MCV4U

Name _____

Date _____

Differentiation Review

I. Differentiate the following.

1) $\omega = (5t - 3t^2)(2 - 3t^2 - 6t^3)$

2) $f(\omega) = \dfrac{3\omega + 5}{1 - \omega^2}$

3) $y = \left(x^3 - \dfrac{4}{x^2} + 5\right)^7$

4) $g(x) = (x^2+3)^4(2x^3-5)^3$

5) $y = \left(\dfrac{3x}{1-x}\right)^5$

6) $y = \left(\dfrac{3x^6}{x^2-5}\right)^{-2}$

7) $h(\omega) = \left[9\omega - 3(1-6\omega^7)^2\right]^4$

8) $y = x^6(5x-3)^2(1-2x)^5$

9) $y = t^{1/3}(4t-5)^{2/3}$

10) $y = \dfrac{x}{\sqrt{2x^2 - x}}$

11) $f(x) = \sqrt{3x + \dfrac{1}{x}}$

12) $y = (2x+3)\sqrt{1-3x-x^2}$

13) $y = \dfrac{x^{3/2} - x^{1/2} + 3}{x^{1/2}}$

Check:

1) $\frac{d\omega}{dt} = 90t^4 - 84t^3 - 45t^2 - 12t + 10$

2) $f'(\omega) = \frac{(3\omega+1)(\omega+3)}{(1-\omega^2)^2}$

3) $\frac{dy}{dx} = \frac{7}{x^3}\left(x^3 - \frac{4}{x^2} + 5\right)^6 (3x^5 + 8)$

4) $g'(x) = 2x(2x^3-5)^2(x^2+3)^3(17x^3+27x-20)$

5) $\frac{dy}{dx} = \frac{1215x^4}{(1-x)^6}$

6) $\frac{dy}{dx} = \frac{-4(x^2-5)(2x^2-15)}{9x^{13}}$

7) $h'(\omega) = 336(1+28\omega^6 - 168\omega^{13})\left[9\omega - 3(1-6\omega^7)^2\right]^3$

8) $\frac{dy}{dx} = -2x^5(5x-3)(1-2x)^4(65x^2 - 53x + 9)$

9) $\frac{dy}{dt} = \frac{12t-5}{3t^{2/3}(4t-5)^{1/3}}$

10) $\frac{dy}{dx} = \frac{-x}{2(2x^2-x)^{3/2}}$

11) $f'(x) = \frac{1}{2}(3x + x^{-1})^{-1/2}(3 - x^{-2})$

12) $\frac{dy}{dx} = -\frac{8x^2 + 24x + 5}{2\sqrt{1-3x-x^2}}$

13) $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-3/2}$

1) $w = (5t - 3t^2)(2 - 3t^2 - 6t^3)$

$\frac{dw}{dt} = (5 - 6t)(2 - 3t^2 - 6t^3) + (5t - 3t^2)(-6t - 18t^2)$

$= 10 - 15t^2 - 30t^3 - 12t + 18t^3 + 36t^4 - 30t^2 - 90t^3 + 18t^3 + 54t^4$

$= 90t^4 - 84t^3 - 45t^2 - 12t + 10$

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2) $f(w) = \frac{3w+5}{1-w^2}$

$f'(w) = \frac{3(1-w^2) + 2w(3w+5)}{(1-w^2)^2}$

$= \frac{3 - 3w^2 + 6w^2 + 10w}{(1-w^2)^2}$

$= \frac{3w^2 + 10w + 3}{(1-w^2)^2}$ $\quad 3w \times 1$, $w \times 3$

$= \frac{(3w+1)(w+3)}{(1-w^2)^2}$

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3) $y = (x^3 - \frac{4}{x^2} + 5)^7$

$\frac{dy}{dx} = 7(x^3 - \frac{4}{x^2} + 5)^6 \times (3x^2 + 8x^{-3})$

$= \frac{7}{x^3}(x^3 - \frac{4}{x^2} + 5)^6 \times (3x^5 + 8)$ → this step not necessary

4) $g(x) = (x^2+3)^4 (2x^3-5)^3$

$g'(x) = 4(x^2+3)^3 \times 2x\,(2x^3-5)^3 + (x^2+3)^4 \times 3(2x^3-5)^2 \times 6x^2$

$= 8x(x^2+3)^3(2x^3-5)^3 + 18x^2(x^2+3)^4(2x^3-5)^2$

$= 2x(x^2+3)^3(2x^3-5)^2[4(2x^3-5) + 9x(x^2+3)]$

$= 2x(x^2+3)^3(2x^3-5)^2(8x^3-20+9x^3+27x)$

$= 2x(2x^3-5)^2(x^2+3)^3(17x^3+27x-20)$

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5) $y = \left(\frac{3x}{1-x}\right)^5$

$\frac{dy}{dx} = 5\left(\frac{3x}{1-x}\right)^4 \times \left[\frac{3(1-x)-(-1)(3x)}{(1-x)^2}\right]$

$= \frac{5(81x^4)}{(1-x)^4} \times \frac{(3-3x+3x)}{(1-x)^2}$

$= \frac{405x^4 \cdot (3)}{(1-x)^6}$

$= \frac{1215x^4}{(1-x)^6}$

6) $y = \left(\frac{3x^6}{x^2-5}\right)^{-2}$

$= \left(\frac{x^2-5}{3x^6}\right)^2$

$= \frac{(x^2-5)^2}{9x^{12}}$

$\frac{dy}{dx} = \frac{2(x^2-5)\times 2x \times 9x^{12} - (x^2-5)^2 \times 108x^{11}}{81x^{24}}$

$= \frac{36x^{13}(x^2-5) - 108x^{11}(x^2-5)^2}{81x^{24}}$

$= \frac{36x^{11}\left[x^2(x^2-5) - 3(x^2-5)(x^2-5)\right]}{81x^{24}}$

$= \frac{4(x^2-3(x^2-5))(x^2-5)}{9x^{13}}$

$= \frac{4(x^2-3x^2+15)(x^2-5)}{9x^{13}}$

$= \frac{4(-2x^2+15)(x^2-5)}{9x^{13}}$

$= \frac{-4(2x^2-15)(x^2-5)}{9x^{13}}$

7) $h(w) = [9w - 3(1-6w^7)^2]^4$

$h'(w) = 4[9w - 3(1-6w^7)^2]^3 \times [9 - 6(1-6w^7)^1 \times -42w^6]$

$= 36[9w - 3(1-6w^7)^2]^3 + 1008w^6(1-6w^7)[9w - 3(1-6w^7)^2]$

$= 36[9w - 3(1-6w^7)^2]^3 (1 + 28w^6(1-6w^7))$

$= 36[9w - 3(1-6w^7)^2]^3 (1 + 28w^6 - 168w^{13})$

8) $y = x^6 (5x-3)^2 (1-2x)^5$ ✓

let $f(x) = x^6 (5x-3)^2 \Rightarrow f'(x) = 6x^5(5x-3)^2 + x^6 \times 2(5x-3) \times 5$

$= 6x^5(5x-3)^2 + 10x^6(5x-3)$

$= 2x^5(5x-3)[3(5x-3) + 5x)$

$= 2x^5(5x-3)(15x-9+5x)$

$= 2x^5(5x-3)(20x-9)$

$g(x) = (1-2x)^5$

$g'(x) = 5(1-2x)^4 \times -2$

$= -10(1-2x)^4$

$y' = f'g + g'f$

$= 2x^5(5x-3)(20x-9)(1-2x)^5 + (-10)(1-2x)^4 \, x^6(5x-3)^2$

$= 2x^5(5x-3)(1-2x)^4 [(20x-9)(1-2x) + (-5x)(5x-3)$ ($-5x = \frac{-10x^6}{2x^5}$)

$= 2x^5(5x-3)(1-2x)^4 (20x - 40x^2 - 9 + 18x - 25x^2 + 15x)$

$= 2x^5(5x-3)(1-2x)^4 (-65x^2 + 53x - 9)$

$= -2x^5(5x-3)(1-2x^4)(65x^2 - 53x + 9)$

9) $y = t^{\frac{1}{3}}(4t-5)^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}t^{\frac{-2}{3}}(4t-5)^{\frac{2}{3}} + t^{\frac{1}{3}} \times \frac{2}{3}(4t-5)^{\frac{-1}{3}} \times 4$$

$$= \frac{t^{\frac{-2}{3}}(4t-5)^{\frac{2}{3}}}{3} + \frac{2t^{\frac{1}{3}} \times 4}{3(4t-5)^{\frac{1}{3}}}$$

$$= \frac{(4t-5)^{\frac{2}{3}}}{3t^{\frac{2}{3}}} + \frac{8t^{\frac{1}{3}}}{3(4t-5)^{\frac{1}{3}}}$$

$$= \frac{(4t-5)^{\frac{2}{3}}(4t-5)^{\frac{1}{3}} + 8t^{\frac{1}{3}} \cdot t^{\frac{2}{3}}}{3t^{\frac{2}{3}}(4t-5)^{\frac{1}{3}}}$$

$$= \frac{(4t-5) + 8t}{3t^{\frac{2}{3}}(4t-5)^{\frac{1}{3}}}$$

$$= \frac{12t-5}{3t^{\frac{2}{3}}(4t-5)^{\frac{1}{3}}}$$

10) $y = \dfrac{x}{\sqrt{2x^2 - x}}$

$$\frac{dy}{dx} = \frac{1(2x^2-x)^{\frac{1}{2}} - x \times \frac{1}{2}(2x^2-x)^{-\frac{1}{2}} \times (4x-1)}{\left[(2x^2-x)^{\frac{1}{2}}\right]^2}$$

$$= \frac{(2x^2-x)^{\frac{1}{2}}\left(1 - \frac{1}{2}x(2x^2-x)^{-1} \times (4x-1)\right)}{2x^2-x}$$

$$= (2x^2-x)^{-\frac{1}{2}}\left[1 - \frac{x(4x-1)}{2(2x^2-x)}\right]$$

$$= (2x^2-x)^{-\frac{1}{2}}\left[\frac{2(2x^2-x) - x(4x-1)}{2(2x^2-x)}\right]$$

$$= \frac{4x^2 - 2x - 4x^2 + x}{2(2x^2-x)^{\frac{3}{2}}}$$

$$= \frac{-x}{2(2x^2-x)^{\frac{3}{2}}}$$

11) $f(x) = \sqrt{3x + \frac{1}{x}}$ ✓

$f(x) = (3x + \frac{1}{x})^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(3x + x^{-1})^{-\frac{1}{2}} \times (3 - x^{-2})$

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12) $y = (2x+3)\sqrt{1-3x-x^2}$ ✓

$y = (2x+3)(1-3x-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = 2(1-3x-x^2)^{\frac{1}{2}} + (2x+3) \times \frac{1}{2}(1-3x-x^2)^{-\frac{1}{2}} \times (-2x-3)$

$= (1-3x-x^2)^{\frac{1}{2}} \left[2 - \frac{1}{2}(2x+3)^2 (1-3x-x^2)^{-1}\right]$

$= (1-3x-x^2)^{\frac{1}{2}} \left[2 - \frac{1(2x+3)^2}{2(1-3x-x^2)}\right]$

$= \frac{(1-3x-x^2)^{\frac{1}{2}}}{1} \times \frac{4(1-3x-x^2) - (2x+3)^2}{2(1-3x-x^2)}$

$= \frac{4-12x-4x^2 - (4x^2+12x+9)}{2(1-3x-x^2)^{\frac{1}{2}}}$

$= \frac{-8x^2-24x-5}{2(1-3-x^2)^{\frac{1}{2}}}$

$= -\frac{8x^2+24x+5}{2\sqrt{1-3-x^2}}$

13) $y = \dfrac{x^{\frac{3}{2}} - x^{\frac{1}{2}} + 3}{x^{\frac{1}{2}}}$

$y = \dfrac{x^{\frac{1}{2}}(x^{\frac{2}{2}} - 1 + 3x^{-\frac{1}{2}})}{x^{\frac{1}{2}}}$

$y = x - 1 + 3x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 1 - 0 - \frac{3}{2}x^{-\frac{3}{2}}$

$\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{3}{2}}$