$$\lim_{s \to t} g(s,t) = \int_{-\infty}^{\infty} S(x-y, t-s)\, f(y,s)\, dy$$

$$= \underbrace{\int_{-\infty}^{\infty} S(x-y, t-s)\, f(y,t)\, dy}_{\longrightarrow f(x,t)} + \underbrace{\int_{-\infty}^{\infty} S(x-y, t-s)\, [f(y,s) - f(y,t)]\, dy}_{(*)}$$

$$(*) \int_{|x-y| \le \tilde{c}} S(x-y, t-s) \underbrace{[f(y,s) - f(y,t)]}_{<\varepsilon} + \int_{|x-y| > \tilde{c}} \underbrace{S(x-y, t-s)}_{e^{-(x-y)}} \underbrace{[f(y,s) - f(y,t)]}_{<2C}$$