Suellen shot JR, w and w' are in the same cell
- $|$Are you the farmer$|^c$ is the two-membered set of sets of worlds (at which the addressee of c exists and there is exactly one farmer of the sort salient in c) such that:
  - All worlds at which the addressee of c exists and is a farmer of the sort salient in c are in one cell
  - All worlds at which the addressee of c exists but is not a farmer of the sort salient in c are in the other cell

Questions and inquiry

- Slogan: *questions structure inquiry*
- Theory to go with the slogan:
  - At a stage of inquiry a number of questions are *live*
  - Any learning that goes on as that stage updates to the next stage is the making of progress at answering a live question
- More formally:
  - $c = \langle i_c, \ell_c \ldots \rangle$
  - $\ell_c \subseteq \mathfrak{Q}(W)$—it represents the set of questions (partitions of subregions of modal space) live at c
  - For some $q \in \ell_c$, for some a in the set of union sets of members of the power set of q, $i_{c'} = i_c \cap a$
    - This is compatible with my not learning anything

Livening things up

- How does the constituency of ℓ_c get updated?
- The most straightforward way is by *accepting an explicit interrogative*
  - Against c, someone asks 'are you the farmer?'
  - Everyone agrees that this is a question worth taking seriously
  - The result is to 'direct-inject' the semantic value of that question to the list of live issues
    - More formally: $\ell_{c+\text{are you the farmer?}} = \ell_c \cup \{|\text{are you the farmer}|^c\}$

Accommodation of an assertion

- Against c, someone says 'goats eat cans'. Two options:
  1. Although this is out of the blue, we decide to accept the assertion. We do so by accommodation: first raise the question then answer it:
     - $c' = c +$ do goats eat cans? $+$ goats eat cans
     - $\ell_{c'} = \ell_c \cup \{|\text{do goats eat cans}|^c\}$
     - $i_{c'} = i_c \cap \lceil \text{goats eat cans} \rceil^c$
     - Public speakers sometimes do this explicitly: 'Will Bob Dole bring prosperity to the American family? Yes he will. Does Bob Dole have the experience needed to make this country grow? Yes he does', says Bob Dole.
  2. Because it is out of the blue, we decide to reject the assertion:
     - Because $|\text{do goats eat cans}|^c \notin \ell_c$ we cannot accept the assertion without accommodating by raising a question to which it is an answer;
     - We do not feel like doing this, so we reject the assertion:
     - 'That's irrelevant', we say, and move on.

Knowledge ascriptions

- Somewhat inclined to think the function of knowledge ascriptions is to regulate channels of information:
  - ‘Sam knows whether goats eat cans’ marks Sam’s opinion on the question ‘do goats eat cans’ as authoritative
  - In the sense that we commit to setting the stance i_c takes on whether goats eat cans in accord with our view of Sam’s opinion on that question
  - So a knowledge ascription presupposes that its embedded question is live
- A test semantics capturing these ideas is available
- Could probably generalize this to ‘there is evidence whether φ’ and the like

Limits of inquiry

- We could think of ‘it is metaphysically necessary that φ’ as establishing a sort of ‘ceiling’ for inquiry for one. In saying that, one signals that one is not willing to consider the negative answer to $?\varphi$: one regards $\neg\varphi$ as lacking a coherent meaning, as failing to describe any way the world could be such that there might be a point to figuring out whether it is that way. One won’t even consider $\neg\varphi$ in supposition—except perhaps within a context of purely formal reasoning.
  - (I realize there are a bunch of metaphysicians in the room, so I don’t expect people to agree with this—anyway, let’s roll with the idea.)
- What would a ‘floor’ to inquiry for one look like? It would be something like a point past which one regards further precision as unattainable. Although both φ and $\neg\varphi$ express coherent situations, the question $?\varphi$ must, by one’s lights, be rejected.

The ultimate question

- Some questions entail others:
  - If we had the answer to 'who shot both JR and Grampa Ewing?', we would thereby have the answer to both 'who shot JR' and 'who shot Grampa Ewing': after all, any cell in the former partition is strictly within some cell in each latter partition (of course the former question has stricter presuppositions than either latter question)
  - The conjunction of two questions—'who shot JR, and are you the farmer'—entails each conjunct in this sense
- Some questions entail very many others:
  - We can imagine an extremely strong question—'what is the case?', perhaps—such that if we had a complete answer to it, we would thereby have the answer to every question
  - We can imagine a slightly weaker question—let us pronounce it 'what is determinately the case?'—such that if we had a complete answer to it, we would thereby have the answer to every question that does not fall below our floor of enquiry
  - Let us call that **our ultimate question**
    - Formally: $c = \langle i_c, \ell_c, \Omega_c, \ldots \rangle$—$\Omega_c \in \mathfrak{Q}(W)$ is the semantic value of our ultimate question
    - For all $q \in \ell_{c'}, \Omega_c \geq q$
    - $q' \geq q$ just if for every cell $a' \in q'$, there is some cell $a \in q$ such that $a' \subseteq a$

Constrained by our ultimate question

- Here's the central clause:
  - For all $q \in \ell_{c'}, \Omega_c > q$
    - Since we don't have a way of getting rid of live issues, that implies the same for all $q \in \ell_c$—we could stipulate that if we felt the need to kill off issues, but perhaps killing off issues is like forgetting stuff
- That says that we can't update by accepting any questions that cut more finely than the ultimate question; in particular:
  - We won't accept any explicit question that cuts more finely
  - We won't accept any assertion that presupposes a question cutting more finely
  - We won't accept any knowledge-ascription that presupposes a question that cuts more finely

Updating our ultimate question?

- So far no restrictions on the relation of Ω_c to $\Omega_{c'}$
- You might have noticed that if $\Omega_c < \Omega_{c'}$ we could get something like a growing block picture ...
- Let's stipulate that $\Omega_c \leq \Omega_{c'}$: otherwise we might be compelled to get rid of information we had already collected

Testing our ultimate question

- Test semantics for D:
  - $[D\omega]^c = \ldots$
    - W iff $\Omega_c \geq |\omega|^c$
    - $\varnothing$ otherwise
- Explicitly: this tests for whether the complement of D crosscuts our ultimate question—whether the answer to the complement lies below our floor of inquiry. If no, the test is passed.

Options

We could vary this along two dimensions:

1. - This requires *strong* determinacy: *every* cell for ω must be in some cell for our ultimate question
   - We could also imagine *weak* determinacy: *some* cell for ω must be in some cell for our ultimate question
     - Probably each would have uses: perhaps the former for free will (if they tie me down what I will do will be determinate); perhaps the latter for vagueness (I couldn't learn anything such that, given it, a man with that hair pattern is determinately bald)
2. - This assumes that we look all throughout modal space to find a pair of worlds sharing a cell for our ultimate question but separated by ω: in that sense it is probably more like a 'it is conceptually necessarily determinate' operator;
   - We could also restrict to worlds in i_c, which would make it more like a 'it is surely determinate' operator
     - Probably each of these would have uses: the latter for the outcome of a quantum experiment (we learned that it was determinately located) the former perhaps for vagueness (it's not even conceivable that I could learn something such that, given it, a man with that hair pattern is determinately bald)

Comments

- $\neg D\omega$ of course performs the opposite test: we say something true just if our ultimate question is crosscut by ω
  - That gets us *weak* indeterminacy—some total packages of ways things might be determinately leave it indeterminate in regard to ω, but maybe others don't;
  - We could also imagine *strong* indeterminacy: all total packages of ways things might be determinately leave it indeterminate
- We will need to talk about how this resolves our various difficulties, but that will require a story about entailment. For the moment, let's do some interpretation in a somewhat more informal key . . .

Does D behave like ‘determinate’?

- It does seem to recapitulate something like the ‘cognitive role’ of much determinacy-discourse. In particular:
  - Suppose that against c, we accept $\neg D?\varphi$:
  - Since c is nondefective, $|?\varphi|^c \nleq \Omega_c$
  - So we cannot update by adding $|?\varphi|^c$ to ℓ_c
  - So we will reject the question $?\varphi$
  - And we will reject an assertion of φ, and we will reject an assertion of $\neg\varphi$
  - And we will reject an assertion of ‘Sam knows whether φ’
  - But it allows us to ‘wait around’ for our ultimate question to change so that we might soften up our position here, open-future style

Am I metaphysical enough for you?

- I like a picture of the world like this:
  - Modal space is classical: for any meaningful question that can be asked from a completely objective point of view, each world (compatible with all presuppositions of that question) gives it a sure answer
  - The total course of history at a world, for example, is fixed; more generally, anything a completely objective God would be able to say about it is either true or false
  - More generally, when we find 'funny business' of any kind (norms, consciousness, math), I'm inclined to make it in some sense a 'creature of the mind' rather than a 'creature of the world'
- Our story is in line with this: 'determinately' doesn't serve to name anything that can serve as an ingredient of the world; more generally, its function is not to represent the world as being any way in particular: it is rather to manifest an aspect of a perspective on the world
- So you might think the talk is off topic: perhaps you think I agree with Williamson; that my D means 'knowable in principle' or some such. (Let me grant that maybe 'in principle' could be patched up so that the operators are equivalent.)

Metametaphysics

- But that wouldn't make me Williamson. The knowledge operator, too, does not function to represent the world, but rather to manifest an aspect of a perspective. The same is true of our Moorean belief operator. (By contrast, Williamson thinks knowledge is a kind of state which is constituted by a belief state being 'safe', more or less).
- The picture is more like Carnap, Schlick, Tractatus, or Kant: the objective world contains *no* funny business—nothing psychological even (though it does contain animals).
- Me and my psychology, my norms, my epistemic channels, my floor of inquiry (including the location of the present): these are 'real', but they are not constituents of the objective world. Rather, they are aspects of the form of my world.
  - Not an especially popular distinction these days, of course, but proof of the pudding and all that . . .
- This feeds in alongside the portentous remarks I made about the subject-matter of logic; to this we now turn.

Classical entailment

- $\varphi \vdash \psi$ just if $\|\varphi\| \subseteq \|\psi\|$
  - At all worlds at which the premiss is true, the conclusion is true
- That's unsuitable for the treatment of test-operators: independent of context, no such thing as 'the worlds at which Bφ is true'
- Fixing a context is also not interesting: sure, relative to c^- in which $\neg\varphi$ is accepted, Bφ entails everything and relative to c^+ in which φ is accepted, Bφ is entailed by everything. So what?
- If our aim is to capture the sense of $\vdash$ we exploited in the first sections, this lacks the generality (it's always OK to say this if you said that) we find in any notion of entailment
  - Compare Kaplan's LD and the move to diagonal propositions

Informational entailment

- Here's a definition:
  - $\varphi \vdash \psi$ just if every c that supports φ supports ψ
  - Where c supports φ just if $\lceil \varphi \rceil^c \subseteq i_c$
- That says that entailment is: no matter what context you start in, if you accept the premiss, you are thereby already in a position from which you accept the conclusion
- Entailment makes for security: assuming the premiss hasn't led you astray, the conclusion won't do so
- Entailment is general: it involves *every* context supporting the premiss

Informational entailment and B

- Let's prove B-I: $\varphi \vdash \mathrm{B}\varphi$
  - Suppose that $i_c \subseteq \lceil\varphi\rceil^c$. Then $\lceil\mathrm{B}\varphi\rceil^c = W \supseteq i_c$, as desired.
- Let's prove B-E: $\mathrm{B}\varphi \vdash \varphi$
  - Suppose that $i_c \subseteq \lceil\mathrm{B}\varphi\rceil^c$. Then if $\lceil\mathrm{B}\varphi\rceil^c = W$, $i_c \subseteq \lceil\varphi\rceil^c$; and if $\lceil\mathrm{B}\varphi\rceil^c = \varnothing$, i_c is a subset of everything: either way, as desired.

Informational entailment and D

- Let's prove D^+-I: $\varphi \vdash D?\varphi$
  - Suppose that $i_c \subseteq \lceil \varphi \rceil^c$: then at some point one learned enough to learn that φ. So the totality of questions one accepts entail the question whether φ; assuming that acceptance of questions is monotonic, $\Omega_c \geq |?\varphi|^c$.
  - (***vague anxiety about disjunctive questions—running late!)

Update entailment

- Here's an alternative definition:
  - $\varphi \vdash \psi$ just if for all c, $i_{c+\varphi} \subseteq i_{c+\varphi+\psi}$
- That says that entailment is: no matter what context you start in, if you update by accepting the premiss, you have thereby done enough so that you already accept the conclusion
  - Here the mark of 'acceptance' of φ is certainty: the content of φ is already part of what one believes
- That gets us the generality of entailment: the relation between the sentences shows up with regard to *any prior context*
- And it gets us the idea of entailment as 'security': no added blame for taking on any epistemic risks can be laid at the feet of the conclusion once the premiss has already been accepted.
- Proving entailments here is harder because we need to accommodate possible differences in meaning of φ between premiss and conclusion.

Update entailment and B-I

- Let's prove B-I: $\varphi \vdash \mathrm{B}\varphi$
  - $i_{c\dotplus\varphi\dotplus\mathrm{B}\varphi} = i_{c\dotplus\varphi} \cap \lceil \mathrm{B}\varphi \rceil^{c\dotplus\varphi}$;
  - Because $i_{c\dotplus\varphi} = i_c \cap \lceil \varphi \rceil^c$, and because $\lceil \mathrm{B}\varphi \rceil^{\kappa}$ is either W or $\varnothing$, the condition for entailment can fail only if

  \# $\lceil \mathrm{B}\varphi \rceil^{c\dotplus\varphi} = \varnothing$; namely
  - $i_c \cap \lceil \varphi \rceil^c \nsubseteq \lceil \varphi \rceil^{c\dotplus\varphi}$.
  - But that would happen only if φ is information-sensitive; so that

  \#\# $\lceil \varphi \rceil^c = W$ (otherwise $c \dotplus \varphi$ would be a crashed context, the information state of which is a subset of everything);
  - But then $i_c = i_{c\dotplus\varphi}$ (because intersecting with W doesn't change anything);
  - And moreover, no accommodation needed to occur in order to accept φ against c;
  - So c and $c \dotplus \varphi$ are alike in all parameters: namely $c = c \dotplus \varphi$;
  - So it follows from (#) that $\lceil \mathrm{B}\varphi \rceil^c = \varnothing$;
  - But that is inconsistent with (##).

Update entailment and B-E

- This one ($\mathrm{B}\varphi \vdash \varphi$) is a lot easier to prove:
  - Suppose $\lceil \mathrm{B}\varphi \rceil^c = \varnothing$: then $i_{c \dotplus \mathrm{B}\varphi} = \varnothing$, which is a subset of everything.
  - Alternatively, suppose $\lceil \mathrm{B}\varphi \rceil^c = W$: then $c \dotplus \mathrm{B}\varphi = c$ and $i_c \subseteq \lceil \varphi \rceil^c$; so $\lceil \varphi \rceil^{c \dotplus \mathrm{B}\varphi} = \lceil \varphi \rceil^c$; so $c \dotplus \mathrm{B}\varphi \dotplus \varphi = c \dotplus \varphi = c$; as desired.
- I find it somewhat plausible that the asymmetry here is associated with the asymmetry between the ease of coming up with counterexamples to my present omniscience and the impossibility of coming up with counterexamples to my present infallibility.

Update entailment and D^+-I/$D^\neg$-I

- Show: $\varphi \vdash D?\varphi$ (proof mutatis mutandis for $D^\neg$-I)
  - Want to show that $i_{c\dotplus\varphi} \subseteq i_{c\dotplus\varphi\dotplus D?\varphi}$
  - If φ is information-sensitive, then either $c \dotplus \varphi$ is crashed and we're done; or $\lceil\varphi\rceil^c = W$, which is always kosher according to the ultimate question.
  - Otherwise, $\lceil\varphi\rceil^c = \lceil\varphi\rceil^{c\dotplus\varphi}$;
  - So $|?\varphi|^c = |?\varphi|^{c\dotplus\varphi}$.
  - Moreover, because *learning is making progress at answering a live question*, for some $q \in \ell_c$, for some a in the set of union sets of members of the power set of q, $i_{c\dotplus\varphi} = i_c \cap a$—so $a = \lceil\varphi\rceil^c$;
  - So $q \geq |?\varphi|^c$;
  - So, because for any $q \in \ell_c, q \leq \Omega_c$, $|?\varphi|^c \leq \Omega_c$;
  - And because asserting φ against c does nothing to manipulate the ultimate question, we may suppose that $\Omega_{c\dotplus\varphi} \geq \Omega_c$ (unequal if, say, the passage of time makes the ultimate question sharper);
  - So $|?\varphi|^{c\dotplus\varphi} \leq \Omega_{c\dotplus\varphi}$;
  - So $\lceil D?\varphi\rceil^{c\dotplus\varphi} = W$;
  - So $i_{c\dotplus\varphi} = i_{c\dotplus\varphi\dotplus D?\varphi}$, as desired.

Valid supposition-free arguments

- A supposition-free argument is a sequence $\mathcal{A}$ of (declarative) sentences $\langle \varphi_1^{\mathcal{A}}, \ldots, \varphi_n^{\mathcal{A}} \rangle$ along with a designation of some (maybe empty) subset of the $\varphi_j^{\mathcal{A}}$ as 'premisses'
- The c-initial sequence of contexts for $\mathcal{A}$ is $\langle c_0^{\mathcal{A}} = c, \ldots, c_{j+1}^{\mathcal{A}} = c_j^{\mathcal{A}} \dot{+} \varphi_{j+1}^{\mathcal{A}}, \ldots, c_n^{\mathcal{A}} = c_{n-1}^{\mathcal{A}} \dot{+} \varphi_n^{\mathcal{A}} \rangle$
- $\mathcal{A}$ is c-risk-free just if, for each $j \in \{1, \ldots, n\}$, either $\varphi_j^{\mathcal{A}}$ is a premiss in $\mathcal{A}$ or $i_{c_{n-1}^{\mathcal{A}}} \subseteq i_{c_n^{\mathcal{A}}}$
- $\mathcal{A}$ is valid just if $\mathcal{A}$ is c-risk-free for all c

Valid arguments with supposition

- An argument is a sequence $\mathcal{A} = \langle x_1^{\mathcal{A}}, \ldots, x_n^{\mathcal{A}} \rangle$, where some of the $x_j^{\mathcal{A}}$ are sentences (some of which are marked as premisses) and the remainder are subproofs
- A subproof is a sequence $\mathcal{S} = \langle x_1^{\mathcal{S}}, \ldots, x_m^{\mathcal{S}} \rangle$, where some of the $x_j^{\mathcal{S}}$ are sentences (some of which are marked as assumptions) and the remainder are subproofs
- The c-initial sequence for $\mathcal{A}$ is such that $c_0^{\mathcal{A}} = c$, while $c_{j+1}^{\mathcal{A}} = \ldots$
  - $c_j^{\mathcal{A}} \dotplus x_{j+1}^{\mathcal{A}}$, if $x_{j+1}^{\mathcal{A}}$ is a sentence;
  - $c_j^{\mathcal{A}} \downarrow x_{j+1}^{\mathcal{A}}$, if $x_{j+1}^{\mathcal{A}}$ is a subproof;
  - Where $i_{c^* \downarrow x} = i_{c^*}$: this reflects the idea that we shouldn't be able to get any information about the objective world by pure reason alone, though we might 'repackage' information we already had

Continued

- If x is a subproof $\mathcal{S} = \langle x_1^{\mathcal{S}}, \ldots, x_m^{\mathcal{S}} \rangle$ somewhere in the hierarchy of $\mathcal{A}$ occurring after an element whose associated context (assuming c-initial evaluation of $\mathcal{A}$) is c^*, then its associated sequence of contexts $\langle c_0 = c^*, \ldots, c_m \rangle$ is such that $c_{j+1} = \ldots$
  - $c_j^{\mathcal{S}} \dot{+} x_{j+1}^{\mathcal{S}}$, if $x_{j+1}^{\mathcal{S}}$ is a sentence;
  - $c_j^{\mathcal{S}} \downarrow x_{j+1}^{\mathcal{S}}$, if $x_{j+1}^{\mathcal{S}}$ is a subproof;
  - Where $i_{c^* \downarrow x} = i_{c^*}$: this reflects the idea that we shouldn't be able to get any information about the objective world by pure reason alone, though we might 'repackage' information we already had
  - Note here that we're doing it for indicative supposition: for subjunctive supposition we would want to not intersect with the content of the sentence but rather replace every world in the context set with the world in the context of the sentence closest to it (something like Will Starr's subjunctive conditional)

Continued

- An argument is *c*-risk-free just if every sentence anywhere in the tree is either (i) a premiss; (ii) a supposition; or (iii) supported by the context that is its predecessor when the whole thing is evaluated *c*-initially
- An argument is valid just if it is *c*-risk-free for every *c*

Supposition and information-sensitivity

- We are now in a position to see how our initial worries about our introduction rules for D and B can be blocked:
- On the apparatus we have developed, the arguments that purport to derive absurdities can be seen to mix fantasy and reality in ways that become illegitimate once information-sensitive operators show up

Counterposed introduction rules

- The derivations of Cp-B-I and Cp-D$^+$-I run like this:

1	$\neg B\varphi$	
2	$\quad \varphi$	
3	$\quad B\varphi$	B-I: 2
4	$\quad \neg B\varphi$	Reit: 1
5	$\neg\varphi$	¬-I: 2, 3, 4

1	$\neg D?\varphi$	
2	$\quad \varphi$	
3	$\quad D?\varphi$	D$^+$-I: 2
4	$\quad \neg D?\varphi$	Reit: 1
5	$\neg\varphi$	¬-I: 2, 3, 4

The trouble with Reit

- Here's the trouble for the B version (let c^j be the context for line (j)):
  - i_{c^1} is either $\varnothing$ (if i_c, the root information state, supports φ) or a subset of $\lceil\varphi\rceil^c$: in the former case the argument is c-risk-free, so let us restrict attention to the latter case
  - In that case i_{c^2} is a subset of $\lceil\varphi\rceil^{c^1=c}$
  - So $i_{c^3} = i_{c^2}$
  - So $\lceil\neg\mathrm{B}\varphi\rceil^{c^3} = \varnothing$;
    - But in that case line (4) (neither a premiss nor a supposition) is not supported by its predecessor context!
    - The use of Reit here renders the argument no longer c-risk-free, and therefore invalid: so in that sense Reit is itself an invalid rule.
- The problem with the D version is largely the same
  - Though there is a complication: in order for the supposition in (2) not to crash the context, we would need a view on how the ultimate question updates under supposition: perhaps that although it does not accommodate at the root level, it is happy to do so under supposition.
  - In that case we could say that $\Omega_{c^2} \geq |?\varphi|^{c^2}$, so that $i_{c^3} = i_{c^2} \neq \varnothing$, while $\lceil\neg\mathrm{D}?\varphi\rceil^{c^3} = \varnothing$, so that (4) (neither a premiss nor a supposition) is unsupported by its predecessor context.

The trouble with dilemma

- Here is the dilemma argument from B-I to opinionation:

1	$\varphi \vee \neg\varphi$	Classical Tautology
2	$\quad \varphi$	
3	$\quad \mathrm{B}\varphi$	B-I: 2
4	$\quad \mathrm{B}\varphi \vee \mathrm{B}\neg\varphi$	∨-I: 3
5	$\quad \neg\varphi$	
6	$\quad \mathrm{B}\neg\varphi$	B-I: 5
7	$\quad \mathrm{B}\varphi \vee \mathrm{B}\neg\varphi$	∨-I: 6
8	$\mathrm{B}\varphi \vee \mathrm{B}\neg\varphi$	∨-E: 1, 4, 7

- The problem here is that:
  - When i_c is neutral on φ (it overlaps both $\lceil\varphi\rceil^c$ and $\lceil\neg\varphi\rceil^c$), the same is true of i_{c^1} (because $\lceil\varphi \vee \neg\varphi\rceil^c = W$);
  - So that $\lceil \mathrm{B}\varphi\rceil^{c^1} = \lceil \mathrm{B}\neg\varphi\rceil^{c^1} = \varnothing$;
  - So that (assuming classical disjunction) $\lceil \mathrm{B}\varphi \vee \mathrm{B}\neg\varphi\rceil^{c^1} = \varnothing$;
  - Since c^1 is the predecessor context of (8), the semantic value of (8) at its predecessor context is $\varnothing$;
  - So that (8) (neither premiss nor supposition) is unsupported by its predecessor.
- Analogous problem for the D version (need to assume that manipulations of UQ by supposition are undone upon discharge)

Conditional proof

- Here's the 'omniscience' direction of Ramsey-Moore-God:

1	φ		
2	$\mathrm{B}\varphi$	B-I: 1
3	$\text{if}\, \varphi, \mathrm{B}\varphi$	if-I: 1, 2

- Suppose c^0 is neutral on φ. Then $[\![\mathrm{B}\varphi]\!]^{c^0} = \varnothing$. Note also that c^0 is the predecessor context of (3).
- To get to the root of the problem we would need a semantic theory for if. Here are two:
  a. $[\![\text{if}\, \varphi, \psi]\!]^c = W$ just if $i_c \cap [\![\varphi]\!]^c \subseteq [\![\psi]\!]^c$; $\varnothing$ otherwise
  b. $[\![\text{if}\, \varphi, \psi]\!]^c = W$ just if $i_c \cap [\![\varphi]\!]^c \subseteq [\![\psi]\!]^{c+\varphi}$; $\varnothing$ otherwise

  (a) goes more with informational entailment, (b) more with update entailment
- Both of these are 'Ramsey-test friendly' for information-insensitive fragments; (b) also for information-sensitive. Accordingly, different results on validity of conditional proof:
  - On theory (a), the semantic value of (3) is $\varnothing$ so that it is unsupported by its predecessor context
  - On theory (b), the semantic value of (3) is W so that it is supported by its predecessor

Conditional proof and D

1		φ	
2		$\mathrm{D}?\varphi$	D^{+}-I: 1
3	$\text{if}\,\varphi, \mathrm{D}?\varphi$		if-I: 1, 2

- Let the root context be one in which the ultimate question is crosscut by whether φ. The supposition (1), as per above, can perhaps force this aspect of the root context around, but it needs to be undone at (3).
- What is the semantic value of (3)? On theory (b) of the conditional, it is W; on theory (a), it is $\varnothing$.
- An alternative approach to the conditional:
  c. $\lceil \text{if}\,\varphi, \psi \rceil^{c} = W$ just if $i_{c \dotplus \varphi} \subseteq \lceil \psi \rceil^{c}$; $\varnothing$ otherwise

  Here we could add the proviso that $\dotplus$ goes by intersection *when it is defined*. We could then claim that $c \dotplus \varphi$ is undefined because the ultimate question is crosscut by whether φ; in that case the semantic value of (3) would be $\varnothing$.

Reductio?

- Are there cases in which reductio is invalidated?
- Perhaps in the derivation of Cp-B-E: $\neg\varphi \vdash \neg B\varphi$?

$$
\begin{array}{r|l|l}
1 & \neg\varphi & \\
\hline
2 & \quad B\varphi & \\
3 & \quad \varphi & \text{B-E: 2} \\
4 & \quad \neg\varphi & \text{Reit: 1} \\
5 & \neg B\varphi & \neg\text{-I: 2, 3, 4}
\end{array}
$$

- Reiteration is unproblematic here when φ is information-insensitive: does reductio fail?
- No reason to think so, because $\neg B\varphi$ is supported by c_1.
- Whether there are other reasons to worry about the mixing of fantasy and reality by reductio is an open question.

Logic and the world

- Let's tie together the portentous remarks about metametaphysics and the subject-matter of logic:
  - Our notion of validity validates the Moore rules; accordingly it has nothing to do with metaphysical necessity—if this is understood as an entirely objective phenomenon
  - Rather, its subject-matter is something more like the boundaries for a coherent stream of consciousness
  - If we are feeling idealistic, we might think that metaphysical necessity should accommodate not only the boundaries of the *contents* of the stream of consciousness (the objective world), but also the boundaries of its *form*, and of the relation of content to form
  - We could then say that our affirmation of LEM is of little metaphysical significance: when it is indeterminate how the disjunction shakes out, its particular way of shaking out is merely noumenal; what matters is that the merely noumenal is beyond the limits of my world
  - That is the sense in which making φ determinate doesn't add anything beyond making φ so.