MAT351 Partial Differential Equations **Fall, Assignment 9**

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Bonus Hand-in Problems (Due till January 11 before lecture, via crowdmark)

1. Prove uniquness of the Dirichlet problem $\Delta u = f$ in $B_a(0)$ with $u = g$ on $\partial B_a(0)$ by the maximum principle. That is, after substracting two solutions $u - v =: w$, apply to maximum principle to w.

2. Solve explicitly $u_{x,x} + u_{y,y} = 0$ on $B_a(0) \subset \mathbb{R}^2$ with the boundary condition

$$u(a\cos\theta, a\sin\theta) = \tilde{u}(a,\theta) = 1 + 3\sin\theta.$$

Write your solution in Euclidean coordinates. *Hint: Apply the Poisson formula.*

Problems for discussion

1. A function $u(\mathbf{x})$ on $U \subset \mathbb{R}^n$ that is open and connected is *subharmonic* in U if $\Delta u \geq 0$ in U. Prove that u attains its maximum value on ∂U. (*Note this is not true for the minimum value*). What does the statement say in the case $n = 1$?

2. Show that there is no solution of

$$\Delta u = f \text{ on } U, \ \frac{\partial u}{\partial N} = g \text{ on } \partial U$$

unless

$$\int_U f(x,y,z)dxdydz = \int_{\partial U} g(s)d\mathcal{S}^{n-1}_{\partial U}(s)$$

where $U \subset \mathbb{R}^3$ has a smooth boundary ∂U and N is the outer uni normal vector along ∂U.

To Read

1. Section 6.1, 6.2, 6.3, 6.4 in *Partial Differential Equations: An Introduction* by W. Strauss.

Happy Holidays and Happy New Year!!
Stay safe