

Miscellaneous Questions

1. If $s(t) = t^3 - 6t^2 + 9t$, $t \geq 0$, is the position function of a particle, where s represents its position in metres and t represents time in seconds, find
 - a) the instantaneous velocity at $t = 4$ s
 - b) the average velocity between $t = 2$ s and $t = 5$ s
2. Is the function $f(x) = x^4 - 2x^2 - 6$ odd, even or neither? Explain.
3. Name two mathematical differences between a secant and a tangent.
4. A spherical balloon is being inflated. The surface area of a sphere is given by the function $A(r) = 4\pi r^2$, where r is the radius in cm. Use two different methods to estimate the instantaneous rate of change in surface area of the balloon when the radius measures 20 cm.
5. Graphs of rational functions look different from those of polynomial functions. Name two significant differences found in graphs of rational functions, and describe how you can determine those differences from the equation.
6. Bobby chooses the horse to ride on a carousel. The horse is located 8 m from the centre of the carousel. If the carousel turns through an angle of $\frac{5\pi}{6}$, determine the length of the arc travelled by the horse, to the nearest tenth of a metre.
7. A bicycle tire revolves at 150 revolutions per minute. What is its angular velocity in radians/s?
8. Prove each identity.
 - a) $\frac{\cos 2x + 1}{\sin 2x} = \cot x$
 - b) $\sec x = \frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x}$
 - c) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$

1. a) 9 m/s; b) 6 m/s; 2. even; 3. see your notes; 4. 502.52 cm³/cm; 5. WA, HA;
6. 20.9 m; 7. 5π rad/s; 9. a=2, b=1, c=1; 10. a) $\frac{\pi}{6}$, $\frac{5\pi}{6}$; b) $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$; 11. a) $\log 49$;
13. a) $A = 2500(1.025)^{2t}$; b) \$3200.21; c) 14 years. 14. a) $f \circ g(x) = x^2 - 2x - 1$;
 b) $g(f(x)) = x^2 - 6x + 9$; c) $g(g^{-1}(x)) = x$, $x \geq 0$; 15. domain: $x \in \mathbb{R}$; range: $\{y \in \mathbb{R}; y \geq -2\}$
9. The expression $2 \cos^2 x + 4 \sin x \cos x$ can be expressed in the form $a \sin 2x + b \cos 2x + c$. Find a, b and c.
10. Find exact values for x on the interval $x \in [0, 2\pi]$
- a) $\csc x = 2$
 b) $2 \cos^2 x - 1 = 0$
11. Write as a single logarithm:
- a) $\frac{\log 9}{\log 4}$
 b) $\log 9 + \log 4$
 c) $\log 16 - \log 2$
 d) $\frac{\log(x+3)}{\log(x-5)}$
12. After 90 days, a sample of silver-110, Ag^{110} , has decayed to about 80% of its original size.
- a) Determine the half-life of Ag^{110}
 b) Describe how the graph of the amount of Ag^{110} remaining as a function of time would change if the half-life were shorter.
 c) How much of a 50 g sample of Ag^{110} will be left after 120 days?
13. A \$2500 investment earns 5% interest compounded semi-annually.
- a) Write an equation for the value of the investment as a function of time in years.
 b) Determine the value of the investment after 5 years.
 c) How long will it take for the investment to double in value?
14. Let $f(x) = x - 2$ and $g(x) = (x - 1)^2$. Determine a simplified algebraic model for
- a) $y = f \circ g(x)$ b) $y = g(f(x))$ c) $y = g(g^{-1}(x))$
15. Using the functions in question 14, what are the domain and range of $y = f \circ g(x)$?

Answers:

1. a) 9 m/s; b) 6 m/s; 2. even; 3. see your notes!; 4. $502.52 \text{ cm}^3 / \text{cm}$; 5. VA, HA;
 6. 20.9 m; 7. $5\pi \text{ rad/s}$; 9. $a=2, b=1, c=1$; 10. a) $\frac{\pi}{6}, \frac{5\pi}{6}$; b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$; 11. a) $\log_4 9$;
 b) $\log 36$; c) $\log 8$; d) $\log_{x-5} x + 3$; 12. a) 280 days; b) steeper; c) 37.1 g;
 13. a) $A = 2500(1.025)^{2t}$; b) \$3200.21; c) 14 years. 14. a) $f \circ g(x) = x^2 - 2x - 1$;
 b) $g(f(x)) = x^2 - 6x + 9$; c) $g(g^{-1}(x)) = x, x \geq 0$; 15. domain: $x \in \mathbb{R}$; range: $\{y | y \in \mathbb{R}; y \geq -2\}$