Near-Optimal Training-Based Estimation of Frequency Offset and Channel Response in OFDM with Phase Noise

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Presented by Darryl Dexu Lin at ICC'06

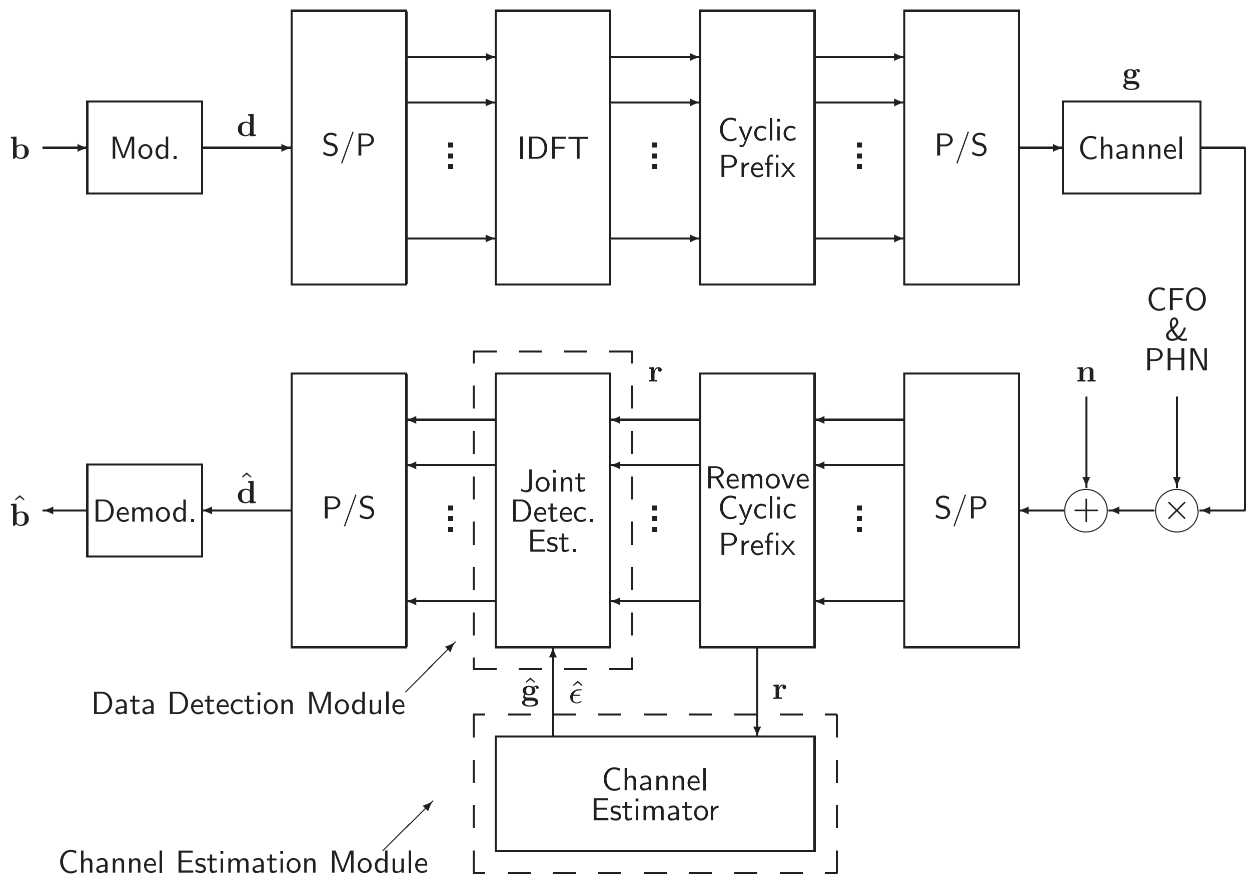
June 12, 2006

Outline

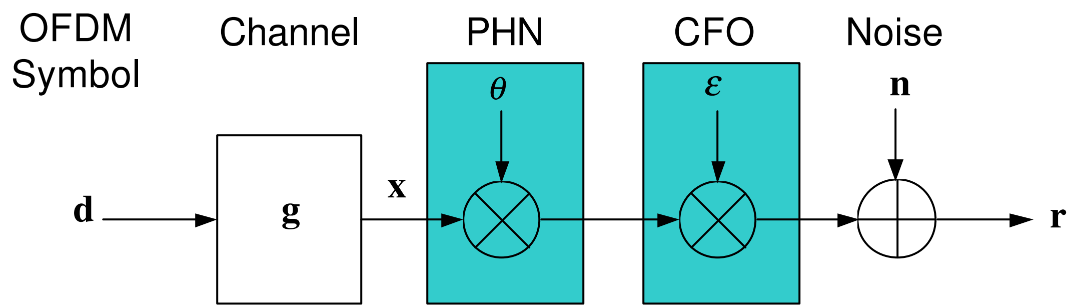
- **Problem description and signal model.**
- Prior statistics of phase noise.
- The optimal joint CFO/PHN/CIR estimator.
- The near-optimal joint CFO/PHN/CIR estimator.
- Complexity reduction using conjugate gradient method.
- Simulation results.

I. Problem Description

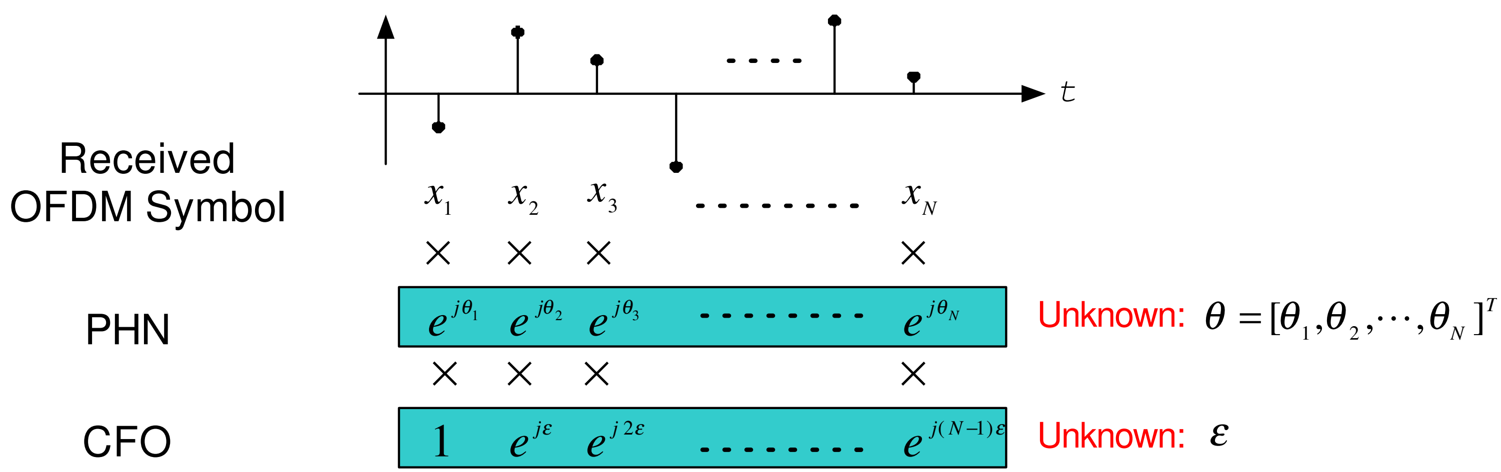
- The role of channel estimation in OFDM receiver design:



- OFDM channel with PHN (Phase Noise) and CFO (Carrier Frequency Offset):



- Distortions caused by PHN and CFO:



I. Signal Model in Matrix Form

- Complex baseband received signal in one OFDM symbol interval:

$$\mathbf{r} = \mathbf{EPGF}^H\mathbf{d} + \mathbf{n}, \tag{1}$$

  - $\mathbf{r} \in \mathbb{C}^{N\times 1}$: received OFDM symbol with cyclic prefix removed;
  - $\mathbf{E} = \text{diag}([1, e^{j2\pi\epsilon/N}, \cdots, e^{j2\pi(N-1)\epsilon/N}]^T)$: CFO matrix;
  - $\mathbf{P} = \text{diag}([e^{j\theta_0}, \cdots, e^{j\theta_{N-1}}]^T)$: PHN matrix;
  - $\mathbf{G}$: channel circular convolution matrix, formed by CIR $\mathbf{g}$;
  - $\mathbf{F} \in \mathbb{C}^{N\times N}$: DFT matrix;
  - $\mathbf{d} \in \mathbb{C}^{N\times 1}$: vector of constant-modulus training symbols;
  - $\mathbf{n} \in \mathbb{C}^{N\times 1}$: complex white Gaussian noise with variance σ^2 per dimension.

- The objective is to, based on received $\mathbf{r}$, estimate three unknowns:
  - (1)ϵ, (2)$\boldsymbol{\theta} = [\theta_0, \cdots, \theta_{N-1}]^T$, (3)$\mathbf{g} = [g_0, \cdots, g_{L-1}]^T$.

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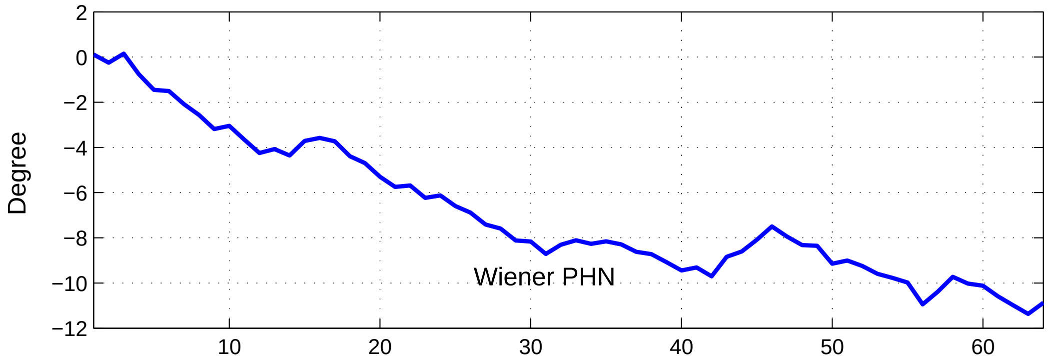
II. Prior Statistics of Phase Noise

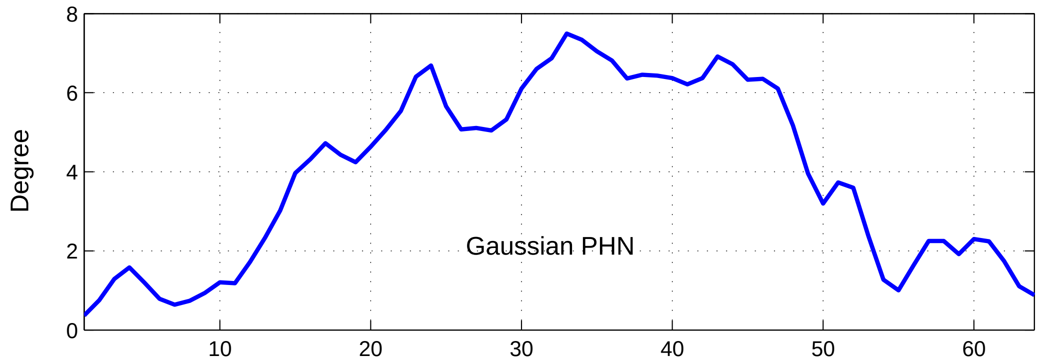
- Two different models of PHN are available:
  - For free-running oscillator at the receiver, we assume a non-stationary Gaussian process, called *Wiener PHN*.
  - For oscillator controlled by a phase-locked loop (PLL), we assume a zero-mean coloured Gaussian process, called *Gaussian PHN*.

- The prior statistics of both types of PHN can be modeled as a multivariate Gaussian distribution:

$$p(\boldsymbol{\theta}) = \mathcal{N}(\mathbf{0}, \boldsymbol{\Phi}), \tag{2}$$

where the covariance matrix $\boldsymbol{\Phi}$ can be determined from the power spectral density (PSD) of the VCO output.





$$\mathbf{\Phi} = \alpha_\phi^2 \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 1 & 2 & & N \end{bmatrix}$$

$$\mathbf{\Phi} = \begin{bmatrix} \phi_0 & \phi_1 & \cdots & \phi_{N-1} \\ \phi_1 & \phi_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \phi_1 \\ \phi_{N-1} & \cdots & \phi_1 & \phi_0 \end{bmatrix}$$

PHN Sample

Covariance Matrix

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III. The Complete Likelihood Function

- The optimal estimator requires joint estimation of three unknowns, $\epsilon, \boldsymbol{\theta}$ and $\mathbf{g}$, in (3), where $\mathbf{F} = [\mathbf{W}|\mathbf{V}]$ and $\mathbf{g}$ is the CIR.

$$\mathbf{r} = \mathbf{E}\mathbf{P}\mathbf{F}^H\mathbf{D}\mathbf{W}\mathbf{g} + \mathbf{n}. \tag{3}$$

- We first write the "complete likelihood function"

$$p(\mathbf{r}, \epsilon, \boldsymbol{\theta}, \mathbf{g}) = p(\mathbf{r}|\epsilon, \boldsymbol{\theta}, \mathbf{g})p(\epsilon)p(\boldsymbol{\theta})p(\mathbf{g}), \tag{4}$$

  which is proportional to the *a posteriori* distribution of the unknowns, $p(\epsilon, \boldsymbol{\theta}, \mathbf{g}|\mathbf{r})$.

- Since we assume no prior knowledge of ϵ and $\mathbf{g}$, $p(\epsilon)$ and $p(\mathbf{g})$ are constants and can be omitted. The prior of $\boldsymbol{\theta}$ is available, which is $p(\boldsymbol{\theta}) = \mathcal{N}(\mathbf{0}, \boldsymbol{\Phi})$ as discussed before.

III. Target of Optimization

- Taking the logarithm, the "complete negative log-likehood function" can be written as

$$\begin{aligned}\mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g}) = & -\log p(\mathbf{r}|\epsilon, \boldsymbol{\theta}, \mathbf{g}) - \log p(\boldsymbol{\theta}) \\ = & \frac{1}{2\sigma^2}(\mathbf{r} - \mathbf{EPF}^H\mathbf{DWg})^H(\mathbf{r} - \mathbf{EPF}^H\mathbf{DWg}) + \frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\Phi}^{-1}\boldsymbol{\theta}.\end{aligned} \tag{5}$$

- The objective is to find the jointly optimal estimates

$$(\hat{\epsilon}, \hat{\boldsymbol{\theta}}, \hat{\mathbf{g}}) = \arg\min_{\epsilon, \boldsymbol{\theta}, \mathbf{g}} \mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g}). \tag{6}$$

- The estimator proposed here is "optimal" in the sense of maximizing the "complete likelihood function". It can be derived in 3 optimization steps.

III. The Optimal Estimator: CIR and PHN Estimation

1. *CIR Estimation*: Solve $\partial \mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g}) / \partial \mathbf{g}^* = \mathbf{0}$, we obtain

$$\hat{\mathbf{g}} = (2\rho^2)^{-1} \mathbf{W}^H \mathbf{D}^H \mathbf{F} \mathbf{P}^H \mathbf{E}^H \mathbf{r}. \tag{7}$$

– Substituting $\mathbf{g} = \hat{\mathbf{g}}$ back into $\mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g})$ produces $\mathcal{L}(\epsilon, \boldsymbol{\theta})$.

2. *PHN Estimation*: Solve $\partial \mathcal{L}(\epsilon, \boldsymbol{\theta}) / \partial \boldsymbol{\theta} = \mathbf{0}$, we obtain

$$\hat{\boldsymbol{\theta}} = [\mathrm{Re}(\mathbf{E}\mathbf{C}\mathbf{C}^H \mathbf{E}^H) + 2\sigma^2 \rho^2 \boldsymbol{\Phi}^{-1}]^{-1} \mathrm{Im}(\mathbf{E}\mathbf{C}\mathbf{C}^H \mathbf{E}^H) \mathbf{1}, \tag{8}$$

where $\mathbf{C} = \mathbf{R}^H \mathbf{F}^H \mathbf{D} \mathbf{V}$ and $\mathbf{R} = \mathrm{diag}(\mathbf{r})$.

– Substituting $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ back into $\mathcal{L}(\epsilon, \boldsymbol{\theta})$ produces $\mathcal{L}(\epsilon)$

III. The Optimal Estimator: CFO Estimation

3. *CFO Estimation*: To minimize $\mathcal{L}(\epsilon)$, we require searching over the range $-0.5 < \epsilon < 0.5$:

$$\begin{aligned}\hat{\epsilon} = \quad & \arg\min_{\epsilon} \mathbf{1}^T \mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H\mathbf{1} - \mathbf{1}^T \mathrm{Im}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H)^T \\ & \times[\mathrm{Re}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H) + 2\sigma^2\rho^2\boldsymbol{\Phi}^{-1}]^{-1}\mathrm{Im}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H)\mathbf{1}.\end{aligned} \tag{9}$$

- Combining the 3 optimization steps leads to the complete *Joint CFO/PHN/CIR Estimation* (JCPCE) algorithm (Proc. WCNC'06):

Step 1: $\hat{\epsilon} = \arg\min_{\epsilon} \mathbf{1}^T \mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H\mathbf{1} - \mathbf{1}^T \mathrm{Im}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H)^T \times[\mathrm{Re}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H) + 2\sigma^2\rho^2\boldsymbol{\Phi}^{-1}]^{-1}\mathrm{Im}(\mathbf{E}\mathbf{C}\mathbf{C}^H\mathbf{E}^H)\mathbf{1}$;

$\hat{\mathbf{E}} = \mathrm{diag}([1, e^{j2\pi\hat{\epsilon}/N}, \cdots, e^{j2\pi(N-1)\hat{\epsilon}/N}]^T)$;

Step 2: $\hat{\boldsymbol{\theta}} = [\mathrm{Re}(\hat{\mathbf{E}}\mathbf{C}\mathbf{C}^H\hat{\mathbf{E}}^H) + 2\sigma^2\rho^2\boldsymbol{\Phi}^{-1}]^{-1}\mathrm{Im}(\hat{\mathbf{E}}\mathbf{C}\mathbf{C}^H\hat{\mathbf{E}}^H)\mathbf{1}$;

$\hat{\mathbf{P}} = \mathrm{diag}([e^{j\hat{\theta}_0}, \cdots, e^{j\hat{\theta}_{N-1}}]^T)$;

Step 3: $\hat{\mathbf{g}} = (2\rho^2)^{-1}\mathbf{W}^H\mathbf{D}^H\mathbf{F}\hat{\mathbf{P}}^H\hat{\mathbf{E}}^H\mathbf{r}$.

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IV. Preliminary: Moose's CFO Estimator

- Assuming only CFO and no PHN, what is the simple way to estimate the CFO without searching over all possible values of ϵ?

- The Moose's method proposes to transmit a training sequence with two identical halves in the time domain, of length $N/2$. Thus, the received vector $\mathbf{r} = [\mathbf{r}_1^T, \mathbf{r}_2^T]^T$, where

$$\mathbf{r}_1 = \mathbf{x} + \mathbf{n}_1; \quad \mathbf{r}_2 = e^{j\pi\epsilon}\mathbf{x} + \mathbf{n}_2 \tag{10}$$

  in which $\mathbf{x} = \mathbf{EGF}^H\mathbf{d} \in \mathbb{C}^{\frac{N}{2}\times 1}$.

- The maximum-likelihood estimate of ϵ is

$$\hat{\epsilon} = \arg\max_{\epsilon} p(\mathbf{r}_1, \mathbf{r}_2|\epsilon) = \arg\max_{\epsilon} p(\mathbf{r}_2|\epsilon, \mathbf{r}_1)p(\mathbf{r}_1|\epsilon). \tag{11}$$

IV. Preliminary: Moose's CFO Estimator (Cont.)

- If we assume the independence between $\mathbf{r}_1$ and ϵ, i.e. $p(\mathbf{r}_1|\epsilon) = p(\mathbf{r}_1)$, we simply have $\hat{\epsilon} = \arg\max_\epsilon p(\mathbf{r}_2|\epsilon, \mathbf{r}_1)$.

- Notice that $\mathbf{r}_2 = e^{j\pi\epsilon}\mathbf{r}_1 - e^{j\pi\epsilon}\mathbf{n}_1 + \mathbf{n}_2 = e^{j\pi\epsilon}\mathbf{r}_1 + \mathbf{z}$.

- Since $\mathbf{n}_1, \mathbf{n}_2 \sim \mathcal{CN}(\mathbf{0}, 2\sigma^2\mathbf{I}) \Rightarrow \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, 4\sigma^2\mathbf{I})$, we have

$$-\log p(\mathbf{r}_2|\epsilon, \mathbf{r}_1) = \frac{1}{4\sigma^2}(\mathbf{r}_2 - e^{j\pi\epsilon}\mathbf{r}_1)^H(\mathbf{r}_2 - e^{j\pi\epsilon}\mathbf{r}_1). \tag{12}$$

- Thus $\hat{\epsilon} = \arg\max_\epsilon p(\mathbf{r}_2|\epsilon, \mathbf{r}_1)$ leads to the near-ML estimate of ϵ

$$\hat{\epsilon} = \frac{1}{\pi}\measuredangle\mathbf{r}_1^H\mathbf{r}_2. \tag{13}$$

IV. Near Optimal Estimator: The New Approach

- How do we make use of the convenience offered by repeating training sequence to efficiently estimate CIR and PHN together with CFO?

- We propose a two stage process:

  1. Estimate $\hat{\epsilon} = \arg\max_{\epsilon} p(\mathbf{r}_1, \mathbf{r}_2|\epsilon)$, motivated by Moose's method.
  2. Estimate $(\hat{\boldsymbol{\theta}}, \hat{\mathbf{g}}) = \arg\min_{\boldsymbol{\theta},\mathbf{g}} \mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta}, \mathbf{g})$, similar to the optimal algorithm proposed earlier, except with $\hat{\epsilon}$ already estimated, where

$$\begin{aligned} & \mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g}) \\ = \; & -\log p(\mathbf{r}|\epsilon, \boldsymbol{\theta}, \mathbf{g}) - \log p(\boldsymbol{\theta}) \\ = \; & \frac{1}{2\sigma^2}(\mathbf{r} - \mathbf{EPF}^H\mathbf{DWg})^H(\mathbf{r} - \mathbf{EPF}^H\mathbf{DWg}) + \frac{1}{2}\boldsymbol{\theta}^T\boldsymbol{\Phi}^{-1}\boldsymbol{\theta}. \end{aligned} \tag{14}$$

IV. Near Optimal Estimator: CFO Estimation

- *CFO Estimation*: In the presence of PHN, CFO estimation using the Moose's method needs to be modified, since

$$\mathbf{r}_1 = \mathbf{P}_1\mathbf{x} + \mathbf{n}_1; \quad \mathbf{r}_2 = e^{j\pi\epsilon}\mathbf{P}_2\mathbf{x} + \mathbf{n}_2, \tag{15}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ contain consequtive PHN sequences $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

- The optimal estimate $\hat{\epsilon}$ is then

$$\arg\max_{\epsilon} p(\mathbf{r}_1, \mathbf{r}_2|\epsilon) = \arg\max_{\epsilon} \int_{\boldsymbol{\theta}_2}\int_{\boldsymbol{\theta}_1} p(\mathbf{r}_1, \mathbf{r}_2, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2|\epsilon) d\boldsymbol{\theta}_1 d\boldsymbol{\theta}_2 \tag{16}$$

- Writing $\mathbf{R}_1 = \text{diag}(\mathbf{r}_1)$ and $\boldsymbol{\Phi}_\triangle$ as the covariance matrix of $\boldsymbol{\theta}_2 - \boldsymbol{\theta}_1$, it can be shown that this is equivalent to maximizing

$$p(\mathbf{r}_2|\mathbf{r}_1, \epsilon) = \mathcal{CN}(e^{j\pi\epsilon}\mathbf{r}_1, \mathbf{R}_1\boldsymbol{\Phi}_\triangle\mathbf{R}_1^H + 4\sigma^2\mathbf{I}). \tag{17}$$

IV. Near Optimal Estimator: CIR Estimation

- Therefore,

$$\hat{\epsilon} = \frac{1}{\pi} \measuredangle\, \mathbf{r}_1^H (\mathbf{R}_1 \mathbf{\Phi}_\triangle \mathbf{R}_1^H + 4\sigma^2 \mathbf{I})^{-1} \mathbf{r}_2. \tag{18}$$

- Substituting $\hat{\epsilon}$ into $\mathcal{L}(\epsilon, \boldsymbol{\theta}, \mathbf{g})$, we get

$$\mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta}, \mathbf{g}) = \frac{1}{2\sigma^2} (\mathbf{r} - \hat{\mathbf{E}} \mathbf{P} \check{\mathbf{F}}^H \mathbf{D} \mathbf{W} \mathbf{g})^H (\mathbf{r} - \hat{\mathbf{E}} \mathbf{P} \check{\mathbf{F}}^H \mathbf{D} \mathbf{W} \mathbf{g}) + \frac{1}{2} \boldsymbol{\theta}^T \mathbf{\Phi}^{-1} \boldsymbol{\theta}. \tag{19}$$

- *CIR Estimation*: Solve $\partial \mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta}, \mathbf{g}) / \partial \mathbf{g}^* = \mathbf{0}$, we obtain

$$\hat{\mathbf{g}} = (4\rho^2)^{-1} \mathbf{W}^H \mathbf{D}^H \check{\mathbf{F}} \mathbf{P}^H \hat{\mathbf{E}}^H \mathbf{r}. \tag{20}$$

  – Substituting $\mathbf{g} = \hat{\mathbf{g}}$ back into $\mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta}, \mathbf{g})$ produces $\mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta})$.

IV. Near Optimal Estimator: PHN Estimation

- *PHN Estimation*: Solve $\partial\mathcal{L}(\hat{\epsilon}, \boldsymbol{\theta})/\partial\boldsymbol{\theta} = \mathbf{0}$, we obtain

$$\hat{\boldsymbol{\theta}} = [\mathrm{Re}(\hat{\mathbf{E}}\mathbf{A}\hat{\mathbf{E}}^H) + 4\sigma^2\rho^2\boldsymbol{\Phi}^{-1}]^{-1}\mathrm{Im}(\hat{\mathbf{E}}\mathbf{A}\hat{\mathbf{E}}^H)\mathbf{1}. \tag{21}$$

- Combining the above steps leads to the *Modified Joint CFO/PHN/CIR Estimation (Modified JCPCE)* algorithm with closed-form CFO estimation:

$$\begin{aligned}
\text{Step 1:} \quad & \hat{\epsilon} = \tfrac{1}{\pi}\measuredangle\ \mathbf{r}_1^H(\mathbf{R}_1\boldsymbol{\Phi}_\Delta\mathbf{R}_1^H + 4\sigma^2\mathbf{I})^{-1}\mathbf{r}_2; \\
& \hat{\mathbf{E}} = \mathrm{diag}([1, e^{j2\pi\hat{\epsilon}/N}, \cdots, e^{j2\pi(N-1)\hat{\epsilon}/N}]^T); \\
\text{Step 2:} \quad & \hat{\boldsymbol{\theta}} = [\mathrm{Re}(\hat{\mathbf{E}}\mathbf{A}\hat{\mathbf{E}}^H) + 4\sigma^2\rho^2\boldsymbol{\Phi}^{-1}]^{-1}\mathrm{Im}(\hat{\mathbf{E}}\mathbf{A}\hat{\mathbf{E}}^H)\mathbf{1}; \\
& \hat{\mathbf{P}} = \mathrm{diag}([e^{j\hat{\theta}_0}, \cdots, e^{j\hat{\theta}_{N-1}}]^T); \\
\text{Step 3:} \quad & \hat{\mathbf{g}} = (4\rho^2)^{-1}\mathbf{W}^H\mathbf{D}^H\check{\mathbf{F}}\hat{\mathbf{P}}^H\hat{\mathbf{E}}^H\mathbf{r}.
\end{aligned}$$

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V. Complexity Reduction: Special Structure of Ψ

- Letting $\mathbf{\Psi} = \frac{1}{4\sigma^2\rho^2}\mathbf{\Phi}$, the dominant complexity of the previous algorithm is associated with the evaluation of $[\text{Re}(\mathbf{EAE}^H) + \mathbf{\Psi}^{-1}]^{-1}$, which in general has complexity $\mathcal{O}(N^3)$.

- Fortunately, complexity reduction is available by noticing the following:
  - For *Wiener PHN*, $\mathbf{\Psi}^{-1}$ is a tridiagonal matrix:

$$\mathbf{\Psi}^{-1} = \frac{4\sigma^2\rho^2}{\alpha_\phi^2} \begin{bmatrix} 2 & -1 & & & \mathbf{0} \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ \mathbf{0} & & & -1 & 1 \end{bmatrix}. \tag{22}$$

  - For *Gaussian PHN*, $\mathbf{\Psi}$ is a Toeplitz matrix, but can be closely approximated as a circulant matrix $\tilde{\mathbf{\Psi}} = \mathbf{F}\mathbf{\Lambda}\mathbf{F}^H$. Therefore, its inverse is easy to evaluate, $\tilde{\mathbf{\Psi}}^{-1} = \mathbf{F}\mathbf{\Lambda}^{-1}\mathbf{F}^H$

V. Complexity Reduction: Conjugate Gradient Method

- Letting $\mathbf{q} = \mathrm{Im}(\hat{\mathbf{E}}\mathbf{A}\hat{\mathbf{E}}^H)\mathbf{1}$, the evaluation of $[\mathrm{Re}(\mathbf{E}\mathbf{A}\mathbf{E}^H) + \mathbf{\Psi}^{-1}]^{-1}\mathbf{q}$ can be accomplished by the conjugate gradient method as follows:

Initialization:	
	$\boldsymbol{\theta}_0 = \mathbf{0}$
	$\boldsymbol{\gamma}_0 = [\mathrm{Re}(\mathbf{E}\mathbf{A}\mathbf{E}^H) + \tilde{\mathbf{\Psi}}^{-1}]\boldsymbol{\theta}_0 - \mathbf{q} = -\mathbf{q}$
	$\boldsymbol{\nu}_0 = -\boldsymbol{\gamma}_0 = \mathbf{q}$
For	$k = 0 : i-1$
	$\alpha_k = \boldsymbol{\gamma}_k^H\boldsymbol{\gamma}_k / (\boldsymbol{\nu}_k^H[\mathrm{Re}(\mathbf{E}\mathbf{A}\mathbf{E}^H) + \tilde{\mathbf{\Psi}}^{-1}]\boldsymbol{\nu}_k)$
	$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \alpha_k\boldsymbol{\nu}_k$
	$\boldsymbol{\gamma}_{k+1} = \boldsymbol{\gamma}_k + \alpha_k[\mathrm{Re}(\mathbf{E}\mathbf{A}\mathbf{E}^H) + \tilde{\mathbf{\Psi}}^{-1}]\boldsymbol{\nu}_k$
	$\beta_{k+1} = \frac{\boldsymbol{\gamma}_{k+1}^H\boldsymbol{\gamma}_{k+1}}{\boldsymbol{\gamma}_k^H\boldsymbol{\gamma}_k}$
	$\boldsymbol{\nu}_{k+1} = -\boldsymbol{\gamma}_{k+1} + \beta_{k+1}\boldsymbol{\nu}_k$
End	

V. Complexity Reduction: Overall Complexity

- By utilizing the special structure of $\mathbf{\Psi}$, we can reduce the complexity from $\mathcal{O}(N^3)$ to $\mathcal{O}(N^2 \log N)$.

- Using the Conjugate Gradient algorithm, we have control over the number of iteration i. Exact matrix inversion corresponds to $i = N$.

- In practice, $i \ll N$. In this case, the complexity becomes $\mathcal{O}(i \times N \log N)$.

- Simulations demonstrate that even for $i = 5$, no significant performance degradation results.

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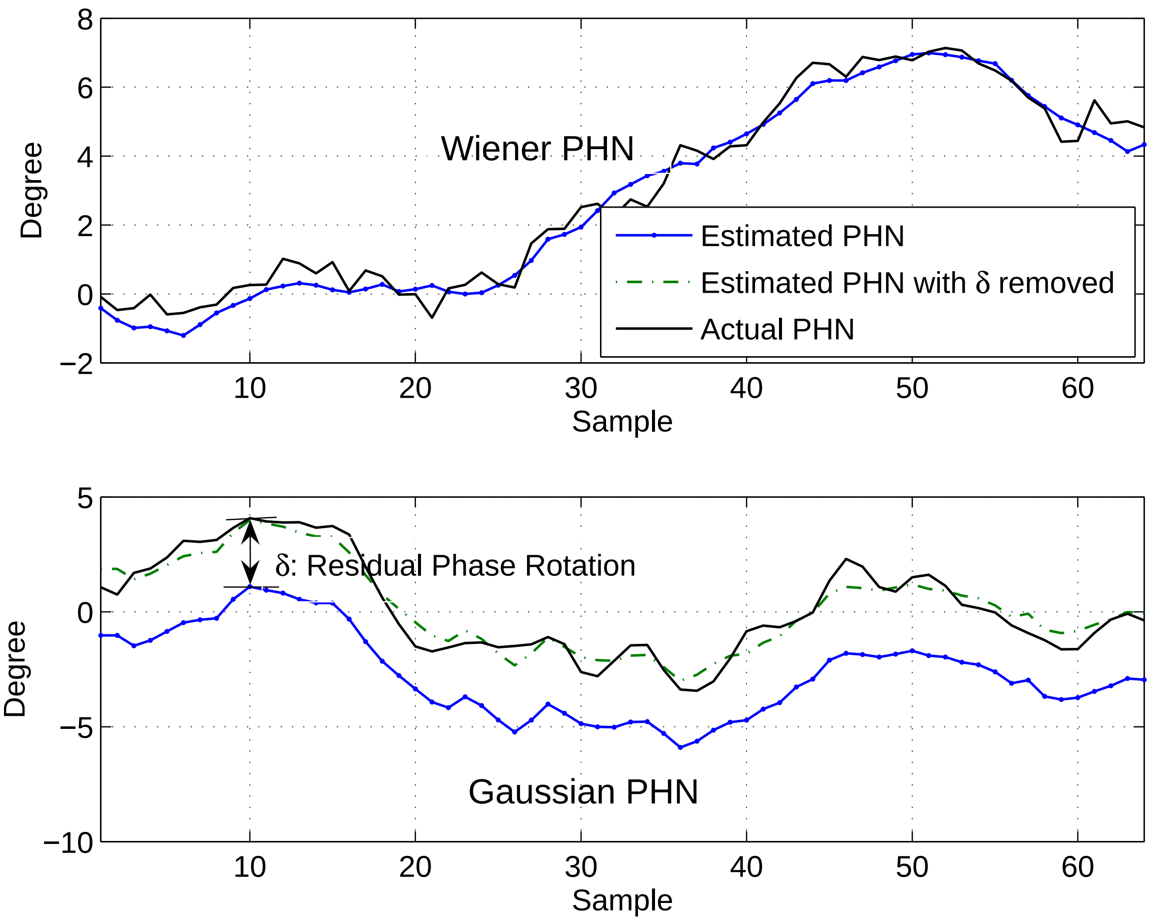
VI. Residual Phase Rotation

- *Unresolvable residual common phase rotation*: A residual phase rotation δ cannot be estimated for Gaussian PHN. $\delta \sim \mathcal{N}(0, \mathbf{1}^T \mathbf{\Phi} \mathbf{1}/N^2)$.



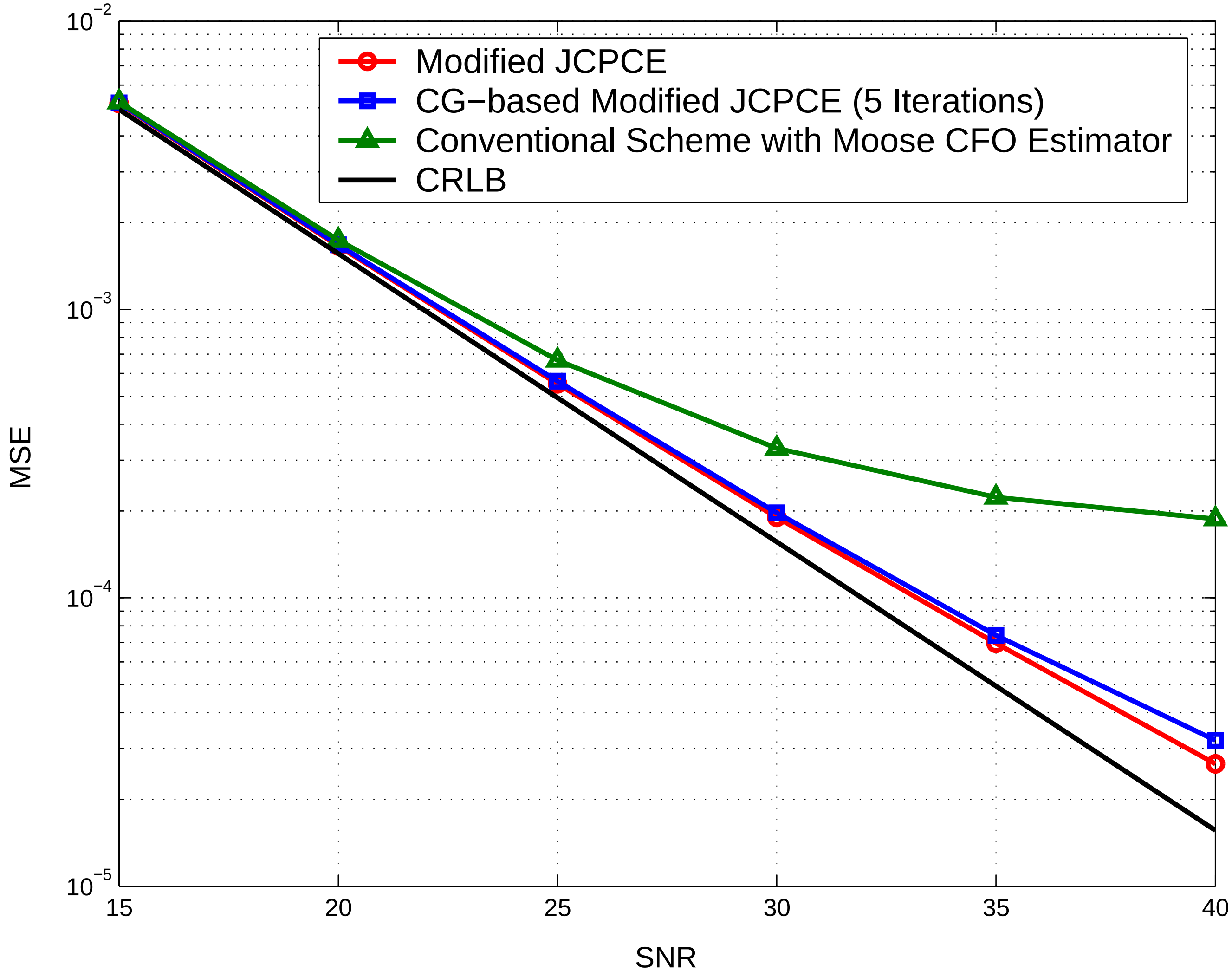
VI. Simulation Settings

- The effect of residual phase rotation is that the channel estimate $\hat{\mathbf{g}}$ is off by a small unknown phase δ, which does not introduce ICI.

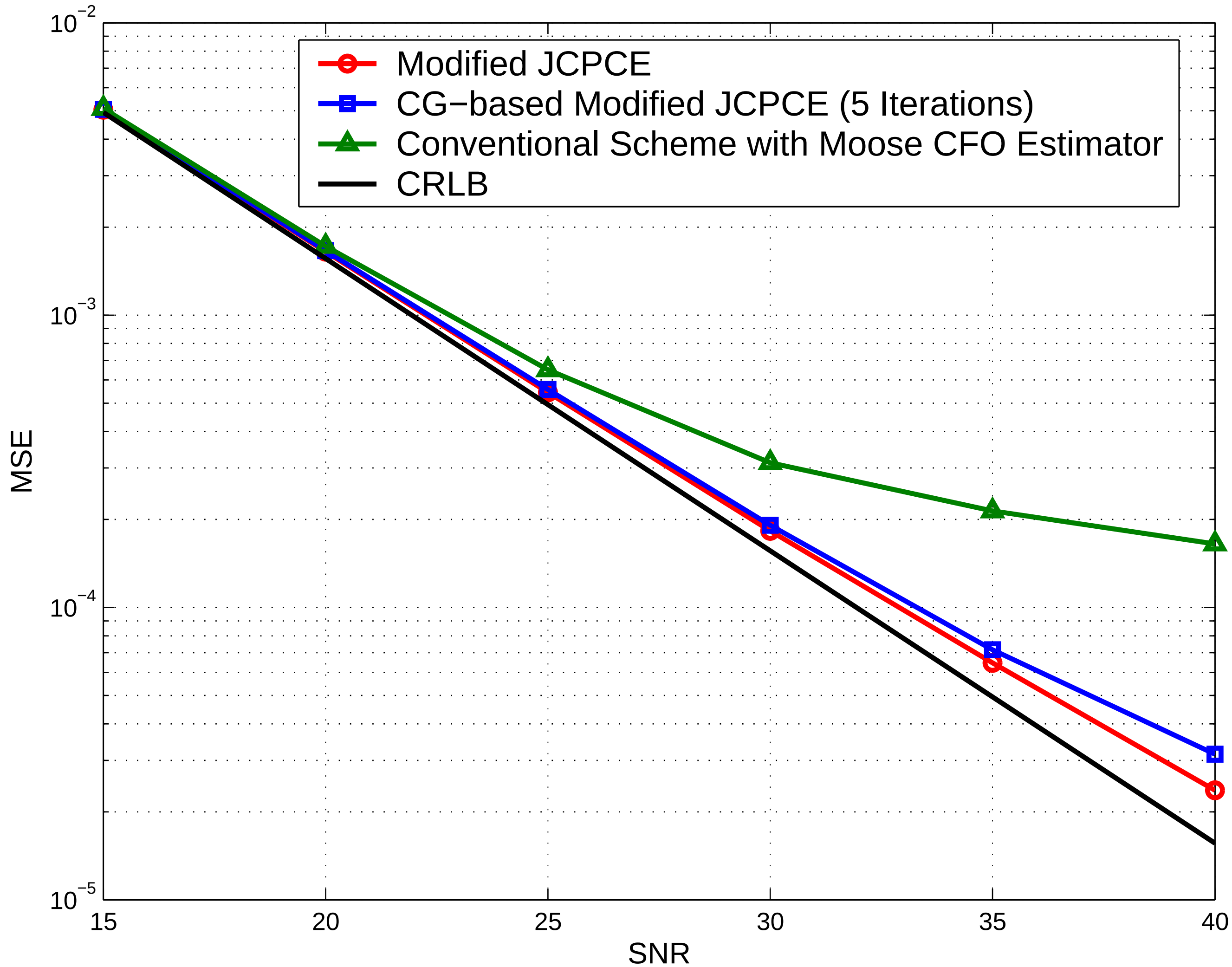
- We assume that δ can be perfectly corrected to facilitate easy assessment of channel estimation mean-squared error (MSE).

- The following system parameters are used in simulations:
  - A Rayleigh multipath fading channel with a delay of $L = 10$ taps;
  - An OFDM training symbol size of $N = 64$ subcarriers with each subcarrier modulated in QPSK;
  - Baseband sampling rate $f_s = 20$ MHz;
  - The Wiener PHN is generated as a random-walk process with incremental PHN of $\alpha_\phi = 1$ deg.
  - The Gaussian PHN has a standard deviation of $\theta_{rms} = 4$ deg. It is generated as i.i.d. Gaussian samples passed through a single pole Butterworth filter of 3dB bandwidth $\Omega_o = 100$ KHz.

- Performance of modified JCPCE in Wiener PHN:



- Performance of modified JCPCE in Gaussian PHN:



Summary

- *Challenge*: How do we optimally estimate the CIR and CFO of an OFDM channel in the presence of unknown PHN?
- *Solution 1*: In a previous work, we derived a *Joint CFO/PHN/CIR Estimation* (JCPCE) algorithm that optimizes the "complete likelihood function".
- *Solution 2*: Here we proposed a suboptimal algorithm, called *Modified JCPCE*, to obtain closed-form estimate of CFO, lowering the computational complexity.
- We may reduce the complexity even further using the Conjugate Gradient method.

Thank You!