Coherent control of population transfer between vibrational states in an optical lattice via two-path quantum interference

**Chao Zhuang**, Christopher R. Paul, Xiaoxian Liu, Samansa Maneshi, Luciano S. Cruz, Aephraim M. Steinberg

Centre for Quantum Information & Quantum Control (CQIQC)  
Institute for Optical Sciences (IOS)  
Department of Physics, University of Toronto

DAMOP, May 26th, 2010
Motivation

- Coherent control of quantum states is important in both physics and chemistry, especially for quantum information processing.
- Optical lattice is a leading candidate for quantum information processing; an important arena for studying condensed-matter and many-body phenomena.
- Two $\omega$ quanta vs. one $2\omega$ quantum interference for coherent control.
  - Realized in different systems: photocurrent in semiconductors, photoionization, and photodissociation.
  - Recent theoretical proposals in different systems: graphene, carbon nanotubes, and molecular wires, quantum wells ...
  - In optical lattices: quantum rachets, control the momentum of atoms.
- Our interest: coherent control of the quantum vibrational states in optical lattices using this two-path quantum interference.
$^{85}\text{Rb}$ atoms in vertical lattice

$^{85}\text{Rb}$ atom cloud temperature $\sim 10\mu\text{K}$
Atomic density in lattice $\sim 10^9\text{atoms/cm}^3$
Photon scattering rate $\sim 20\text{Hz}$
Effect Recoil Energy $E_r = \hbar \omega_{re}$

$\omega_{re} = 2\pi \times 685\text{Hz}$
Gravity $\sim 2.86$ Effect Recoil Energy

30GHz red detuned from $^{85}\text{Rb}$ D2 line ($\lambda = 780\text{nm}$)

$0 \text{r.m.s radius } = 1.5\text{mm}$
Some Facts

$^{85}$Rb atom cloud temperature $\sim 10\mu K$

$\Rightarrow$ de Broglie wavelength $\sim 60\text{nm}$  $<<$ Lattice constant $= 930\text{nm}$

$\Rightarrow$ no coherence between the wells

Atomic density in lattice $\sim 10^9\text{atoms/cm}^3$

$\Rightarrow$ interactions between atoms can be neglected

Photon scattering rate $\sim 20\text{Hz}$

$\Rightarrow$ Lifetime in lattice $\sim 50\text{ms}$  $>>$ Experiment time $\sim 1\text{ms}$

Dimensionless lattice Hamiltonian: $\tilde{\mathbf{H}}_0 = \tilde{p}^2 + r \sin^2 \tilde{x} + \frac{s}{\pi} \tilde{x}$

The eigenstates for $\tilde{\mathbf{H}}_0$ are Wannier-Stark states.
State preparation and measurement

- **Initial Lattice**
- **After adiabatic decrease**

- **Lattice Depth**
  - operation
  - preparation
  - measurement
  - Time

- **Ground State** $P_{\text{Gnd}}$
- **1st Excited State** $P_{\text{Exc}}$
- **Unbound States** $P_{\text{Loss}}$

- **2 bound states**
- **1 bound state**
**ω-phonon Excitation**

**Phase Modulation (PM)**

~ Resonant Drive

~ Displace the potential

~ Displace the wavefunction

**Spectrum of PM**

\[ \begin{align*}
\omega_L - 2\omega & \quad \omega_L - \omega & \quad \omega_L & \quad \omega_L + \omega & \quad \omega_L + 2\omega
\end{align*} \]
Add another excitation and make them interfere!
2ω-phonon Excitation

Amplitude Modulation (AM)

~ Parametric Drive

~ Change the potential depth

~ Squeeze the wavefunction

Spectrum of AM

\[ \omega_L - 2\omega \quad \omega_L \quad \omega_L + 2\omega \]
Two $\omega$-phonons vs. One $2\omega$-phonon

$\Delta \phi = \phi_{AM} - 2\phi_{PM} = 2\omega \Delta \tau$

$P_{Loss}^{AM+PM} = \left| e^{i2\phi_{PM}} \sqrt{P_{Loss}^{PM}} + e^{i\phi_{AM}} \sqrt{P_{Loss}^{AM}} \right|^2$

PM: $\theta(t) = A_{PM}(1 - \cos \omega t)$, number of PM cycle: $n$;
AM: $\eta(t) = A_{AM} \sin[2\omega(t - \Delta \tau)]$, number of AM cycle: $2n$. 
Populations depend on Relative phase!

- $P^{AM+PM}_{Loss}$: 
  - minima at $\Delta \phi = 2l\pi$,
  - maxima at $\Delta \phi = (2l + 1)\pi$
  ($l$ is an integer)

- some points:
  - $P^{AM+PM}_{Loss}$ lower than $P^{PM}_{Loss}$

- $P^{AM+PM}_{Loss}$ fringe
  - mean: $0.274 \pm 0.001$
  - amplitude: $0.073 \pm 0.002$

- $P^{AM+PM}_{Loss}$ two-path model
  - mean:
    
  $P^{PM}_{Loss} + P^{AM}_{Loss} = 0.29 \pm 0.01$
  - amplitude:
    
  $2\sqrt{P^{PM}_{Loss} P^{AM}_{Loss}} = 0.219\pm0.02$

\[
\begin{align*}
P^{PM}_{Loss} & = 0.24 \pm 0.01 \text{ (4-cycle-PM, } A_{PM} = 8^\circ) \\
P^{AM}_{Loss} & = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) 
\end{align*}
\]
**Maxima and Minima**

\[ P_{Loss}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]

different \( P_{Loss}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)
**Motivation**

**Experiment Setup**

**Experiment Results**

**Conclusion**

---

**Maxima and Minima**

\[ P_{Loss}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]

different \( P_{Loss}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)
Maxima and Minima

\[ P^{AM}_{\text{Loss}} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]
different \[ P^{PM}_{\text{Loss}} \text{ (4-cycle-PM, } A_{PM} \text{ is varied) } \]
Maxima and Minima

\[ P_{Loss}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]
derifferent \[ P_{Loss}^{PM} \text{ (4-cycle-PM, } A_{PM} \text{ is varied) \]

\begin{align*}
\text{Loss with PM alone} & \quad \text{Max, Min Loss with PM + AM} \\
\text{0} & \quad \text{0} \\
\text{0.2} & \quad \text{0.2} \\
\text{0.4} & \quad \text{0.4} \\
\text{0.6} & \quad \text{0.6} \\
\text{0.8} & \quad \text{0.8} \\
\text{1} & \quad \text{1}
\end{align*}

\begin{align*}
A_{PM} = 5^\circ & \quad P_{Loss}^{PM} = 0.06 \\
\text{Time of AM delayed after PM, } \Delta \tau \text{ (µs)} & \quad \text{P}_{Loss}
\end{align*}
Maxima and Minima

\[ P_{\text{Loss}}^{\text{AM}} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{\text{AM}} = 10\%) \]
different \( P_{\text{Loss}}^{\text{PM}} \) (4-cycle-PM, \( A_{\text{PM}} \) is varied)
Maxima and Minima

\[ P_{\text{Loss}}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{\text{AM}} = 10\%) \]

Different \[ P_{\text{Loss}}^{PM} \text{ (4-cycle-PM, } A_{\text{PM}} \text{ is varied)} \]
Maxima and Minima

\[ P_{\text{Loss}}^{\text{AM}} = 0.05 \pm 0.01 \quad (8\text{-cycle-AM, } A_{\text{AM}} = 10\%) \]
different \[ P_{\text{Loss}}^{\text{PM}} \quad (4\text{-cycle-PM, } A_{\text{PM}} \text{ is varied}) \]
Maxima and Minima

\[ P_{\text{Loss}}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]

different \( P_{\text{Loss}}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)

\[
\begin{array}{c|c}
\text{Max Loss} & \text{Min Loss} \\
\hline
0.2 & 0.1 \\
0.4 & 0.3 \\
0.6 & 0.5 \\
0.8 & 0.7 \\
1 & 0.9 \\
\end{array}
\]

\[ A_{PM} = 9^\circ \quad P_{\text{Loss}}^{PM} = 0.31 \]

\[ \text{Time of AM delayed after PM, } \Delta \tau (\mu s) \]

\[ 0 \quad 200 \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]
Maxima and Minima

\[ P_{\text{loss}}^{\text{AM}} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{\text{AM}} = 10\%) \]
different \( P_{\text{loss}}^{\text{PM}} \) (4-cycle-PM, \( A_{\text{PM}} \) is varied)

\[ P_{\text{loss}}^{\text{max}} \text{ Exp} \]
\[ P_{\text{loss}}^{\text{min}} \text{ Exp} \]

\[ A_{\text{PM}} = 10^o \quad P_{\text{loss}}^{\text{PM}} = 0.39 \]

Time of AM delayed after PM, \( \Delta \tau \) (\( \mu s \))
Maxima and Minima

\[ P_{\text{Loss}}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\% \text{)} \]

different \[ P_{\text{Loss}}^{PM} \] \( (4\text{-cycle-PM, } A_{PM} \text{ is varied}) \)

![Graph showing maxima and minima with PM and AM](image)
**Maxima and Minima**

\[ P_{\text{Loss}}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]

different \( P_{\text{Loss}}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)
Maxima and Minima

\[ P_{\text{Loss}}^{\text{AM}} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{\text{AM}} = 10\%) \]
different \( P_{\text{Loss}}^{\text{PM}} \) (4-cycle-PM, \( A_{\text{PM}} \) is varied)
Maxima and Minima

\[ P_{\text{Loss}}^{\text{AM}} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{\text{AM}} = 10\%) \]
different \( P_{\text{Loss}}^{\text{PM}} \) (4-cycle-PM, \( A_{\text{PM}} \) is varied)
Maxima and Minima

\( P_{Loss}^{AM} = 0.05 \pm 0.01 \) (8-cycle-AM, \( A_{AM} = 10\% \))
different \( P_{Loss}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)

\[
P_{Loss}^{AM} = 0.05 \pm 0.01 \quad \text{(8-cycle-AM, } A_{AM} = 10\% \text{)}
\]
different \( P_{Loss}^{PM} \) (4-cycle-PM, \( A_{PM} \) is varied)
Maxima and Minima

\[ P_{\text{Loss}}^{AM} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%) \]
different \[ P_{\text{Loss}}^{PM} \text{ (4-cycle-PM, } A_{PM} \text{ is varied) } \]

\[ P_{\text{Loss}}^{\text{AM+PM}} = \left| e^{2i\phi_{PM}} \sqrt{P_{\text{Loss}}^{PM}} + e^{i\phi_{AM}} \sqrt{P_{\text{Loss}}^{AM}} \right|^2 \]
Motivation

Experiment Setup

Experiment Results

Conclusion

Maxima and Minima

$P^{AM}_{Loss} = 0.05 \pm 0.01 \text{ (8-cycle-AM, } A_{AM} = 10\%)$

different $P^{PM}_{Loss}$ (4-cycle-PM, $A_{PM}$ is varied)

- two-path interference model:

$$P^{AM+PM}_{Loss} = \left| e^{i2\phi_{PM}} \sqrt{P^{PM}_{Loss}} + e^{i\phi_{AM}} \sqrt{P^{AM}_{Loss}} \right|^2$$

- lattice simulation:

  sum over Gaussian distribution of lattice depths with mean depth $r_0 = 18$ and r.m.s radius $\sigma = 3$.

  $$\tilde{H}(t) = \tilde{p}^2 + r[1 + \eta(t)] \sin^2 \left[ \tilde{x} + \frac{\theta(t)}{2} \right] + \frac{s}{\pi} \tilde{x}$$
Try to find the optimal point

- Search in the parameter space of $\Delta \phi$, $A_{PM}$, $A_{AM}$, and $n$. (number of PM cycle: $n$, number of AM cycle: $2n$)
- $\Delta \phi$ is set to 0
- Branching ratio
  $B = \frac{P_{Exc}}{P_{Loss}}$ versus $P_{Exc}$ on a log-log graph
- At optimal point, branching ratio improved by a factor of $3.5 \pm 0.7$ with respect to PM alone ($A_{PM} = 8^\circ$, $A_{AM} = 10\%$)
Conclusion

- First to demonstrate coherent control of population transfer between Wannier-Stark states in an optical lattice by using interference between a two $\omega$-phonons transition and a one $2\omega$-phonon transition
- Simultaneously reduce the loss and increase the first excited state population
- At optimal point, branching ratio is increased by a factor of $3.5 \pm 0.7$
- Deviation from two-path interference model: inter-well coupling, anharmonicity, inhomogeneous broadening ...

We thank Ardavan Darabi for useful help with lattice simulation.
Time dependent lattice Hamiltonian

Time dependent Hamiltonian:
Lab frame:
\[ \tilde{H}(t) = \tilde{p}^2 + r[1 + \eta(t)] \sin^2 \left[ \tilde{x} + \frac{\theta(t)}{2} \right] + \frac{s}{\pi} \tilde{x} \]

Accelerated lattice frame:
\[ \tilde{H}_U(t) = \tilde{p}^2 + r \sin^2 \tilde{x} + \frac{s}{\pi} \tilde{x} - \frac{\dot{\theta}(t)}{2} \tilde{x} + r\eta(t) \sin^2 \tilde{x} \]

- \[ -\frac{\dot{\theta}(t)}{2} \tilde{x} \sim \text{opposite parity} \]
- \[ r\eta(t) \sin^2 \tilde{x} \sim \text{same parity} \]
- \[ \tilde{H}(t) \text{ and } \tilde{H}_U(t) \text{ are equivalent,} \]
  when \[ \theta(0) = \theta(t_p) = 0, \quad \dot{\theta}(0) = \dot{\theta}(t_p) = 0 \]
Determine $\omega$ and inhomogeneous broadening

Ramsey interference method:

$$P_{Gnd}(t) = \left| \langle 0 | D(-\alpha)R(t)D(\alpha)|0 \rangle \right|^2$$

$D(\alpha)$ displacement operator with spatial displacement $\alpha$

$R(t)$ free evolution with time $t$

- Oscillation curve fit using:
  $$P_{Gnd}(t) = a \cdot e^{-\frac{1}{2}(\frac{t}{T^*})^2} \cos[\omega t + \phi_0] + b$$

- $\omega_{exp} = 2\pi \times (4.99 \pm 0.01)kHz$
  corresponds to a lattice depth of $18E_r$

- $T^*_2 = 370 \pm 10\mu s$
  corresponds to an r.m.s radius of $3E_r$, if the lattice depths distribution is Gaussian
State preparation and measurement

Initial Lattice

After adiabatic decrease

Ground State $P_{Gnd}$
1st Excited State $P_{Exc}$
Unbound States $P_{Loss}$

Lattice Depth

Time

preparation measurement operation

2 bound states 1 bound state

40ms