$$u_{tt} = c^2 \Delta u \quad \text{on } \mathbb{R}^3 \times [0, \infty)$$

$$\lim_{(x,t) \to (x_0, 0)} u(x,t) = \phi(x) \quad \text{on } \mathbb{R}^3$$

$$\lim_{(x,t) \to (x, 0)} u_t(x,t) = \psi(x)$$

$(*)$

Kirchhoff's formula

$u \in C^2(\mathbb{R}^3 \times [0, \infty))$ solves $(*)$

$$\Longrightarrow \quad u(x,t) = t\, \bar{\Phi}(x,t) + \frac{\partial}{\partial t}\left(t\, \bar{\Psi}(x,t)\right)$$

where $\bar{\Psi}(x,t) = \fint_{\partial B_{ct}(x)} \psi(y)\, dS(y) = \frac{1}{4\pi c^2 t^2} \int_{\partial B_{ct}(x)} \psi(y$

$\bar{\Phi}(x,t) = \ldots.$

Remark (Alternative form)

$$\frac{\partial}{\partial t}\left(t\,\bar{\Phi}(x,t)\right) = \frac{\partial}{\partial t}\left(t \cdot \frac{1}{4\pi c^2 t^2}\int_{\partial B_{ct}(x)} \Phi(y)\, ds(y)\right.$$

$$= \frac{\partial}{\partial t}\left(t\,\frac{1}{4\pi c^2 t^2}\int_{\partial B_1(0)} \Phi(x+cty)\cdot c^2 t^2\, d\right.$$

$$= \bar{\Phi}(x,t) + t\cdot\frac{1}{4\pi}\int_{\partial B_1(0)} \langle \nabla\Phi(x+cty),\right.$$

$$= \bar{\Phi}(x,t) + \frac{tc}{4\pi c^2 t^2}\int_{\partial B_{ct}(x)} \langle \nabla\Phi(y), \frac{y-x}{ct}\right.$$

$$= \fint_{\partial B_{ct}(x)} \left(\Phi(y) + \langle \nabla\Phi(y), y-x\rangle\right) ds(y)$$

$$u(x,t) = \fint_{\partial B_{ct}(x)} \left(t\Psi(y) + \Phi(y) + \langle\nabla\Phi(\cdot\right.$$

Similar for $n = 2$:

$$u(x,t) = \frac{1}{2} \int_{B_{ct}(x)} \frac{t\psi(y) + \phi(y) + t\langle \nabla\phi(y), y-\dots}{\sqrt{t^2 - |y-x|^2}}$$

Theorem: (Solution for wave equa. in 3 D...

$n = 3$, $\phi, \psi \in C^2(\mathbb{R}^3)$. Define u by K. ho...

Then $u \in C^2(\mathbb{R}^3 \times [0,\infty))$ and solves $(*)$ ...

Proof: Assume $\phi = 0$

$$\Longrightarrow u(x,t) = t \cdot \Psi(x,t) = t \cdot \frac{1}{4\pi c^2 t^2} \int_{\partial B_{ct}(x)} \psi(\dots$$

$$= t \cdot \frac{1}{4\pi} \int_{\partial B_1(0)} \psi(x + cty)\, ds(y)$$

$\Longrightarrow u \in C^2(\mathbb{R}^3 \times [0,\infty))$

[$\lim_{t\to 0} \int f(x,t) \dots$

$= \int \lim_{t\to \dots} \dots$]

- $u_t(x,t) = \bar{\Psi}(x,t) + t\,\bar{\Psi}_t(x,t)$

$$u_{tt}(x,t) = \bar{\Psi}_t(x,t) + \bar{\Psi}_t(x,t) + t\,\bar{\Psi}_{tt}$$

$$= \frac{1}{t}\,\frac{d}{dt}\left(t^2 \cdot \bar{\Psi}_t(x,t)\right)$$

$$\bar{\Psi}_t(x,t) = \frac{1}{4\pi}\cdot c \int_{\partial B_1(0)} \langle \nabla \psi(x+cty), y \rangle \, d$$

$$= \frac{c}{4\pi c^2 t^2} \int_{\partial B_{ct}(x)} \langle \nabla \psi(y), \underbrace{\frac{y-x}{ct}}_{N} \rangle \, d$$

$$= \frac{c}{4\pi c^2 t^2} \int_{B_{ct}(x)} \Delta \psi(y)\, dy$$

$$t^2\, \bar{\Psi}_t(x,t) = \frac{1}{4\pi c}\cdot \int_{B_{ct}(x)} \Delta\psi(y)\, dy = \frac{1}{4\pi c}\int_0^{ct}\int_{\partial B_r(x)} \widehat{\Delta\psi}$$

$$\frac{d}{dt}\left(t^2 \overline{\varphi}_t(x,t)\right) = \frac{1}{4\pi c}\cdot c \int_{\partial B_1(x)} \widetilde{\Delta\varphi}(ct\cdot\theta)\, c$$

$$= \frac{1}{4\pi}\int_{\partial B_1(0)} \Delta\varphi(x+y)\, c^2 t^2\, d$$

$$= \Delta_x\Big(\frac{1}{4\pi}\int_{\partial B_1(0)} \varphi(x+y)\, c t^2$$

$$\frac{1}{t}\frac{d}{dt}\left(t^2\overline{\varphi}_t(x,t)\right) = c^2\Delta_x\Big(t\cdot\frac{1}{4\pi c^2 t^2}\cdot\int_{\partial B_{ct}(} t$$

$$= c^2\Delta_x\left(t\,\overline{\varphi}(x,t)\right)$$

(-) Caso: $u_t(x,t) = \underline{\Phi(x,t)} + t\,\Phi_t(x,t)$

$$\Phi(x,t) = \frac{1}{4\pi}\int_{\partial B_1(0)} \varphi(x+ct\,y)\,ds(y) \xrightarrow[x\to x_0]{t\to 0} \psi$$

$$|t\,\Phi_t(x,t)| \leq t\cdot\frac{1}{4\pi}\left|\int_{\partial B_1(0)} \langle \nabla\varphi(x+ct\,y \ldots\right.$$

$$\leq t\,\frac{1}{4\pi}\underbrace{\underbrace{\int_{\partial B_1(0)} |\nabla\varphi|(x+ct \ldots}_{\xrightarrow[x\to x_0]{t\to 0}}}_{\xrightarrow[t\to 0]{} 0}$$

Similiar for $0 = \phi \leftarrow u(x,t)$

Remark: $n = 2k + 1 \geq 3$

solution formula is

$$u(x,t) = \frac{1}{\gamma_n} \left[\frac{\partial}{\partial t} \left(\left(\frac{1}{t} \frac{\partial}{\partial t} \right)^{\frac{n-3}{2}} t^{n-} \right. \right.$$

$$+ \left(\frac{1}{t} \frac{\partial}{\partial t} \right)^{\frac{n-3}{2}} t^2 \int_{\partial B_{ct}(x)} \psi_i$$