Diversity-Assisted Active User Identification

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Abstract — **We propose a novel "subspace averaging" technique for subspace-based active user identification with multiple receiving antennas. It is shown that the subspace averaging utilizes more diversity information than the conventional covariance averaging technique and offers significantly improved identification performance with almost no additional complexity.**

I. Introduction

In a CDMA ad hoc network, each node is responsible for detecting the spreading codes modulating the messages sent to itself [1] before subsequently decoding the messages. The problem of active user identification arises in such circumstances. A feasible solution to this problem is a MUSIC-based technique [2], where the candidate spreading codes are projected onto the signal subspace of the received vector and the output of the projection, compared with a preset threshold, determines whether the candidate spreading code is in use (active) or not (inactive).

For the single-transmitting-antenna and single-receiving-antenna scenario, we assume a slotted chip-synchronous short code system and a slow flat fading channel. The network allows random access from all users in the form of packets of length L symbols in each packet slot. Given that the packets are synchronized, at each symbol interval, the received vector at the receiver node can be expressed as

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n}, \tag{1}$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]$ is the spreading code matrix consisting of the spreading sequences of the K active packets in the current packet slot, and $\mathbf{b} = [b_1, b_2, \cdots, b_K]^H$ represents the transmitted information symbols in the current symbol interval. The channel matrix $\mathbf{A} = diag(A_1, \cdots, A_K)$ contains independent channel coefficients that are complex Gaussian distributed with independent real and imaginary parts each with distribution $\mathcal{N}(\mu, \sigma^2)$. In other words, the magnitudes of $\{A_k\}_{k=1}^K$ are Rician distributed with pdf:

$$f_R(r) = 2(1+\kappa) r e^{-(1+\kappa)r^2 - \kappa} I_0(2\sqrt{\kappa(1+\kappa)}r), \tag{2}$$

where I_0 is the zeroth order modified Bessel function of the first kind. The Rice factor is $\kappa = \mu^2/\sigma^2$ and the parameters are normalized such that $2\mu^2 + 2\sigma^2 = 1$. The Rician channel model is chosen because a line-of-sight signal component is likely to exist in a close-range ad hoc network environment. Also notice that when $\kappa = 0$, the Rician channel reduces to a Rayleigh channel. It is also important to note that although $\mathbf{S}$ and $\mathbf{A}$ may vary from packet to packet, they are considered constants within the duration of a packet.

The autocovariance matrix of the received vector is

$$\mathbf{R} = E\{\mathbf{r}\mathbf{r}^H\} = \mathbf{S}\mathbf{A}^2\mathbf{S}^H + \sigma^2\mathbf{I}. \tag{3}$$

We then perform eigenvalue decomposition (EVD) and separate the signal subspace ($\mathbf{U}_s \in \mathbb{C}^{N\times K}$) from the noise subspace ($\mathbf{U}_n \in \mathbb{C}^{N\times(N-K)}$) and obtain

$$\mathbf{R} = \mathbf{U\Lambda U}^H = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & 0 \\ 0 & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}. \tag{4}$$

We shall assume the receiver knows the pool of spreading codes used by other nodes to transmit packets to itself, as well as the correspondence between the spreading codes and the transmitting nodes. These candidate spreading codes are placed in the set $\{\mathbf{s}_i\}_{i=1}^M$, where i labels the identity of the transmitting nodes. Whether or not node i is transmitting a packet to the receiver can be determined by comparing the identifier output d_i to a threshold value d_{th}, where

$$d_i = \|\mathbf{U}_s^H \mathbf{s}_i\|^2 = \mathbf{s}_i^H \mathbf{U}_s \mathbf{U}_s^H \mathbf{s}_i \quad (i = 1, \cdots, M). \tag{5}$$

In [1] we presented a thorough performance analysis for the above technique in terms of the constraints on false alarm rate (P_f) and miss rate (P_m) in active user identification.

II. Multiple Antenna Schemes

The goal of this paper is to investigate how to improve the performance of the active user identifier using multiple receiver antennas. We assume on the transmitter side, each node transmits using a single antenna; and each signal goes through n_R independent Rician channels to arrive at n_R receiving antennas. Thus each receiver node has n_R received vectors

$$\mathbf{r}_i = \mathbf{SA}_i\mathbf{b} + \mathbf{n}_i \quad (i = 1, \cdots, n_R). \tag{6}$$

We stack up the received vectors from all the receiver antennas and obtain $\check{\mathbf{r}} = [\mathbf{r}_1^H, \mathbf{r}_2^H, \cdots, \mathbf{r}_{n_R}^H]^H$. The autocovariance matrix of $\check{\mathbf{r}}$ thus takes the form:

$$\check{\mathbf{R}} = E\{\check{\mathbf{r}}\check{\mathbf{r}}^H\} = \begin{bmatrix} \mathbf{R}_{1,1} & \cdots & \mathbf{R}_{1,n_R} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{n_R,1} & \cdots & \mathbf{R}_{n_R,n_R} \end{bmatrix} \in \mathbb{C}^{Nn_R \times Nn_R}, \tag{7}$$

where $\mathbf{R}_{i,i} = E\{\mathbf{r}_i\mathbf{r}_i^H\}$ represents the autocovariance matrix at the ith antenna, and $\mathbf{R}_{i,j} = E\{\mathbf{r}_i\mathbf{r}_j^H\}$ $(i \neq j)$ represents the crosscovariance matrix between the ith and jth antennas.

The rest of the section will discuss and compare different approaches for active user identification involving multiple receiver antennas.

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Scheme 1: Dimension Scaling

A straightforward scheme can be derived by making use of (7) directly. We consider the same approach as the single antenna case, but the signal dimension is increased by n_R times, where n_R is the number of receiving antennas. Performing EVD on $\check{\mathbf{R}}$:

$$\check{\mathbf{R}} = E\{\check{\mathbf{r}}\check{\mathbf{r}}^H\} = \left[\check{\mathbf{U}}_s \ \check{\mathbf{U}}_n\right] \begin{bmatrix} \check{\mathbf{\Lambda}}_s & 0 \\ 0 & \check{\mathbf{\Lambda}}_n \end{bmatrix} \begin{bmatrix} \check{\mathbf{U}}_s^H \\ \check{\mathbf{U}}_n^H \end{bmatrix}. \quad (8)$$

Lemma 1 *If $\mathbf{s}_i$ is a column in $\mathbf{S}$ in (6), then $\check{\mathbf{s}}_i = [\mathbf{s}_i^H, \mathbf{s}_i^H, \cdots, \mathbf{s}_i^H]^H \in Span(\check{\mathbf{U}}_s)$*

Proof: Since $\check{\mathbf{r}}$ can be expressed as

$$\begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_{n_R} \end{bmatrix} = \begin{bmatrix} \mathbf{S} & & \\ & \ddots & \\ & & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_{n_R} \end{bmatrix} \mathbf{b} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_{n_R} \end{bmatrix}, \quad (9)$$

or in the compact form,

$$\check{\mathbf{r}} = \check{\mathbf{S}}\check{\mathbf{A}}\mathbf{b} + \check{\mathbf{n}}, \quad (10)$$

it is easy to see that $Span(\check{\mathbf{S}}) = Span(\check{\mathbf{U}}_s)$. It also follows that any linear combination of the columns of $\check{\mathbf{S}}$ is in the column space of $\check{\mathbf{U}}_s$. Therefore, the vector $\check{\mathbf{s}}_i$, which is the sum of exactly n_R columns in $\check{\mathbf{S}}$, is in the span of $\check{\mathbf{U}}_s$. □

Utilizing the above property, we can use the output of the following subspace projection as the decision metric:

$$\check{d}_i = \|\check{\mathbf{U}}_s^H \check{\mathbf{s}}_i\|^2 \quad (i = 1, \cdots, M), \quad (11)$$

where $\check{\mathbf{U}}_s \in \mathbb{C}^{Nn_R \times Kn_R}$.

Scheme 2: Covariance Averaging

Instead of resorting to higher dimensions, which implies higher complexity, we can easily do better in lower dimensions. Realizing that the individual autocovariance matrices have the following structure,

$$\mathbf{R}_{i,i} = \mathbf{S}\mathbf{A}_i\mathbf{A}_i^H\mathbf{S}^H + \sigma^2\mathbf{I} = \mathbf{S}\mathbf{A}_i^2\mathbf{S}^H + \sigma^2\mathbf{I}, \quad (12)$$

we may define an average autocovariance matrix as

$$\bar{\mathbf{R}} = \frac{1}{n_R}\sum_{i=1}^{n_R} \mathbf{R}_{i,i} = \frac{1}{n_R}\sum_{i=1}^{n_R} \mathbf{S}\mathbf{A}_i^2\mathbf{S}^H + \sigma^2\mathbf{I}. \quad (13)$$

Essentially, the effect of averaging the autocovariance matrices is equivalent to treating the received signal vectors at each antenna as additional independent samples. In other words, with n_R receiving antennas and L samples, effectively we have $n_R \times L$ samples at a single antenna. By the law of large numbers, increasing the number of samples improves the accuracy of the estimated autocovariance matrix, and hence the accuracy of the estimated signal subspace. We perform EVD on the average autocovariance matrix and obtain

$$\bar{\mathbf{R}} = \left[\bar{\mathbf{U}}_s \ \bar{\mathbf{U}}_n\right] \begin{bmatrix} \bar{\mathbf{\Lambda}}_s & 0 \\ 0 & \bar{\mathbf{\Lambda}}_n \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}}_s^H \\ \bar{\mathbf{U}}_n^H \end{bmatrix}, \quad (14)$$

and the decision metric becomes

$$\bar{d}_i = \|\bar{\mathbf{U}}_s^H \mathbf{s}_i\|^2 \quad (i = 1, \cdots, M), \quad (15)$$

where $\bar{\mathbf{U}}_s \in \mathbb{C}^{N \times K}$.

Scheme 3: Subspace Averaging

The effectiveness of an active user identification algorithm is measured by its ability to separate the identifier output for the inactive users (d_{in}) and active users (d_{ac}). It was shown in [1] that for a "random" spreading code scheme, the distribution of d_{in} is independent of the accuracy of the estimated signal subspace, as its pdf takes on a fixed analytical form which is only a function of N and K. The distribution of d_{ac}, on the other hand, is determined by the accuracy of the estimated signal subspace, which is in turn influenced by many factors besides N and K, such as SNR, sample size, and channel model. The more accurate the estimated signal subspace is, the closer the distribution of d_{ac} is to unity. If the signal subspace is estimated perfectly, d_{ac} only takes on the value 1.

In Scheme 2, we demonstrated that by averaging autocovariance matrices from different antennas, we can improve the signal subspace estimate. The question is, if we have already obtained individual signal subspace estimates from the individual autocovariance matrices, can we combine these estimates and obtain a better estimate for the signal subspace?

The answer to this question is the nontrivial concept of "average subspace" developed by Krzanowski [3].

Lemma 2 *Let $\mathbf{b}$ be an arbitrary unit vector in an N-dimensional space. Let $\mathbf{M} \in \mathbb{C}^{N\times K}$ be an orthonormal basis of a K-dimensional subspace of the original N-dimensional space. The angle $\delta \in [0, \pi]$ between the vector $\mathbf{b}$ and the subspace $\mathbf{M}$ satisfies $\cos^2\delta = <\mathbf{b}, \mathbf{M}\mathbf{M}^H\mathbf{b}>$.*

Proof: It is easy to see that the projection of $\mathbf{b}$ onto the subspace $\mathbf{M}$ is $\mathbf{M}\mathbf{M}^H\mathbf{b}$ with a magnitude $\| \mathbf{M}\mathbf{M}^H\mathbf{b} \| = \cos\delta$. Thus $< \mathbf{b}, \mathbf{M}\mathbf{M}^H\mathbf{b} > = \| \mathbf{M}\mathbf{M}^H\mathbf{b} \| \cos\delta = \cos^2\delta$. □

Theorem 1 *Let $\{\mathbf{L}_t\}_{t=1}^g$ be g orthonormal bases of K-dimensional subspaces. The vector $\mathbf{b}$ that maximizes $V = \sum_{t=1}^g \cos^2\delta_t$, where $\cos^2\delta_t = \mathbf{b}^H\mathbf{L}_t\mathbf{L}_t^H\mathbf{b}$, is given by the eigenvector $\mathbf{b}_1$ that correspond to the largest eigenvalue μ_1 of $\mathbf{H} = \sum_{t=1}^g \mathbf{L}_t\mathbf{L}_t^H$.*

Proof: From Lemma 2, for any unit vector $\mathbf{b}$, we have $V = \sum_{t=1}^g \cos^2\delta_t = \mathbf{b}^H \sum_{t=1}^g (\mathbf{L}_t\mathbf{L}_t^H)\mathbf{b} = \mathbf{b}^H\mathbf{H}\mathbf{b}$. The rest of the proof then follows from the fact that $sup_{\|\mathbf{x}\|=1}\mathbf{x}^H\mathbf{C}\mathbf{x}$ is given by the largest eigenvalue λ_1 of $\mathbf{C}$ and is attained when $\mathbf{x}$ is the eigenvector corresponding to λ_1. □

Similarly, we may argue that $\mathbf{b}_2$, the eigenvector corresponding to the second largest eigenvalue μ_2, gives the direction, orthogonal to $\mathbf{b}_1$, in which V takes the next largest value. With some imagination, we may relate the subspace spanned by the first K eigenvectors of $\mathbf{H}$ to the "average" subspace of the subspaces in $\{\mathbf{L}_t\}_{t=1}^g$, i.e. the subspace that most closely resembles the g subspaces.

Our ability to combine different subspace estimates motivates us to find more of such subspace estimates, since the accuracy of the average increases with the averaging population. Additional subspace estimates may be obtained from the crosscovariance matrices that have the following form:

$$\mathbf{R}_{i,j} = E\{\mathbf{r}_i\mathbf{r}_j^H\} = \mathbf{S}\mathbf{A}_i\mathbf{A}_j^H\mathbf{S}^H, \quad (16)$$

where $\mathbf{A}_i$ and $\mathbf{A}_j$ represent the channel coefficients from the K active users to the ith and jth receiving antenna, respectively. Performing singular value decomposition (SVD) on $\mathbf{R}_{i,j}$ we have

$$\mathbf{R}_{i,j} = \mathbf{W}\mathbf{\Sigma}\mathbf{V}^H. \tag{17}$$

It can be shown that the first K columns of $\mathbf{W}$ and $\mathbf{V}$ (which shall be denoted as $\mathbf{W}_s$ and $\mathbf{V}_s$) are estimates of the signal subspace. In the ideal case, $Span(\mathbf{W}_s) = Span(\mathbf{V}_s) = Span(\mathbf{S})$. We may extract all subspace estimates from the individual crosscovariance and autocovariance matrices in (7) and obtain n_R^2 separate subspace estimates.

Hence, we arrive at an improved signal subspace estimate $\tilde{\mathbf{U}}_s$ that consists of the first K eigenvectors corresponding to the K largest eigenvalues of $\mathbf{H} = \sum_{t=1}^{n_R^2} \mathbf{L}_t \mathbf{L}_t^H$, where $\{\mathbf{L}_t\}_{t=1}^{n_R^2}$ are derived from the autocovariance and crosscovariance matrices.

Consequently, the decision metric is

$$\tilde{d}_i = \|\tilde{\mathbf{U}}_s^H \mathbf{s}_i\|^2 \quad (i = 1, \cdots, M), \tag{18}$$

where $\tilde{\mathbf{U}}_s \in \mathbb{C}^{N \times K}$.

Scheme 4: Weighted Subspace Averaging

The shortcoming of Scheme 3 is its excessively high complexity, because every time a subspace estimate is needed, an SVD or EVD has to be performed. A simplified subspace averaging technique is thus devised.

Let $\{\mathbf{R}_{i,j}\}_{1 \le i,j \le n_R}$ be the set containing the autocovariance and crosscovariance matrices in (7), and define $\mathbf{F} = \sum_{1 \le i,j \le n_R} \mathbf{R}_{i,j}\mathbf{R}_{i,j}^H$. The alternative estimate of the signal subspace $\hat{\mathbf{U}}_s$ consists of the first K eigenvectors of $\mathbf{F}$. The reason for calling it a "weighted subspace averaging" technique is demonstrated as follows. Note that

$$\begin{aligned}\mathbf{R}_{i,j}\mathbf{R}_{i,j}^H &= \mathbf{U}_{i,j}\mathbf{\Sigma}_{i,j}\mathbf{V}_{i,j}^H\mathbf{V}_{i,j}\mathbf{\Sigma}_{i,j}^H\mathbf{U}_{i,j}^H \\ &= \mathbf{U}_{i,j}\mathbf{\Sigma}_{i,j}\mathbf{\Sigma}_{i,j}^H\mathbf{U}_{i,j}^H \\ &= [\mathbf{u}_1, \cdots, \mathbf{u}_N] \begin{bmatrix} |\sigma_1|^2 & & 0 \\ & \ddots & \\ 0 & & |\sigma_N|^2 \end{bmatrix} [\mathbf{u}_1, \cdots, \mathbf{u}_N]^H,\end{aligned} \tag{19}$$

where the subscripts i, j are understood and omitted in the last line. We then have, for an arbitrary unit vector $\mathbf{b} \in \mathbb{C}^{N \times 1}$,

$$\mathbf{b}^H \mathbf{R}_{i,j}\mathbf{R}_{i,j}^H \mathbf{b} = \sum_{n=1}^{N} |\sigma_n|^2 < \mathbf{b}, \mathbf{u}_n >, \tag{20}$$

which is a weighted projection of the vector $\mathbf{b}$ onto the space spanned by $\mathbf{R}_{i,j}$, where the weights correspond to the energy in the projected direction. Similar to the proof for Scheme 3, $\hat{\mathbf{U}}_s$ contains the first K vector directions that maximize the sum of these weighted projections. The decision metric is therefore,

$$\hat{d}_i = \|\hat{\mathbf{U}}_s^H \mathbf{s}_i\|^2 \quad (i = 1, \cdots, M). \tag{21}$$

Although this scheme is based on Scheme 3, one may also be interested in finding its resemblance to Scheme 2 and the difference that makes it better. We will show

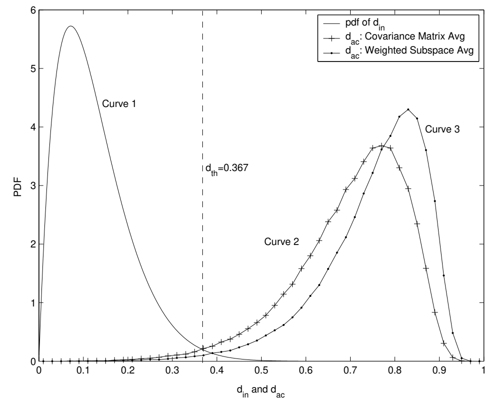

Fig. 1: PDF of d_{in} and d_{ac}.

just that by using a two antenna example. With Scheme 2, we estimate the signal subspace by extracting the first K eigenvectors of the sum of autocovariance matrices at each antenna, i.e. $\mathbf{R}_{1,1} + \mathbf{R}_{2,2}$. This is equivalent to extracting the first K eigenvectors of $(\mathbf{R}_{1,1}+\mathbf{R}_{2,2})^2$, since this product does not change the eigenvectors and their order. And we have

$$(\mathbf{R}_{1,1}+\mathbf{R}_{2,2})^2 = \mathbf{R}_{1,1}\mathbf{R}_{1,1}^H + \mathbf{R}_{2,2}\mathbf{R}_{2,2}^H + \mathbf{R}_{1,1}\mathbf{R}_{2,2}^H + \mathbf{R}_{2,2}\mathbf{R}_{1,1}^H. \tag{22}$$

Notice that the similarity of the above expression to the $\mathbf{F}$ matrix we used in Scheme 4,

$$\mathbf{F} = \mathbf{R}_{1,1}\mathbf{R}_{1,1}^H + \mathbf{R}_{2,2}\mathbf{R}_{2,2}^H + \mathbf{R}_{1,2}\mathbf{R}_{1,2}^H + \mathbf{R}_{2,1}\mathbf{R}_{2,1}^H. \tag{23}$$

The only difference between the two is the replacement of the autocovariance matrices with crosscovariance matrices in the last two terms. The performance is enhanced due to the more effective use of diversity information.

As we are no longer required to perform SVD on every $\mathbf{R}_{i,j}$, this scheme is much more efficient than Scheme 3. Yet both of them yield very similar performance.

III. Simulation Examples

In this section, we will compare Scheme 2 and Scheme 4 through simulations to demonstrate the power of the subspace averaging technique. We chose these two schemes to compare because they are low in complexity and are the most representative of the different approaches. In contrast, Scheme 1 has poor performance and high complexity, and Scheme 3 is too computationally expensive.

Firstly, we consider a system of spreading gain $N = 32$ and $K = 4$ concurrent users whose spreading codes are generated randomly. Each packet slot has a length of $L = 100$ symbols. The signals are received by $n_R = 4$ receiver antennas through independent Rician channels with the same Rice factor $\kappa = 6$dB. Fig. 1 shows three curves. Curve 1 is the theoretical pdf of the identifier output for inactive users, d_{in}. Its shape is only a function of K and N. Since an extensive analysis is already presented in [1], we can use this theoretical result instead of actual simulations. Curve 2 and 3 are the pdf's of the identifier output for active users using Scheme 2 and 4, respectively. The pdf's are approximated by the statistics of 100,000 independent trials. It is clear that the performance can

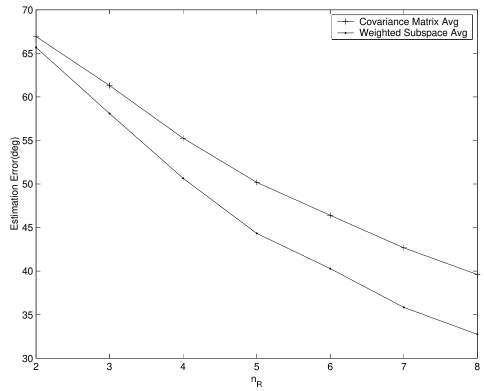

Fig. 2: Subspace estimation error as a function of n_R.

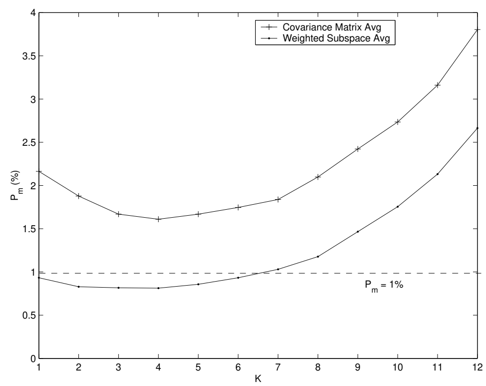

Fig. 3: P_m v.s. K with $P_f = 1\%$.

be visually measured by how far the pdf's for inactive and active users are separated.

Secondly, we will quantitatively assess the difference in performance as a function of number of antennas. We make use of the definition of the angle between two equidimensional subspaces in [4]:

Let $\mathbf{L}$ and $\mathbf{M}$ be the orthonormal bases of two subspaces of dimension K in an N-dimensional space. And let $\{\sigma_k\}_{k=1}^K$ be the K singular values of $\mathbf{L}^H\mathbf{M}$, ordered from largest to smallest in magnitude. The subspace angle between the subspaces $\mathbf{L}$ and $\mathbf{M}$ is defined as:

$$\theta(\mathbf{L},\mathbf{M}) = \cos^{-1}|\sigma_{min}|(\mathbf{L}^H\mathbf{M}) = \cos^{-1}|\sigma_K|. \tag{24}$$

The subspace estimates derived using Schemes 2 and 4 are compared with the ideal subspace which is spanned by the columns of the spreading code matrix $\mathbf{S}$. The difference, represented by a subspace angle, is used as the measure of error within each subspace estimate. The settings for this simulation is the same as the previous, except that the number of receiver antennas is varied. The estimation errors in terms of subspace angles are plotted as a function of the number of receiving antennas in Fig. 2. It is shown that Scheme 4 consistently outperforms Scheme 2 and the improvement in performance grows as the number of antennas increases.

To see how a more accurate subspace estimate translates to performance improvements in active user identification, we will examine the miss rate (P_m, the probability that an active user is categorized as inactive) as a function of K, under fixed false alarm rate (P_f, the probability that an inactive user is categorized as active). The false alarm rate can be easily fixed by choosing an appropriate set of identifier threshold values (d_{th}) using the analytical results derived in [1]. Specifically, the threshold value d_{th} and the false alarm rate P_f satisfy the relationship

$$P_f(N,K) = 1 - I\left(d_{th}(N,K); \frac{K}{2}, \frac{N-K}{2}\right), \tag{25}$$

where $I(z;a,b)$ is the incomplete Beta function defined as

$$I(z;a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^z t^{a-1}(1-t)^{b-1}\,dt \tag{26}$$

We will set the threshold values $\{d_{th}(N,K)\}_{K<N}$ such that the false alarm rate is 1%. The dashed line in Fig. 1 shows $d_{th}(32,4) = 0.367$ for $P_f = 1\%$. With a fixed $\{d_{th}(N,K)\}$, we plot the miss rate P_m as a function of K for both Scheme 2 and 4 in Fig. 3. The settings for this simulation is the same as the first simulation, except that the number of concurrent users K is varied. If we consider $P_m = 1\%$ as a satisfactory performance criterion, we see that Scheme 4 allows maximum 6 concurrent users while Scheme 2 fails to satisfy this criterion for any K, and yields an error probability of approximately $P_m = 2\%$ for $K = 6$. This shows that Scheme 4 increases active user identification capability of the multiuser receiver and, subsequently, the throughput of the ad hoc network.

IV. Conclusions

Our study presents four different schemes for active user identification using multiple antennas, where the "subspace averaging" technique (scheme 3 and 4) significantly improve the identification performance. Scheme 4, in particular, introduces almost no additional complexity compared to the simple covariance averaging technique in Scheme 2. The novel idea of "subspace averaging" not only is applicable for the particular problem of active user identification, but also has implications for other subspace-based identification problems in general.

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