MCV4U

Pretend Quiz - Derivatives

1. Find each derivative. Express your answer in simplified, factored form with only positive exponents.

a) $y = \sqrt[3]{x^2}$

b) $y = \dfrac{x^2 + 3}{2x - 1}$

c) $y = (x^2 - 1)^4 (2x + 1)^3$

d) $y = \sqrt{\dfrac{1 + x}{1 + x^2}}$

2. Find $\dfrac{dy}{dx}\Big|_{x=2}$ if $y = u^2 - u^3 + 2u^4$ and $u = \dfrac{x}{2x - 1}$.

3. Find the points on the curve $y = \dfrac{1}{2x - 1}$ where the tangent line is perpendicular to the line $x - 2y = 1$.

4. For the function $f(x) = ax^4 + bx^3 - 4x^2 + 2cx + 14$, determine the values of a and b so that $f(-2) = 2$, $f'(-2) = 16$, $f''(-2) = -8$.

Answers to Pretend Quiz

1a)
$$y=\sqrt[3]{x^2}$$
$$=x^{\frac{2}{3}}$$
$$y'=\frac{2}{3}x^{\frac{-1}{3}}$$
$$=\frac{2}{3\sqrt[3]{x}}$$

b)
$$y=\frac{x^2+3}{2x-1}$$
$$y'=\frac{(2x-1)(2x)-(x^2+3)(2)}{(2x-1)^2}$$
$$=\frac{4x^2-2x-2x^2-6}{(2x-1)^2}$$
$$=\frac{2x^2-2x-6}{(2x-1)^2}$$
$$=\frac{2(x^2-x-3)}{(2x-1)^2}$$

c)
$$y=(x^2-1)^4(2x+1)^3$$
$$y'=(2x+1)^3 4(x^2-1)^3(2x)+(x^2-1)^4 3(2x+1)^2(2)$$
$$=8x(2x+1)^3(x^2-1)^3+6(x^2-1)^4(2x+1)^2$$
$$=2(2x+1)^2(x^2-1)^3\left[4x(2x+1)+3(x^2-1)\right]$$
$$=2(2x+1)^2(x^2-1)^3\left[8x^2+4x+3x^2-3\right]$$
$$=2(2x+1)^2(x^2-1)^3\left[11x^2+4x-3\right]$$

d)
$$y=\sqrt{\frac{1+x}{1+x^2}}$$
$$=\left(\frac{1+x}{1+x^2}\right)^{\frac{1}{2}}$$
$$y'=\frac{1}{2}\left(\frac{1+x}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{(1+x^2)1-(1+x)2x}{(1+x^2)^2}\right]$$
$$=\frac{1}{2}\left(\frac{1+x}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{1+x^2-2x-2x^2}{(1+x^2)^2}\right]$$
$$=\frac{1}{2}\left(\frac{1+x}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{-x^2-2x+1}{(1+x^2)^2}\right]$$
$$=\frac{(-x^2-2x+1)\sqrt{1+x^2}}{2(1+x^2)^2\sqrt{1+x}}$$
$$=\frac{-x^2-2x+1}{2(1+x^2)^{\frac{3}{2}}\sqrt{1+x}}$$

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2.
$$y=u^2-u^3+2u^4 \text{ and } u=\frac{x}{2x-1}$$
$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$
$$=(2u-3u^2+8u^3)\left(\frac{(2x-1)(1)-x(2)}{(2x-1)^2}\right)$$
$$=(2u-3u^2+8u^3)\left(\frac{2x-1-2x}{(2x-1)^2}\right)$$
$$=(2u-3u^2+8u^3)\left(\frac{-1}{(2x-1)^2}\right)$$

$$u(2)=\frac{2}{2(2)-1}$$
$$=\frac{2}{3}$$

$$\frac{dy}{dx}\Big|_{x=2}=\left(2\left(\frac{2}{3}\right)-3\left(\frac{2}{3}\right)^2+8\left(\frac{2}{3}\right)^3\right)\left(\frac{-1}{(2(2)-1)^2}\right)$$
$$=\left(\frac{64}{27}\right)\left(\frac{-1}{9}\right)$$
$$=\frac{-64}{243}$$

3.
$$y=\frac{1}{2x-1}$$
$$y'=-1(2x-1)^{-2}(2)$$
$$=\frac{-2}{(2x-1)^2}$$

slope of $x-2y=1$:
$$-2y=-x+1$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$m=\frac{1}{2}$$
$$m_p=-2$$

$$\frac{-2}{(2x-1)^2}=-2$$
$$-2(2x-1)^2=-2$$
$$(2x-1)^2=1$$
$$4x^2-4x+1=1$$
$$4x^2-4x=0$$
$$4x(x-1)=0$$

$x=0$ or 1

y - values:

$y(0)=-1$
$y(1)=1$

$(x,y)=(0,-1)$ or $(1,1)$

4. $$f(x) = ax^4 + bx^3 - 4x^2 + 2cx + 14$$
$$f'(x) = 4ax^3 + 3bx^2 - 8x + 2c$$
$$f''(x) = 12ax^2 + 6bx - 8$$

$$f(-2) = 2$$
$$(-2)^4 + b(-2)^3 - 4(-2)^2 + 2c(-2) + 14 = 2$$
$$16a - 8b - 16 - 4c + 14 = 2$$
$$16a - 8b - 4c = 4$$
$$4a - 2b - c = 1$$

$$f'(-2) = 16$$
$$4a(-2)^3 + 3b(-2)^2 - 8(-2) + 2c = 16$$
$$-32a + 12b + 16 + 2c = 16$$
$$-32a + 12b + 2c = 0$$
$$16a - 6b - c = 0$$

$$f''(-2) = -8$$
$$12a(-2)^2 + 6b(-2) - 8 = -8$$
$$48a - 12b - 8 = -8$$
$$48a - 12b = 0$$
$$12b = 48a$$
$$b = 4a$$

$$4a - 2b - c = 1$$
$$4a - 2(4a) - c = 1$$
$$-4a - c = 1$$
$$c = -4a - 1$$

$$-16a - 6b - c = 0$$
$$-16a - 6(4a) + (-4a - 1) = 0$$
$$-16a + 24a - 4a - 1 = 0$$
$$4a = 1$$
$$a = \frac{1}{4}$$

$$b = 4a = 4\left(\frac{1}{4}\right) = 1$$

$$c = -4a - 1 = -4\left(\frac{1}{4}\right) - 1 = -2$$