Name: MA Date: March 29, 2011.

MCV4U1-03 Chapter 3 Curve Sketching Test [K: /15 A: /11 C: /6 T: /5]

Demonstrate your understanding of the concepts learned in Chapter 3 by showing full solutions for the following questions. Round to two decimal places when rounding is required. If you would like a bonus mark for reading these instructions, draw a self portrait beside your name.

Part A: Curve Sketching

1. Given the function $f(x)=-\frac{x^3}{x^2-1}$ and its derivatives $f'(x)=-\frac{x^4-3x^2}{(x^2-1)^2}$ and $f''(x)=-\frac{2x^3+6x}{(x^2-1)^3}$ determine

a) its symmetry [K1]

$$f(-x)=\frac{-(-x)^3}{(-x)^2-1}=\frac{x^3}{x^2-1}=-f(x)$$

odd function → rotational symmetry

b) the domain [K1]

$\{x \mid x\in\mathbb{R},\ x\neq\pm1\}$

c) the intercepts [K1]

x-int $0=\frac{-x^3}{x^2-1}$

$-x^3=0$

$x=0$

y-int $y=\frac{-(0)^3}{(0)^2-1}=0$

$y=0$

d) the equations of vertical asymptotes [K1]

$x=1$, $x=-1$

e) the one sided limits of the vertical asymptotes [K2]

$$\lim_{x\to-1^-}-\frac{x^3}{x^2-1}\doteq-\frac{(-1.001)^3}{(-1.001)^2-1}=\frac{-(-)}{+\text{ small }\#}=+\infty$$

$$\lim_{x\to-1^+}\frac{-x^3}{x^2-1}=\frac{-(-)}{-\text{ small }\#}=-\infty$$

$$\lim_{x\to1^-}\frac{-x^3}{x^2-1}=\frac{-}{-\text{ small }\#}=+\infty$$

$$\lim_{x\to1^+}\frac{-x^3}{x^2-1}=\frac{-}{+\text{ small }\#}=-\infty$$

f) the oblique asymptote[K2]

$$\begin{array}{r|l} & -x \\ x^2+0x-1 & -x^3+0x^2+0x+0 \\ & -)\ -x^3+0x^2+1x \\ & \qquad -1x \end{array}$$

∴ O.As is $y=-x$

$$f(x)=-x-\frac{x}{x^2-1}$$

Name: ______________________

g) end behaviour (whether f(x) is above / below the oblique asymptote) [K2]

$\lim_{x \to +\infty} \frac{x}{x^2-1} = +$ small # below $y = -x$

$\lim_{x \to -\infty} \frac{x}{x^2-1} = -$ small # above $y = -x$

h) the critical points [K2]

$x^4 - 3x^2 = 0$

$x^2(x^2-3) = 0$

$x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$

$f(0) = \frac{-(0)^3}{(0)^2-1} = 0$

$f(-\sqrt{3}) = \frac{-(-\sqrt{3})^3}{(-\sqrt{3})^2-1} = \frac{3}{2}\sqrt{3} \doteq 2.60$

$f(\sqrt{3}) = \frac{-(\sqrt{3})^3}{(\sqrt{3})^2-1} = -\frac{3}{2}\sqrt{3} \doteq 2.60$

$(-\sqrt{3}, \frac{3\sqrt{3}}{2})$ $(-1.73, 2.60)$

$(\sqrt{3}, -\frac{3\sqrt{3}}{2})$ $(1.73, -2.60)$

$f(0) = 0$

i) the point(s) of inflection [K1]

$2x^3 + 6x = 0$

$2x(x^2+6) = 0$

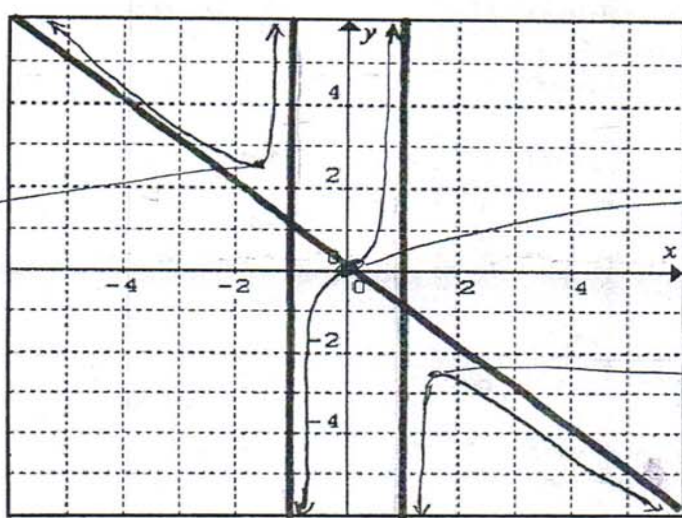
$x = 0$ no real roots

j) intervals of increase/decrease and concavity by filling out the following table [A4]

Intervals	$x < -\sqrt{3}$	$x = -\sqrt{3}$	$-\sqrt{3} < x < -1$	$x = -1$	$-1 < x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x < \sqrt{3}$	$x = \sqrt{3}$	x
Test values	$x = -2$		$x = -1.5$		$x = -0.5$		$x = 0.5$		$x = 1.5$		
f'(x)	−	0	+		+	0	+		+	0	−
f''(x)	+	+	+		−	0	+		−	−	−
f(x)	dec con ↑	local min	inc con ↑	V.A.	inc con ↓	POI	inc con ↑	V.A.	inc con ↓	local max	d Co

k) Sketch the curve. Label all the asymptotes, turning point(s) and point(s) of inflection. [A4]

Local min $(-1.73, 2.60)$



$(0,0)$ POI

$(1.73, -2.60)$ local max

Name: ______________________

Multiple Choice

Identify the choice that best completes the statement or answers the question.

D 2. [K1]Which of the following functions has a vertical asymptote at $x = 2$ and a horizontal asymptote of $y = 1$?

a. $y = \dfrac{x^2 - 6x + 9}{x^2 + 3x + 2}$ $(x+2)(x+1)$

b. $y = \dfrac{3}{x - 2}$

c. $y = \dfrac{x + 3}{x^2 - 4}$

d. $y = \dfrac{x^2 - 9}{x^2 - 4x + 4}$ $(x-2)(x-2)$

B 3. [K1]Determine the absolute maximum value(s) of $f(x) = 4x^3 + 12x^2 + 12x$, where $-2 \le x \le 2$.

a. 40

b. 104

c. -4

d. 28

$f(2) = 4(2)^3 + 12(2)^2 + 12(2)$

$= 32 + 48 + 24$

Short Answer

4. [A3]Graph a function $f(x)$ that has all of the following characteristics.

$\lim_{x \to 3^+} f(x) = -\infty$

$\lim_{x \to -1^-} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -2$

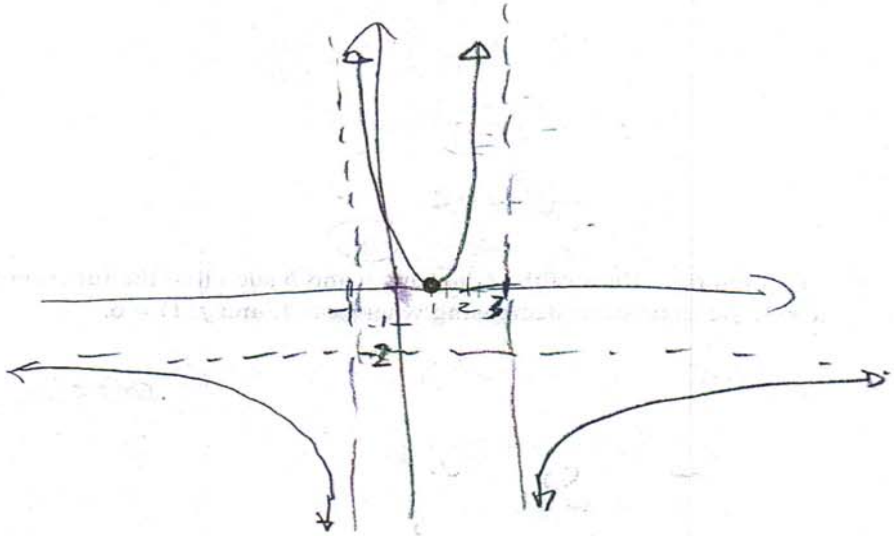
$\lim_{x \to -\infty} f(x) = -2$

$f(1) = 0$

$f'(1) = 0$

$f''(1) < 0$ local max

$>$



5. [C1]Under what condition does a rational function have an oblique asymptote?

deg of num = deg of denom + 1

[C1]Under what condition does a rational function have a horizontal asymptote?

deg of num ≤ deg of denom

Name: ____________________

6. [T2]Show that, for any cubic function of the form $f(x) = ax^3 + bx^2 + cx + d$, there is a single point of inflection, and the slope of the curve at that point is $c - \frac{b^2}{3a}$.

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$0 = 6ax + 2b$

$6ax = -2b$

$x = -\frac{2b}{6a}$

$x = \frac{-b}{3a}$ (1) $\Longrightarrow$ SINGLE POINT of POI

$f'(-\frac{b}{3a}) = 3a(-\frac{b}{3a})^2 + 2b(\frac{-b}{3a}) + c$

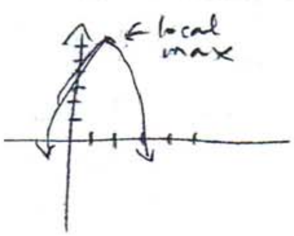
$= 3a\left(\frac{b^2}{9a^2}\right) - \frac{2b^2}{3a} + c$

$= \frac{b^2 - 2b^2}{3a} + c$

$= \frac{-b^2}{3a} + c$

$= c - \frac{b^2}{3a}$ (1)

7. [T3]Find the values of the constants a and b such that the function $f(x) = x^3 + ax^2 + b$ is increasing where $x < 1$, the function is decreasing where $x > 1$, and $f(1) = 6$.



$f(x) = x^3 + ax^2 + b$

$6 = (1)^3 + a(1)^2 + b$

$6 = 1 + a + b$

$a + b = 5$ (1)

$f'(x) = 3x^2 + 2ax$

$0 = 3(1)^2 + 2a(1)$

$0 = 3 + 2a$

$\boxed{a = \frac{-3}{2}}$ (1)

sub $a = \frac{-3}{2}$ int $a + b = 5$

$\frac{-3}{2} + b = 5$

$b = 5 + \frac{3}{2}$

$b = \frac{10 + 3}{2}$

$\boxed{b = \frac{13}{2}}$ (1)

Use of appropriate mathematical symbols and conventions	/2
Clarity in explanations and justifications in reporting	/2

Name: ______________________ Date: ______________________

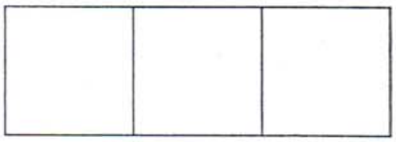
MCV4U1-06 Optimization Question

[A: /6 T: /6 C: /4]

Time: 50 minutes

Short Answer

1. A rectangular region is to be enclosed and subdivided with fencing as illustrated. Find the dimensions of the maximized area of the region if 640 m of fencing is available.



a) Declare your variables. "Let..." [C1]
b) Deterimine a function in ONE variable that represents the quantity to be optimized.[A2]
c) Determine the domain of the function to be optimized, write the restrictions in the form a<x<b [A1]
d) Find the critical point(s) - not just the critical number(s).[A2]
e) Use the second derivative test to determine the nature of the critical point(s). [A1]
f) Concluding statement. Answer the original question! [C1]

Let x be the width
y be the length.

$P = 4x + 2y$
$640 = 4x + 2y$
$2y = 640 - 4x$
$y = 320 - 2x$

$\therefore$ the dimensions are
80 m $\times$ 160 m
area is 12 800 m^2

$A = xy$
$A(x) = x(320 - 2x)$ 2
$A(x) = 320x - 2x^2$
$A'(x) = 320 - 4x$
$0 = 320 - 4x$
$4x = 320$
$x = 80$ 1

$y = 320 - 2(80)$
$= 160$ 1

$A = 80 \times 160$
$= 12800 m^2$ 1

$A''(x) = -4 < 0 \Rightarrow$ LOCAL MAX 1

Domain
$0 < x < 160$ m 1

Name: ______________________

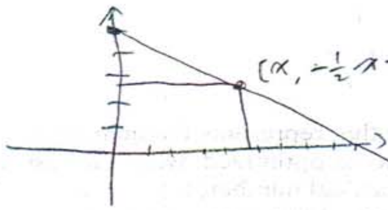
Problem

2. A rectangle lies in the first quadrant, with one vertex at the origin and two of the sides along the coordinate axes. If the fourth vertex lies on the line defined by x + 2y – 10 = 0, find the ~~the~~ dimensions and the area of the largest rectangle. [T6, C2]

$x + 2y - 10 = 0$

$2y = -x + 10$

$y = -\frac{1}{2}x + 5$



$\boxed{0 < x < 10.}$ 1

$\boxed{A(x) = x(-\frac{1}{2}x + 5)}$ 2

$A(x) = -\frac{1}{2}x^2 + 5x$

$A'(x) = -x + 5$

$0 = -x + 5$

$\boxed{x = 5}$ 1

$\boxed{A''(x) = -1 < 0}$ local Max 1

$y = -\frac{1}{2}(5) + 5$

$\boxed{A(5) = \frac{5}{2}}$

$A = xy$

$= 5 \times \frac{5}{2}$

$= \frac{25}{2}$

$\boxed{A = 12.5 \text{ sq. units}}$ 1