MAT351 Partial Differential Equations **Fall, Assignment $3\frac{1}{2}$**

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Practice Problems

1. (*Hammer blow*) Let $\phi(x) \equiv 0$, $x \in \mathbb{R}$ and $\psi(x) = 1$ for $|x| < a$ and $\psi(x) = 0$ for $|x| \geq a$. Consider the solution $u(x,t)$ of the wave equation $u_{t,t} = c^2 u_{x,x}$ with initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$.

   (a) Sketch the profile of the function $u(x,t)$, $x \in \mathbb{R}$, for the successive instants $t = a/2c, a/c, 3a/2c, 2a/ac$ and $5a/c$.

   Hint: Calculate

   $$u(x,t) = \frac{1}{c}\int_{x-ct}^{x+ct} \psi(s)ds.$$

   (b) Find $\max_{x\in\mathbb{R}} u(x,t)$ as a function of t.

2. Solve the wave equation $u_{t,t} = c^2 u_{x,x}$ on $\mathbb{R} \times [0,\infty)$ with initial values $u(x,0) = \log(1+x^2)$ and $u_t(x,0) = 4 + x$.

3. Solve

   $$\begin{cases} u_{x,x} - 3u_{x,t} - 4u_{t,t} = 0 & \text{in } \mathbb{R} \times [0,\infty) \\ u(x,0) = x^2, \quad u_t(x,0) = e^x & \text{for } x \in \mathbb{R}. \end{cases}$$

   Hint: Factor the differential operator into two first order operators.

4. Recall that function $\phi : \mathbb{R} \to \mathbb{R}$ iscalled odd if $\phi(-x) = -\phi(x)$ $\forall x \in \mathbb{R}$ and is called even if $\phi(x) = \phi(-x)$.

   Let $u : \mathbb{R} \times [0,\infty)$ is a solution of the wave equation $u_{t,t} = c^2 u_{x,x}$ with initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$.

   If ϕ and ψ are odd in x, show that u is odd in x.

To Read

1. Section 2.2, 2.3, 2.4, 2.5 in *Partial Differential Equations: An Introduction* by W. Strauss.