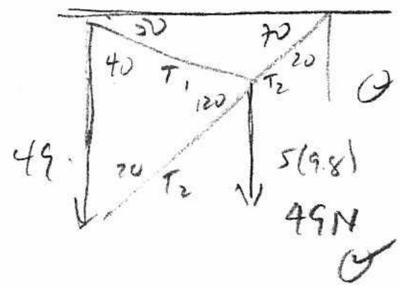
1. A mass of 5kg is suspended by two wires that make angles of 40^0 and 20^0 respectively with the vertical. Find the tension of each wire. (6 marks)



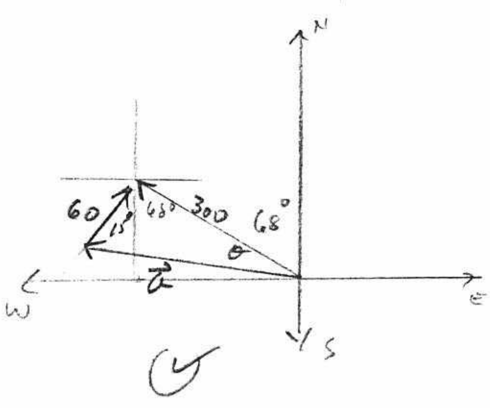
$$\frac{T_1}{\sin 20} = \frac{49}{\sin 120} \quad \& \quad \frac{T_2}{\sin 40} = \frac{49}{\sin 120}$$

6

$$T_1 = \frac{49 \sin 20}{\sin 120} \qquad T_2 = \frac{49 \sin 40}{\sin 120}$$

$$T_1 = 19.3\,N \qquad T_2 = 36\,N$$

2. A navigator wishes to fly an aircraft on a bearing of $W22^0N$ with a ground speed of 300km/h. There is a 60km/h wind from the direction $S15^0W$. Calculate the heading and airspeed that the navigator should use. (6 marks)



$$|\vec{a}|^2 = 60^2 + 300^2 - 2(60)(300)\cos(15+68)$$

$$|\vec{a}| = 298.7 \text{ km/h}$$

$$\sin\theta = \frac{60 \sin 83^\circ}{298.7}$$

6

$$\angle\theta = 11.5^\circ$$

∴ the navigator should use an airspeed of 298.7 km/h at a heading of N79.5°W or W10.5°N.

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3. A force of 50N acts at the end of a wrench 18cm long.

a) What is the maximum torque that can be produced? (3 marks)

→ to produce max torque the force must be $\perp$ to the wrench

3 $\therefore \tau = (0.18)(50)\sin 90^\circ$

$= 9J$

b) At what angle will the force produce half the maximum force? (2 marks)

$4.5 = (0.18)(50)\sin\theta$

2 $\sin\theta = \frac{1}{2}$

$\measuredangle\theta = 30^\circ$

4. Given $\vec{p} = [1,2,-3]$ and $\vec{q} = [2,-1,4]$, determine:

a) A unit vector perpendicular to both $\vec{p}$ and $\vec{q}$. (4 marks)

$\vec{p}\times\vec{q} = [1,2,-3]\times[2,-1,4]$

$= [2(4)-(-3)(-1),\ (-3)(2)-(1)(4),\ (1)(-1)-2(2)]$

$= [5,-10,-5]$

4 $= [1,-2,-1]$

$|\vec{p}\times\vec{q}| = \sqrt{1^2+(-2)^2+(-1)^2}$

$= \sqrt{6}$

$\therefore$ a unit vector $\perp$ to both $\vec{p}$ & $\vec{q}$ =

$\frac{1}{\sqrt{6}}[1,-2,1] = \left[\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right]$

b) The angle between $\vec{p}$ and $\vec{q}$. (3 marks)

$\cos\theta = \dfrac{\vec{p}\cdot\vec{q}}{|\vec{p}||\vec{q}|}$

3

$= \dfrac{1(2)+2(-1)+(-3)(4)}{\sqrt{1^2+2^2+(-3)^2}\ \sqrt{2^2+(-1)^2+4^2}}$

$= \dfrac{-12}{\sqrt{294}}$

12 $\therefore \measuredangle\theta = 124.4^\circ$

5. Determine if the plane and the line intersect. If so, state the solution. (6 marks)

$$-4x - 5y + 6z = 34$$

$$[x, y, z] = [4, 2, 6] + t[1, -2, 3]$$

$\vec{m} = [1, -2, 3]$ & $\vec{n} = [-4, -5, 6]$

$$\begin{aligned}\vec{m} \cdot \vec{n} &= [1, -2, 3] \cdot [-4, -5, 6] \\ &= 1(-4) + (-2)(-5) + (3)(6) \\ &= 24 \\ &\neq 0\end{aligned}$$

$\therefore$ $\vec{m}$ is not $\perp$ to $\vec{n}$, hence they intersect.

$$\left.\begin{aligned}x &= 4 + t \\ y &= 2 - 2t \\ z &= 6 + 3t\end{aligned}\right\}$$

$$-4(4+t) - 5(2-2t) + 6(6+3t) = 34$$
$$-16 - 4t - 10 + 10t + 36 + 18t = 34$$
$$24t = 24$$
$$t = 1$$

$$\therefore \left.\begin{aligned}x &= 4 + 1 = 5 \\ y &= 2 - 2(1) = 0 \\ z &= 6 + 3(1) = 9\end{aligned}\right\} (5, 0, 9) \text{ is the soln}$$

6

6. Describe the system of planes, using normal vectors. If possible, solve the system. (7 marks)

$$x + 2y + z = -4$$
$$x + 4y + 5z = -18$$
$$4x - z = -4$$

$n_1 = [1, 2, 1]$
$n_2 = [1, 4, 5]$
$n_3 = [4, 0, -1]$
} none parallel ✓

$(n_1 \times n_2) \cdot n_3$
$= [1,2,1] \times [1,4,5] \cdot [4,0,-1]$
$= [2(5)-(1)(4),\ 1(1)-(1)(5),\ (1)(4)-(2)(1)] \cdot [4,0,-1]$
$= [6, -4, 2] \cdot [4, 0, -1]$
$= 24 - 2$
$= 22$
$\neq 0 \quad \therefore$ non-coplanar ✓, ~~has~~ one pt of ∩ ✓

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -4 \\ 1 & 4 & 5 & -18 \\ 4 & 0 & -1 & -4 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - R_1 \\ R_3 + 4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & -4 \\ 0 & 2 & 4 & -14 \\ 0 & -8 & -5 & 12 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 + 4R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & -4 \\ 0 & 2 & 4 & -14 \\ 0 & 0 & 11 & -44 \end{array}\right] ✓$$

$\therefore 11z = -44$
$z = -4$ → ② ✓

$2y + 4(-4) = -14$
$2y - 16 = -14$
$2y = 2$
$y = 1$ → ① ✓

$x + 2(1) - 4 = -4$
$x = -2$ ✓

$\therefore (-2, 1, -4)$ is the pt of ∩.

$\frac{}{7}$

7. Use first principles to determine y' for $y = \sqrt{x^2 - 1}$. (7 marks)

$$y' = \lim_{h \to 0} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{x^2 - 1}}{h} \cdot \frac{\sqrt{(x+h)^2 - 1} + \sqrt{x^2 - 1}}{\sqrt{(x+h)^2 - 1} + \sqrt{x^2 - 1}}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h\left(\sqrt{(x+h)^2 - 1} + \sqrt{x^2 - 1}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}\left(\sqrt{(x+h)^2 - 1} + \sqrt{x^2 - 1}\right)}$$

7 $$= \frac{2x + 0}{\sqrt{(x+0)^2 - 1} + \sqrt{x^2 - 1}}$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2 - 1}} \quad \therefore \; y' = \frac{x}{\sqrt{x^2 - 1}}$$

8. Differentiate the following. **DO NOT SIMPLIFY** (12 marks)

a) $y = \dfrac{2^{\sqrt{x}}}{\sin(x^2 - x + 1)}$

6 $$y' = \frac{2^{\sqrt{x}} \ln 2 \left(\frac{1}{2}x^{-\frac{1}{2}}\right) \sin(x^2 - x + 1) - 2^{\sqrt{x}}\left(\cos(x^2 - x + 1)\right)(2x - 1)}{\sin^2(x^2 - x + 1)}$$

b) $y = \left(\sqrt[4]{x}\right)e^{\cos e^{3x}}$

6

$$y' = \frac{1}{4}x^{-3/4} e^{\cos e^{3x}} + \sqrt[4]{x}\, e^{\cos e^{3x}}\left(-\sin e^{3x}\right)\left(e^{3x}\right)(3)$$

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9. Find the equation of the tangent line to the curve $y = -e^{-x}$ at the point where $x = \ln\left(\frac{1}{2}\right)$. No decimals please!!!!!!! (4 marks)

$y' = -e^{-x}(-1)$

when $x = \ln\frac{1}{2} = -\ln 2$

$y' = e^{-(-\ln 2)}$

$= e^{\ln 2}$

$= 2.$

4

$\therefore y = -e^{-(-\ln 2)}$

$= -e^{\ln 2}$

$= -2$

$m(x - x_1) = y - y_1$

$2(x + \ln 2) = y + 2$

$2x + 2\ln 2 = y + 2$

$y = 2x + 2\ln 2 - 2$

or

$2x - y + 2\ln 2 - 2 = 0$

10. A jet car traveling at 200m/s needs to stop quickly by steadily increasing the braking force. Its position is given by $s(t) = 200t - 0.4t^3$, where t is measured in seconds and s in metres. Determine the car's stopping distance and maximum braking deceleration. (Show all work for full marks.) (5 marks)

$v(t) = 200 - 1.2t^2$

$200 - 1.2t^2 = 0$

$t = 12.9$ sec

$a(t) = -2.4t$

$a(12.9) = -2.4(12.9)$

$= -31\ m/s^2$

5

$s(12.9) = 200(12.9) - 0.4(12.9)^3$

$= 1721.3$ m

$\therefore$ it will take 1721.3 m for the car to stop with a max deceleration of 31 m/s²

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11. The manager of 120 condo complex knows from experience that all units will be occupied if rent is \$400 a month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should she charge to maximize revenue? (5 marks)

$R(x) = (120-x)(400+10x)$, where x rep the number of \$10 increases ✓

$= 48000 + 1200x - 400x - 10x^2$

$= -10x^2 + 800x + 48000$ ✓

$R'(x) = -20x + 800$ ✓

5

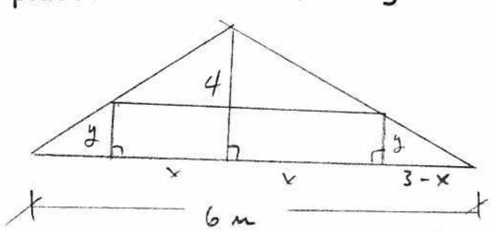
$-20x + 800 = 0$

$x = 40$ ✓

$\therefore$ price/unit $= 400 + 10(40)$

$= \$800.00$ ✓

12. The ends of a long greenhouse is the shape of an isosceles triangle with base 6m long and height 4m. What are the dimensions of the largest rectangular door that can be placed in the end of the greenhouse? (8 marks)



✓

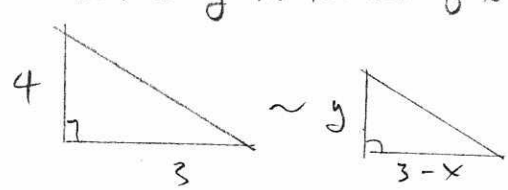
Let $2x$ rep the width

" y " " height

$\therefore A = 2xy$ ✓

now write y in terms of x

8



$\frac{4}{3} = \frac{y}{3-x}$

$y = \frac{4(3-x)}{3}$ ✓

$\therefore A(x) = 2x\left(\frac{4(3-x)}{3}\right)$

$= 8x - \frac{8x^2}{3}$ ✓

$A'(x) = 8 - \frac{16}{3}x$ ✓

$\frac{24-16x}{3} = 0$

$16x = 24$

$x = \frac{3}{2}$ ✓

$\therefore y = \frac{4\left(3-\frac{3}{2}\right)}{3}$

$y = 2$ ✓

$\therefore$ the garage door must be $2\left(\frac{3}{2}\right) = 3m$ ✓

& 2m high

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13. For the curve $y = \dfrac{3x^2 - 1}{x^3}$ and whose derivatives are $y' = \dfrac{3 - 3x^2}{x^4}$ and $y'' = \dfrac{6x^2 - 12}{x^5}$, determine the following. (17 marks)

a) intercepts and asymptotes (3 marks)

x-inter

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

y-inter

none

• VA $\Rightarrow x = 0$

• HA $\Rightarrow y = \lim_{x \to \infty} \dfrac{\frac{3x^2}{x^3} - \frac{1}{x^3}}{\frac{x^3}{x^3}}$

$y = 0$

b) intervals of increase & decrease (4 mks)

C.N. $3 - 3x^2 = 0$ $x = \pm 1$

$x^4 = 0$ $x = 0 \leftarrow$ V.A

interval	f'
$x < -1$	−
$-1 < x < 0$	+
$0 < x < 1$	+
$x > 1$	−

∴ increasing on $(-1, 0)$ & $(0, 1)$

decreasing on $(-\infty, -1)$ & $(1, +\infty)$

c) maximum & minimum points (2 marks)

• local min at $x = -1$

$f(-1) = -2$

∴ min pt at $(-1, -2)$

• local max at $x = 1$

$f(1) = 2$

∴ max pt at $(1, 2)$

d) points of inflection (2 marks)

$\dfrac{6x^2 - 12}{x^5} = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$f(\sqrt{2}) \doteq 1.8$

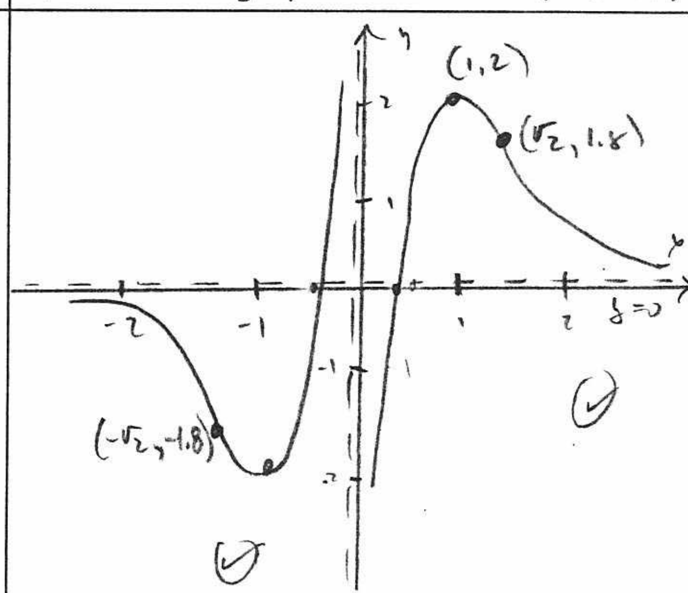
$f(-\sqrt{2}) \doteq -1.8$

∴ POI's are $(\sqrt{2}, 1.8)$ & $(-\sqrt{2}, -1.8)$

e) intervals of concavity (4 marks)

interval	f''	concavity
$x < -\sqrt{2}$	−	CD
$-\sqrt{2} < x < 0$	+	CU
$0 < x < \sqrt{2}$	−	CD
$x > \sqrt{2}$	+	CU

f) sketch the graph (2 marks)



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14. An automatic door has been programmed so that the angle, in degrees, that the door opens has the equation $a(t) = 180t(2)^{-t}$, where t is the time in seconds. How fast is the door closing after 5 seconds? (5 marks)

OR

A radar antenna is located on a ship that is 4km from a straight shore. It is rotating at 32 rev/min. How fast does the radar beam sweep along the shore when the angle between the beam and the shortest distance to the shore is $\frac{\pi}{4}$? (5 marks)

$$a'(t) = 180(2)^{-t} + 180t(2)^{-t}\ln 2(-1)$$

$$= \frac{180(1 - t\ln 2)}{2^t}$$

$$a'(5) = \frac{180(1-5\ln 2)}{2^5}$$

$$= -13.9^\circ/s$$

$$\frac{x}{4} = \tan\theta$$

$$x = 4\tan\theta$$

$$\frac{dx}{dt} = 4\sec^2\theta\,\frac{d\theta}{dt}$$

$$= 4\left(\sec\frac{\pi}{4}\right)^2\left(32(2\pi)\right)$$

$$= 4(\sqrt{2})^2(64\pi)$$

$$= 512\pi \text{ km/min}$$

5