

2W p. 334 # 1bd, 2bdr, 3, 4bdr, 5bd, 6bd, 7bd, 11bdr, 10

6.3 Multiplying a Vector by a Scalar

3 properties - Commutative $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
 Associative $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
 ID $\vec{0} + \vec{a} = \vec{a} = \vec{a} + \vec{0}$

3 new properties for scalar multiplication

For any vectors $\vec{a}$ and $\vec{b}$, and scalars $k, m \in \mathbb{R}$:

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b} \quad (\text{distributive property})$$

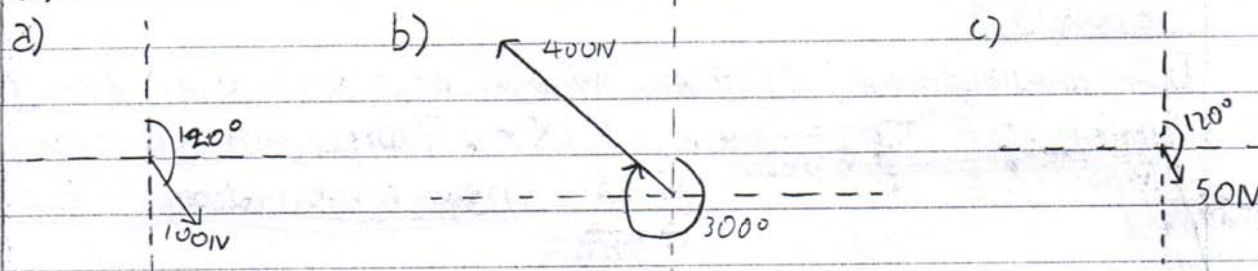
$$k(m\vec{a}) = (km)\vec{a} \quad (\text{associative})$$

$$1\vec{a} = \vec{a} \quad (\text{ID})$$

Example 1

Suppose $|\vec{x}| = 200\text{N}$, at a true bearing of 120° . Draw a scale diagram to represent each vector. Scale $1\text{cm} = 100\text{N}$

a) $0.5\vec{x}$ b) $-2\vec{x}$ c) $\frac{1}{4}\vec{x}$



Example 2

Simplify algebraically

$$\begin{aligned} \text{a) } \vec{x} + \vec{x} + 2\vec{x} - \vec{x} \\ = 3\vec{x} \end{aligned}$$

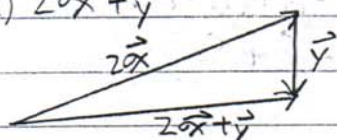
$$\begin{aligned} \text{b) } -2(\vec{x} + \vec{y}) + 3\vec{y} + \vec{x} \\ = -2\vec{x} - 2\vec{y} + 3\vec{y} + \vec{x} \\ = -\vec{x} + \vec{y} \end{aligned}$$

Example 3

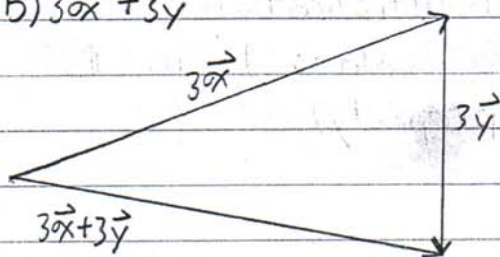
Copy vectors $\vec{x}$ and $\vec{y}$. Then draw a vector diagram to illustrate each combination of vectors.



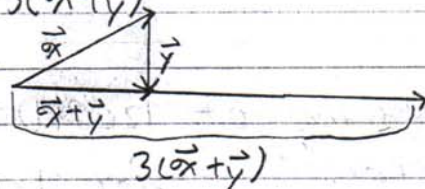
a) $2\vec{x} + \vec{y}$



b) $3\vec{x} + 3\vec{y}$

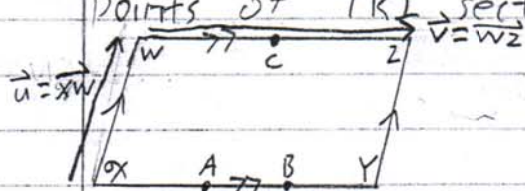


c) $3(\vec{x} + \vec{y})$



Example 4

In the diagram, C is the midpoint of WZ and A and B are the points of TRI section of XY. Express each vector in terms of a linear combination of $\vec{u} = \vec{xw}$ and $\vec{v} = \vec{wz}$



a) $\vec{AB} = \frac{1}{3} \vec{v}$

b) $\vec{AZ} = \frac{2}{3} \vec{v} + \vec{u}$

c) $\vec{WB} = -\vec{u} + \frac{2}{3} \vec{v}$ $\vec{v} - \vec{u} - \frac{1}{3} \vec{v} = \frac{2}{3} \vec{v} - \vec{u}$

d) $\vec{AC} = \frac{2}{6} \vec{v} + \vec{u} + \frac{3}{6} \vec{v}$
 $= \vec{u} + \frac{1}{6} \vec{v}$

Collinear

$\vec{d}$ → tail to tail one vector on top of each other.

Linear Combination

Given $\vec{u}$ and $\vec{v}$ are two vectors and s and t are scalars and $t \in \mathbb{R}$
 $s\vec{u} + t\vec{v}$ is called a linear combination of $\vec{u}$ and $\vec{v}$