MAT351 Partial Differential Equations **Fall, Assignment 2**

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Hand-in Problems (Due till October 5 before lecture, via crowdmark)

H1 Use the method of characteristics to find he solution of

$$u_t - yu_x + xu_y = f(x, y) \text{ in } \mathbb{R}^2 \times [0, \infty) \quad \text{with} \quad u(x, y, 0) = g(x, y),\ g, f \in C^1(\mathbb{R}^2).$$

H2 Use the method of charateristics to find the solution of

$$u_x + xyu_y = x \text{ in } \mathbb{R}^2 \ \text{ with } \quad u(0, y) = \sin y.$$

Problems for discussion

1. Recall the exact statement of the general existence and uniqueness theorem for ODEs.
2. State the inverse function theorem for continuously differentiable maps $f : \Omega \to \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$. Likewise the implicite function theorem.
3. Use the method of characteristics to find the solution of

   $$u_t + \nabla u \cdot V(\mathbf{x}) = u + f(\mathbf{x}) \text{ in } \mathbb{R}^n \times [0, \infty) \quad \text{with} \quad u(\mathbf{x}, 0) = g(\mathbf{x}),\ g, f \in C^1(\mathbb{R}^n)$$

   where $V \in C^1(\mathbb{R}^n, \mathbb{R}^n)$ with compact support.
4. Use the method of characteristics to find the solution of

   $$u_x + yu_y = 0 \text{ in } \mathbb{R}^2 \text{ with } \quad u(y, 0) = \frac{1}{1 + y^2}.$$

To Read

1. Section 4.3 and 4.4 in *Partial Differential Equations: A First Course* by R. Choksi.