MAT351 Partial Differential Equations **Fall, Assignment 6**

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Hand-in Problems (Due till November 02 before lecture, via crowdmark)

H1 (a) Consider the diffusion equation on the whole real line with the initial condition $u(x,0) = \phi(x)$. ϕ is odd if $\phi(x) = -\phi(-x)$ and even if $\phi(x) = \phi(-x)$.

If ϕ is odd, show that also the solution $u(x,t)$ is an odd function in x. (*Hint: Consider* $u(-x,t) + u(x,t)$ *and use uniqueness.*)

(b) Show that the same is true if "odd" is replaced by "even".

(c) Show that the analogous statements are true for the wave equation.

H2 Solve the inhomogeneous diffusion equation on the half-line with Dirichlet boundary condition

$$\begin{aligned} u_t - ku_{x,x} &= f(x,t) \quad \text{on } (0,\infty)\times(0,\infty),\\ u(0,t) &= 0 \quad \text{on } (0,\infty),\\ u(x,0) &= \phi(x) \quad \text{on } u(x,0) = \phi(x). \end{aligned}$$

using the method of reflection.

Problems for discussion

1. Solve the diffusion equation $u_t = ku_{x,x}$ on $\mathbb{R}\times(0,\infty)$ with the initial condition $\phi(x) = e^{-x}$.
2. Solve the diffusion equation on the half-line $(0,\infty)$ with Dirichlet boundary condition and initial condition e^{-x} for $x > 0$.
3. Derive the solution formula for the half-line Neumann problem $u_t - kw_{x,x} = 0$ on $\mathbb{R}\times(0,\infty)$ with $u_x(0,t) = 0$ and $u(x,0) = \phi(x)$.

To Read

1. Section 3.3, 3.4 and 3.5 in *Partial Differential Equations: An Introduction* by W. Strauss.