3.6 Optimization Problems

p.202 #17

Given running cost $C(v) = 0.9 + 0.0016v^2$ per 100 km

Let T be Brenda's costs for the trip

$T =$ Running cost + her own salary.

$= \frac{1500}{100} C(v) \quad + \$30/h$

$$T(v) = \frac{1500}{100}(0.9 + 0.0016v^2) + 30 \times \left(\frac{1500}{v}\right) \quad t = \frac{d}{v}$$

$$= 15(0.9 + 0.0016v^2) + \frac{45000}{v}$$

$$= 13.5 + 0.024v^2 + 45000v^{-1}$$

$$T'(v) = 0.048v - 45000v^{-2}$$

$$0 = 0.048v - \frac{45000}{v^2}$$

$$0 = \frac{0.048v^3 - 45000}{v^3}$$

$$45000 = 0.048v^3$$

$$v^3 = 937500$$

$$v \doteq 97.9 \text{ km/h}.$$

Double Derivative Test:

$$T''(v) = 0.048 + 90000v^{-3}$$

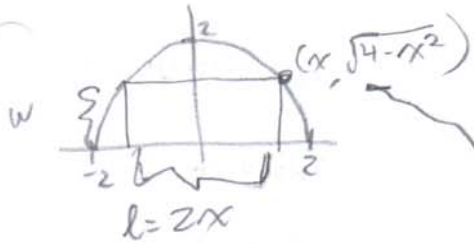
$$T''(97.9) > 0 \Rightarrow \text{Local min}.$$

$\therefore$ The speed that will minimize Brenda's cost is 97.9 km/h.

#18)

Recall

Equation of circle



$$x^2 + y^2 = r^2.$$
$$x^2 + y^2 = 2^2$$
$$y^2 = 4 - x^2$$
$$y = \pm\sqrt{4-x^2}$$
$$y = \sqrt{4-x^2} \text{ (only the top half)}$$

Let A represent the area

$$A = l \times w$$
$$A(x) = (2x)(\sqrt{4-x^2})$$
$$A(x) = 2x(4-x^2)^{\frac{1}{2}}$$
$$A'(x) = 2(4-x^2)^{\frac{1}{2}} + 2x\left(\tfrac{1}{2}\right)(4-x^2)^{-\frac{1}{2}} \times (-2x)$$
$$= 2(4-x^2)^{\frac{1}{2}} - \frac{2x^2}{(4-x)^{\frac{1}{2}}}$$
$$= \frac{2(4-x^2) - 2x^2}{(4-x)^{\frac{1}{2}}}$$
$$= \frac{8-2x^2-2x^2}{(4-x)^{\frac{1}{2}}}$$
$$A'(x) = \frac{8-4x^2}{(4-x)^{\frac{1}{2}}}$$

$$0 = 8 - 4x^2$$
$$4x^2 = 8$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$
$$x = \sqrt{2} \text{ (reject } -\sqrt{2}\text{)}$$

I am skipping the second derivative test. SORRY.

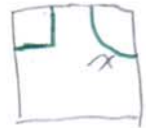
$\therefore$ Dimensions are

$$l = 2x = 2(\sqrt{2}) = 2\sqrt{2}$$
$$W = \sqrt{4-x^2} = \sqrt{4-(\sqrt{2})^2} = \sqrt{2}$$

Max area $= 2\sqrt{2} \times \sqrt{2} =$ 4 sq units.

#19) Diagram seems complicated in the textbook

- the edging refers to the green lines!



a) Let x be the edging used for the $\frac{1}{4}$ circle. the length of edging used for each side of the square garden is:

$$\frac{20 - x}{2}$$

b) $C = 2\pi r \rightarrow$ Circumference of a circle.

$x = \frac{2\pi r}{4} \rightarrow$ length of edging $= \frac{1}{4}$ of circumference.

$4x = 2\pi r$

$r = \frac{4x}{2\pi}$

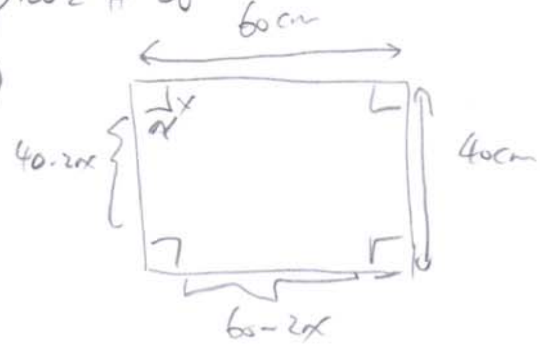
$r = \frac{2x}{\pi}$

c) Combined area: $A(x) = \left(\frac{20-x}{2}\right)^2 + \frac{1}{4}\pi\left(\frac{2x}{\pi}\right)^2$

d) Don't worry. It's kind of irrelevant to what we're learning.

p.202 # 20

a)



b)

$V = lwh$

$V(x) = (60-2x)(40-2x)x$

c) Domain

$\{x \mid x \in \mathbb{R},\ 0 < x < 20\,cm\}$ $\frac{40}{2} = 20$

d) $V(x) = (60-2x)(40-2x)x$

$= (2400 - 120x - 80x + 4x^2)x$

$= (2400 - 200x + 4x^2)x$

$V(x) = 4x^3 - 200x^2 + 2400x$

$V'(x) = 12x^2 - 400x + 2400$

$0 = 4(3x^2 - 100x + 600)$

$0 = 3x^2 - 100x + 600$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\cancel{x = 25.49}$ or $x = 7.85$

rejected
outside domain

$V''(x) = 24x - 400$

$V''(7.85) = 24(7.85) - 400$

$= 188.4 - 400$

$= -211.6$

< 0

$\Rightarrow$ Local max.

$\therefore$ Max Volume at these dimensions.

$h = x = 7.85\,cm$

$w = 40-2x = 24.3\,cm$

$l = 60-2x = 44.3\,cm$.

Volume $= 7.85 \times 24.3 \times 44.3$

$= 8450\,cm^3$

Back of textbook had exact answer, but as this is an application that involves measurement, approximate answer is fine. It's hard to measure $\frac{50-10\sqrt{3}}{3}$ cm to cut!