$$e^{ix} = \cos x + i \sin x \quad \text{Euler's Id}$$

$$e^{i(x+y)} = \underline{\cos(x+y)} + i\, \underline{\sin(x+y)}$$

$$e^{ix} \cdot e^{iy} = (\cos x + i \sin x)(\cos y + i \sin y)$$

$$= \underline{\cos x \cos y + \underset{-1}{i^2} \sin x \cdot \sin y}$$

$$+ i(\underline{\cos x \sin y + \sin x \cos y})$$

$$\int_0^{\ell} \sin\left(\frac{n\pi}{\ell} x\right) dx$$

$$= \frac{\ell}{n\pi} \int_0^{n\pi} \sin x \; dy$$

$$= \frac{\ell}{n\pi} \; -\cos x \Big|_0^{n\pi}$$

$$\Phi(x) = 1 \qquad x \in [0, \ell]$$

$$\frac{2}{\ell}\int_0^{\ell} \phi(x)\cdot \sin\left(\frac{n\pi}{\ell}x\right) dx = \frac{2}{\ell}\int_0^{\ell} \sin\left(\frac{n\pi}{\ell}x\right) dx$$

$$= \frac{2}{n\pi}\int_0^{n\pi} \sin(x)\, dx = \frac{2}{n\pi}\left[-\cos x \Big|_0^{n\pi}\right]$$

$$= \frac{2}{n\pi}\left[-\cos n\pi + \cos 0\right] = \frac{2}{n\pi}\left(-(-1)^n + 1\right)$$

$$\Rightarrow \quad S(\phi) = \sin\left(\frac{\pi}{\ell}x\right)\cdot\frac{4}{\pi} + \sin\left(\frac{3\pi}{\ell}x\right)\cdot\frac{4}{3\pi} + \dots$$

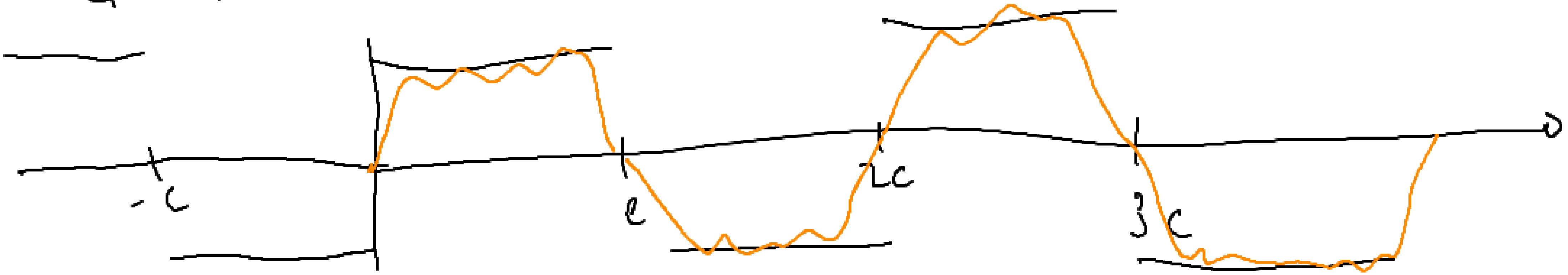
Fourier sin series

if convergent

converges to periodic odd

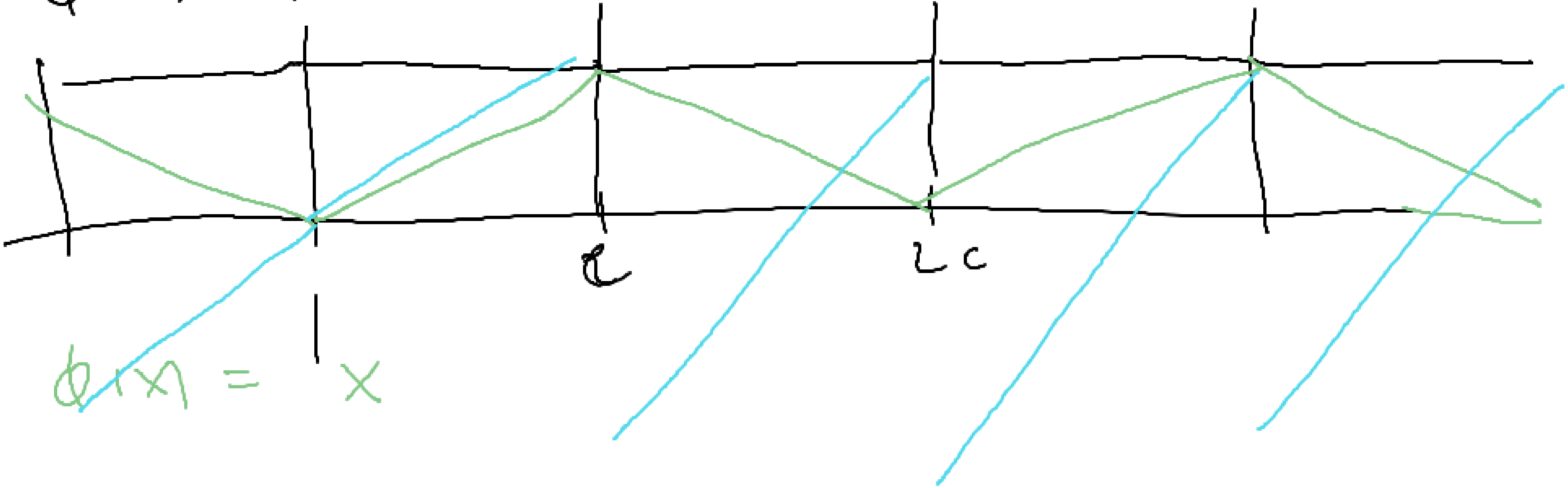
extension of ϕ

$\phi(x) = 1$ on $[0, c]$

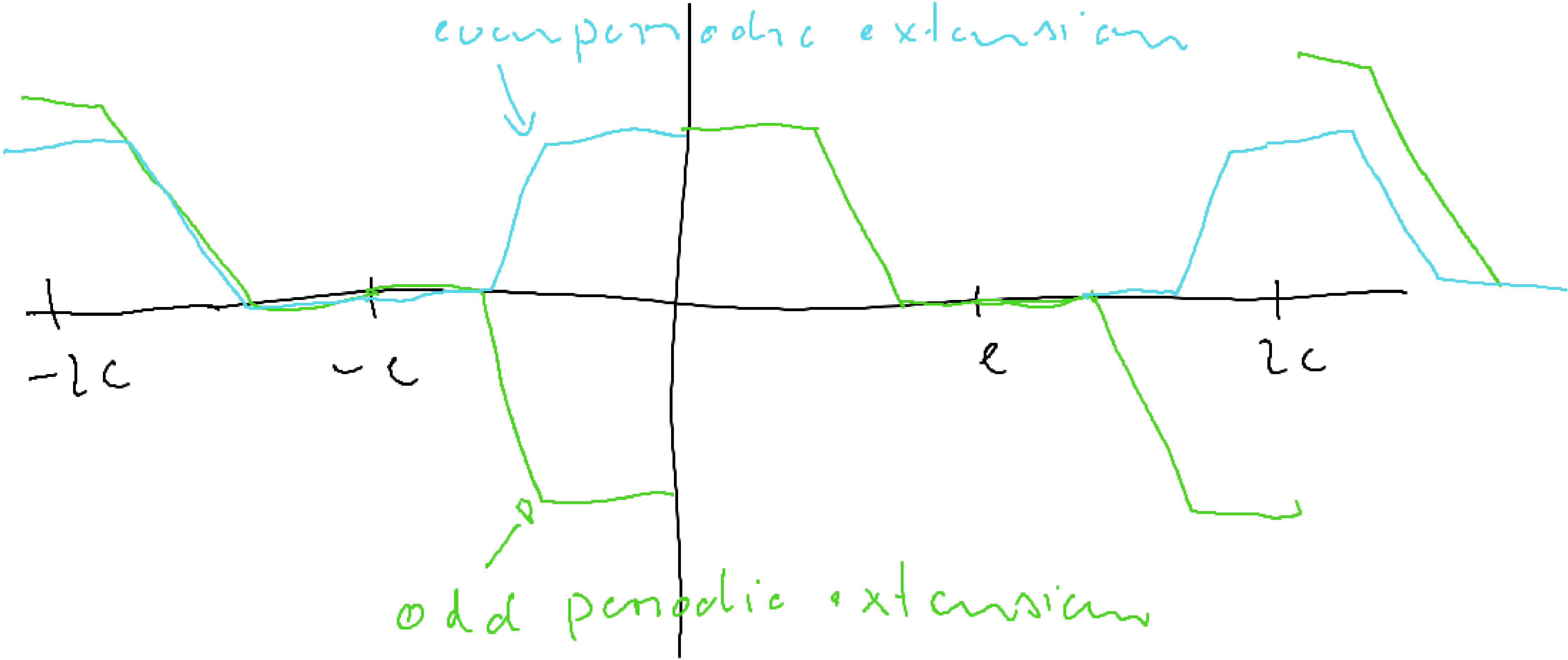


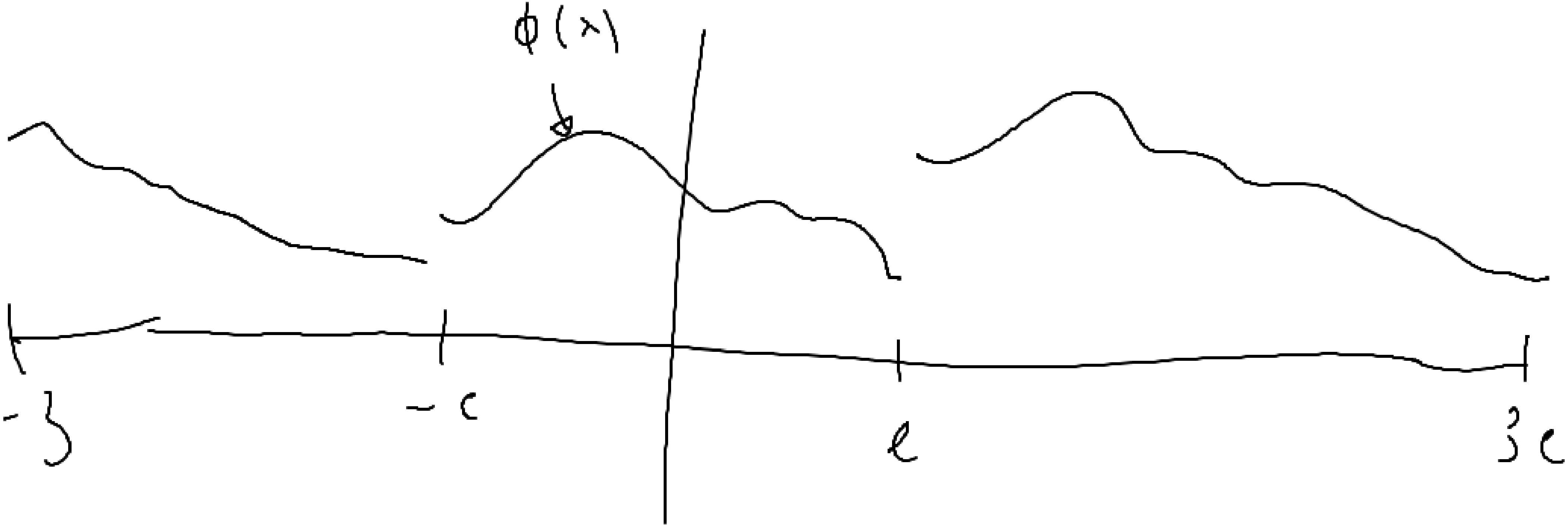
Fourier cosine series converges to the periodic even extension

$\phi(x) = 1 \quad [0, c]$



$\phi(x) = x$


even periodic extension
odd periodic extension
-2c
-c
c
2c



φ(x)
-3
-c
c
3c