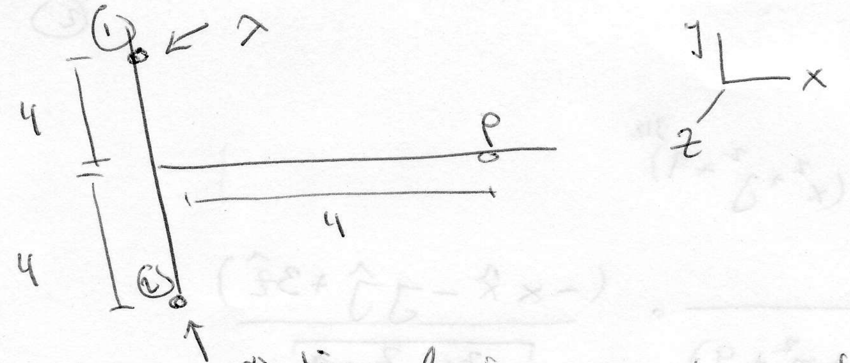
1.



∞ line of charge with charge density $= \lambda = 4\,nC/m$

Find $\vec{E}$ @ point P

$|\vec{E}|$ from single ∞ line is $\dfrac{\lambda}{2\pi\epsilon_0 r}$

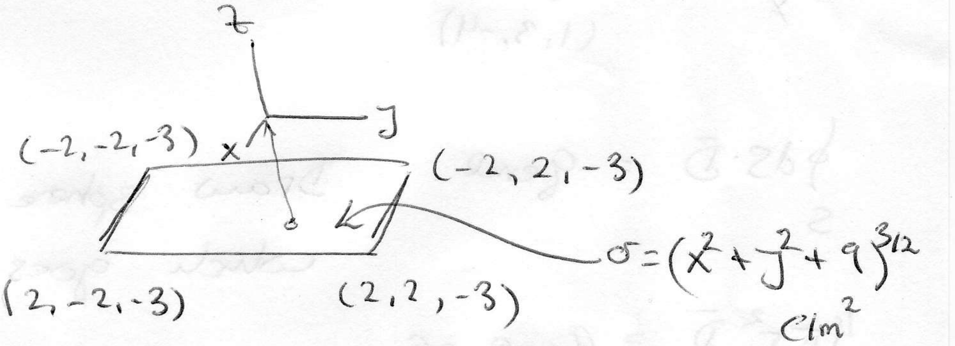
Superposition of both cancels $\hat{y}$ component, so

$$E_{Total} = \frac{\lambda}{2\pi\epsilon_0 r}\frac{(4\hat{x} - 4\hat{y})}{\sqrt{32}} + \frac{\lambda}{2\pi\epsilon_0 r}\frac{(4\hat{x} + 4\hat{y})}{\sqrt{32}}, \quad r = \sqrt{32}$$

$$= \frac{2\lambda \cdot 4\hat{x}}{2\pi\epsilon_0 r^2}$$

$$= 18\,\hat{x}\ V/m$$

2 Charge on sheet:
(sheet is at $z = -3$)



Find $\vec{E}$ @ origin.

$$\vec{E} = \frac{Kq}{r^2}\hat{r} \quad \text{and} \quad \hat{r} = \frac{-x\hat{x} - y\hat{y} + 3\hat{z}}{\sqrt{x^2+y^2+9}}$$

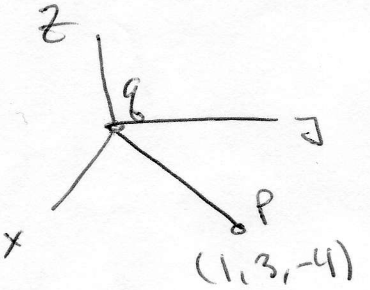
and $q = \int_{-2}^{2} dx \int_{-2}^{2} dy \, \sigma$

$$\therefore \bar{E} = \int_{-2}^{2} dx \int_{-2}^{2} dy \, \frac{\sigma}{\underbrace{4\pi\epsilon_0 (x^2+y^2+9)}} \cdot \frac{(-x\hat{x} - y\hat{y} + 3\hat{z})}{\sqrt{x^2+y^2+9}} \qquad = (x^2+y^2+9)^{3/2}$$

by symmetry, x and y components will cancel.

So $\bar{E} = \int_{-2}^{2} dx \int_{-2}^{2} dy \cdot \frac{3}{4\pi\epsilon_0} \hat{z} = \frac{48}{4\pi\epsilon_0} \hat{z}$

3. Point charge q at origin. Find electric flux density at $(1, +3, -4)$



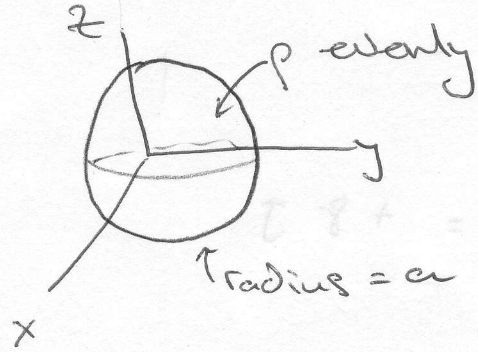
$\oint_S d\bar{s} \cdot \bar{D} = q_{encl}$. Draw sphere as Gaussian surface which goes through point P

$4\pi r^2 D = q_{encl.} = q$

$|D| = \frac{q}{4\pi r^2}$, $r = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}$

and $\hat{D} = \hat{r}$ (i.e. points radially outward)

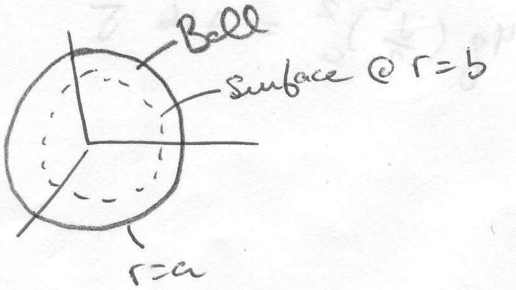
4. Have uniform ball of charge with density ρ



ρ evenly distributed inside.

Find $\bar{D}$ at points $r = b$ $(b < a)$ and $r = c$ $(c > a)$

for $r = b$, draw Gaussian surface inside ball:



$$\int D \cdot dS = q_{encl}$$

$$|\bar{D}| \cdot 4\pi b^2 = \frac{4}{3}\pi b^3 \cdot \rho$$

$$|D| = \rho\frac{b}{3}$$

for $r = c$, draw surface outside ball.

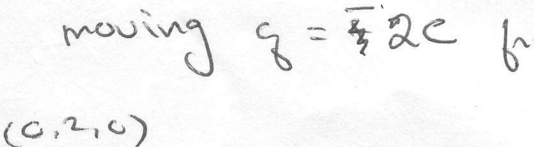
Then $$\int \vec{D} \cdot d\vec{S} = q_{encl}$$

$$4\pi c^2 |\vec{D}| = \frac{4}{3}\pi a^3 \rho$$

$$|\vec{D}| = \rho\frac{a^3}{c^2}$$

5. Have $E = 2x\,\hat{x} - 4y\,\hat{y}$ V/m. Find work done in moving $q = 2C$ from a) $(2,0,0)$ to $(0,0,0)$ to $(0,2,0)$

b) $(2,0,0)$ to $(0,2,0)$ directly



c) From $(2,0,0)$ to $(0,0,0)$

$W = \bar{F} \cdot \bar{d}$

$= q\bar{E} \cdot \bar{d}$ Here $y = 0$, so

$W = \int_2^0 q(2x\hat{x})\,dx\,\hat{x} = 2q\frac{x^2}{2}\Big|_2^0 = +8\text{ J}$

From $(0,0,0)$ to $(0,2,0)$

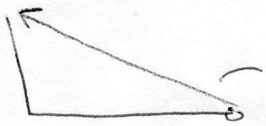
$W = q\bar{E} \cdot \bar{d}$ $x = 0$, so

$= \int_0^2 q(-4y\hat{y}) \cdot dy\,\hat{y} = -4q\left(\frac{y^2}{2}\right)_0^2 = +16\text{ J}$

So $\text{Work}_{\text{Total}} = 24\text{ J}$

b)

(0,2)



line in $y = 2 - x$

(2,0)

$W = q\bar{E} \cdot \bar{d}$

$= q\int_{(2,0)}^{(0,2)} (2x\hat{x} - 4y\hat{y}) \cdot (dx\,\hat{x} + dy\,\hat{y})$

put everything in terms of $\underline{\underline{x}}$

i.e. $y = 2 - x$

and $\frac{dy}{dx} = -1 \rightarrow dy = -dx$

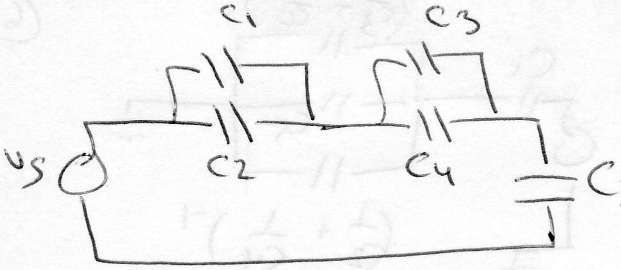
$\therefore W = q\left[\int_2^0 2x\,dx - 4(2-x)(-dx)\right]$

$= q\int_2^0 2x\,dx + q\int_2^0 8\,dx - q\int_2^0 4x\,dx$

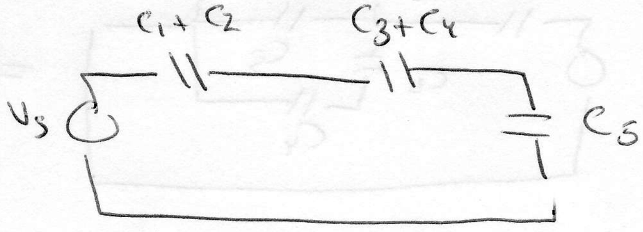
$= q\left[x^2\Big|_2^0 + 8x\Big|_2^0 - 2x^2\Big|_2^0\right] = q(-4 - 16 + 8) = \underline{\underline{+24\text{ J}}}$ again

6.

a)

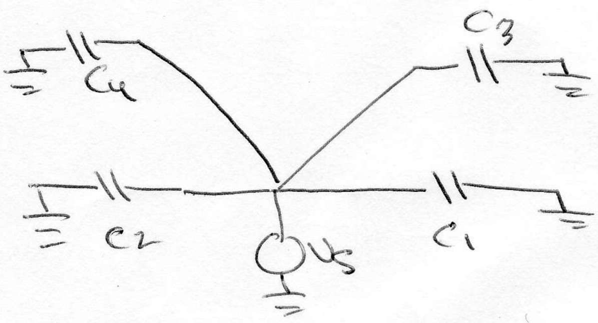


$\Rightarrow$



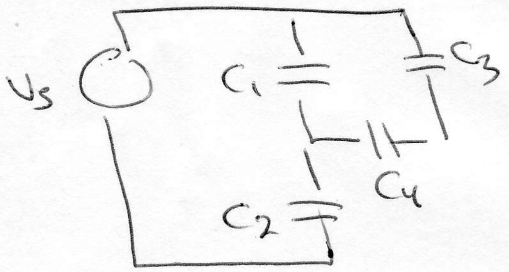
$$C_T = \left(\frac{1}{C_1+C_2} + \frac{1}{C_3+C_4} + \frac{1}{C_5}\right)^{-1}$$

b)



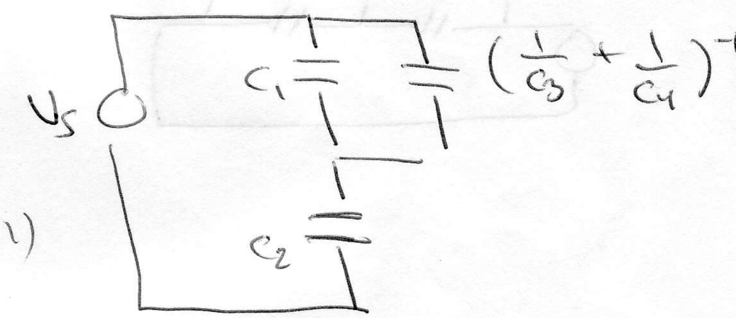
$$C_T = C_1 + C_2 + C_3 + C_4$$

c)

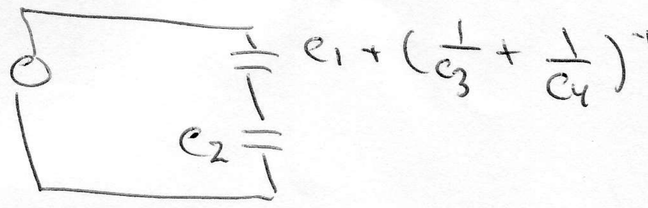


$\Rightarrow$

1)



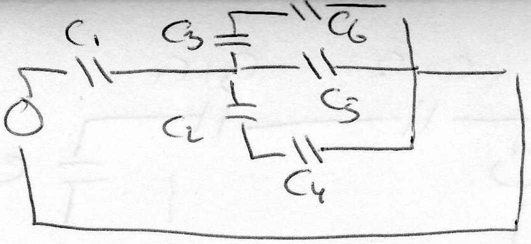
2)



3)

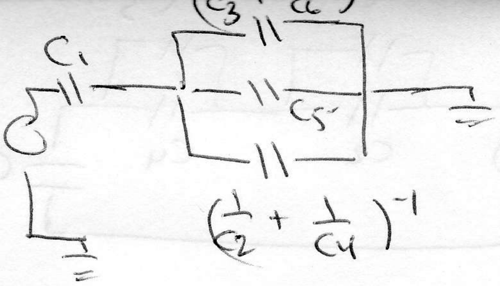
$$C_T = \left[\frac{1}{C_1+\left(\frac{1}{C_3}+\frac{1}{C_4}\right)^{-1}} + \frac{1}{C_2}\right]^{-1}$$

d)



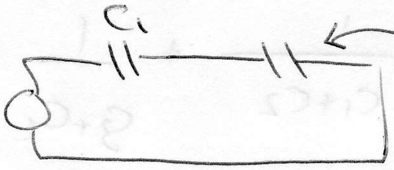
$\Rightarrow$

1)



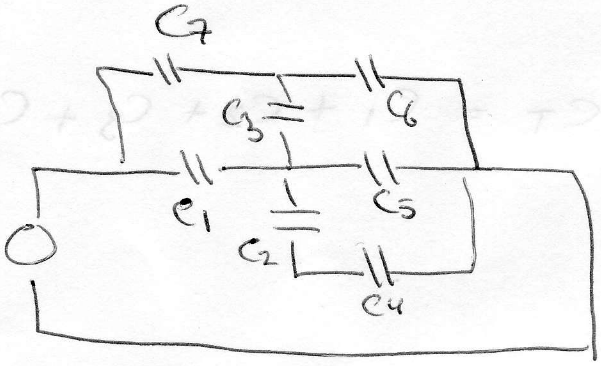
$\left(\frac{1}{C_2} + \frac{1}{C_4}\right)^{-1}$

2)



$\left(\frac{1}{C_3} + \frac{1}{C_6}\right)^{-1} + C_5 + \left(\frac{1}{C_2} + \frac{1}{C_4}\right)^{-1}$

e)



1)