# A Subspace-Based Active User Identification Scheme for CDMA Ad Hoc Networks

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Abstract — We propose a novel spreading code scheme, Transmitter-Receiver-Based Code, for wireless ad hoc networks. A subspace-based active user identification algorithm based on the proposed spreading code design is introduced. The performance of the active user identifier is also studied by investigating the false alarm rate  $P_f$  and miss rate  $P_m$  with respect to the identifier threshold value  $d_{th}$ .

## I. INTRODUCTION

In a non-centralized CDMA ad hoc network where transmissions between any pairs of nodes are allowed, designing a good spreading code scheme is challenging. Simple code schemes such as receiver-based code or transmitter-based code are either susceptible to packet collisions or require high complexity receivers [1]. We propose a novel spreading code design, the Transmitter-Receiver-Based Code (TRBC), that is collision free and also enables each receiver to identify and decode the packets transmitted to itself, while suppressing the interfering packets blindly.

For the TRBC code, each node in the network is assigned two unique binary  $(\pm 1)$ 's spreading codes  $\mathbf{w}_m$  and  $\mathbf{p}_m$   $(m = 1, \dots, M)$  of processing gain N. The transmission from node i to j uses spreading code  $\mathbf{s}_{i,j} = \mathbf{w}_i \circ \mathbf{p}_j$ , where  $\circ$  denotes the Hadamard (or element-wise) product of two matrices.

# II. ACTIVE USER IDENTIFICATION

The task of active user identification at a receiver is defined as the identification of packets arriving at that receiver that are actually destined for it. Other packets received are treated as interference. A feasible solution for active user identification is a MUSIC-based technique [2], where the candidate spreading codes are projected onto the signal subspace of the received vector and the output of the projection, compared with a preset threshold, determines whether the candidate spreading code is in use (active) or not (inactive).

We assume a slotted chip-synchronous short code system and a slow flat fading AWGN channel. The network allows random access from all users in the form of packets of length L symbols in each packet slot. Given that the packets are synchronized, at each symbol interval, the received vector at the receiver node can be expressed as

$$\mathbf{r} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n},\tag{1}$$

where  $S = [s_1, s_2, \dots, s_K]$  is the spreading code matrix consisting of the spreading sequences of the K active packets in the current packet slot, and  $\mathbf{b} = [b_1, b_2, \dots, b_K]^T$  represents

the transmitted information symbols in the current symbol interval. The channel matrix  $\mathbf{A} = diag(A_1, \dots, A_K)$  contains independent channel coefficients.

Averaging over the received symbols, the autocovariance matrix of the received vector is  $\mathbf{R} = E\{\mathbf{r}\mathbf{r}^T\}$ . By performing eigenvalue decomposition (EVD), we may extract the signal subspace ( $\mathbf{U}_s \in \mathbb{R}^{N \times K}$ ).

We shall assume the receiver knows the pool of spreading codes used by other nodes to transmit packets to itself. These candidate spreading codes are placed in the set  $\{s_i\}_{i=1}^M$ . Whether or not  $s_i$  is active can be determined by comparing the identifier output  $d_i$  to a threshold value  $d_{th}$ , where

$$d_i = \|\mathbf{U}_s^T \mathbf{s}_i\|^2 = \mathbf{s}_i^T \mathbf{U}_s \mathbf{U}_s^T \mathbf{s}_i \quad (i = 1, \dots, M).$$
 (2)

#### III. PERFORMANCE ANALYSIS

The outputs of the active user identifier  $d_i$  fall into two categories: that of active users  $(d_{ac})$  and that of inactive users  $(d_{in})$ . The statistics of  $d_{in}$  and  $d_{ac}$  determine the best value to set the identifier threshold  $d_{th}$ , given the constraints on false alarm rate  $P_f$  and miss rate  $P_m$ , where  $P_f = P(d_{in} \geq d_{th})$  and  $P_f = P(d_{ac} \leq d_{th})$ .

Through rigorous statistical analysis we arrived at a close form approximation (exact for large N) for the pdf of  $d_{in}$  as a Beta distribution

$$f_Y(y) = \frac{\Gamma(\frac{N}{2})}{\Gamma(\frac{K}{2})\Gamma(\frac{N-K}{2})} y^{\frac{K}{2}-1} (1-y)^{\frac{N-K}{2}-1} . \tag{3}$$

The false alarm rate  $P_f$  is thus determined by the identifier threshold values  $(d_{th})$  through

$$P_f(N,K) = 1 - F_Y(d_{th}) = 1 - I(d_{th}(N,K); \frac{K}{2}, \frac{N-K}{2}),$$
 (4)

where I(z; a, b) is the incomplete Beta function.

Similar analysis applies also to the miss rate  $P_m$ , which is determined by the distribution of  $d_{ac}$ ,  $f_Z(z)$ , through  $P_m = F_Z(d_{th})$ . However, the distribution of  $d_{ac}$  is influenced by the accuracy of the estimated signal subspace, which in turn associates with many factors beside N and K, such as SNR, sample size, and channel model. Thus the analysis of  $P_m$  can only be studied on a case-by-case basis.

### REFERENCES

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<sup>&</sup>lt;sup>1</sup>This research was sponsored in part by a research grant from the Bell Canada University Laboratories at the University of Toronto.