ECE 467 Midterm 2

University of Toronto

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1. (4 points) Consider the following grammar.

```
S -> A
S -> B
A -> C A
A -> a
B -> D B
B -> b
C -> d
D -> d
```

Nonterminal	First
S	a, b, d
A	a, d
B	b, d
C	d
D	d

(a) (2 points) Compute the LR(1) start state.

- -0.5 for each incorrect or missing item (no negative marks).

(b) (2 points) Compute all the possible next states for the following state only (compute the GOTO for each symbol, and take the closure of each resulting kernel).

```
A -> C . A, $
A -> . C A, $
A -> . a, $
C -> . d, a/d
```

- -0.5 for each incorrect or missing state (no negative marks).

2. (4 points) Consider the following grammar, and the following LR(1) states (these are just a subset of all the LR(1) states for this grammar).

```
// Grammar
1. S -> E
2. E -> E - T
3. E -> T
4. T -> n
5. T -> l E r

// States
1. {[S -> E ., $], [E -> E . - T, $/-]}
2. {[E -> T ., $/-]}
3. {[T -> n ., $/-]}
4. {[T -> l E . r, $/-], [E -> E . - T, r/-]}
5. {[E -> T ., r/-]}
6. {[T -> l E r ., $/-]}
7. {[T -> l E . r, r/-], [E -> E . - T, r/-]}
```

   (a) (2 points) Draw the LR(1) parsing table for the above states, filling in just the reduces.
      - -0.3 for each incorrect or missing row (no negative marks).

   (b) (2 points) Draw the merged LALR(1) states from those LR(1) states.
      - -0.5 for each incorrect or missing state (no negative marks).

3. (2 points) Draw a control-flow graph for the following code.

```
int i;
for (i = 0; i < 10; i = i + 1) {
    t = i * i;
    if (t > 100) {
        break;
    }
    i = t;
}
print(i);
```

   - -0.5 for each mistake.

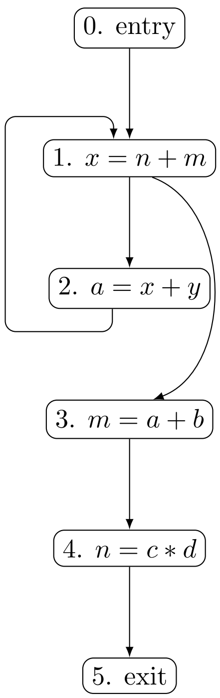
4. (4 points) For busy expressions analysis:
   - A value in the domain is a set of expressions that appear in the program.
   - It is a backwards analysis.
   - $f_s(x) = \text{gen}(s) \cup (x \setminus \text{kill}(s))$.
   - $\text{gen}(s)$ is the set containing the expression of s.
   - $\text{kill}(s)$ is the set of all expressions in the program that have an operand that is assigned to by s.
   - Meet is set intersection.
   - Initialize IN[exit] to be empty. Initialize IN of every other node to be the set of all expressions.

   The iterative algorithm for backwards analysis is as follows.

```
// initialization
while changed {
    for each node s in the CFG {
        OUT[s] = meet IN[s'] for all successors s';
        IN[s] = f_s(OUT[s]);
    }
}
```

Compute 1 iteration of busy expressions analysis, going in *backwards order* (5, 4, 3, 2, 1, 0). *Reminder*: you don't need to do anything for OUT[exit] or IN[entry].

- -0.5 for each incorrect IN or OUT.



5. (2 points) A semilattice consists of the following.

- A domain (a set of values) V.
- A binary meet $\wedge$ operator.
- A distinguished "top" value $\top$.

The meet operator must satisfy the following relations, for all $x, y, z \in V$.

1. $x \wedge x = x$.
2. $x \wedge y = y \wedge x$.
3. $(x \wedge y) \wedge z = x \wedge (y \wedge z)$.

Additionally, for all values $x \in V$, we must have that $\top \wedge x = x$.

Suppose $\top' \in V$ also satisfies $\top' \wedge x = x$ for all $x \in V$. Can $\top'$ be different from $\top$ (does the property for "top" in a lattice uniquely define an element)? Prove or disprove.

- 1 point if you leave this blank; otherwise
- -1 for incorrect reasoning, -0.5 for unclear or incomplete reasoning.

6. (4 points) *In general*, for any set S, a partial order $\leq$ is a binary relation on S such that for all $x, y, z \in S$, the following hold.

   1. $x \leq x$.
   2. $x \leq y$ and $y \leq x$ implies $x = y$.
   3. $x \leq y$ and $y \leq z$ implies $x \leq z$.

   In general, given a partial order $\leq$ on a set S, and a function $f: S \to S$, we say f is *monotonic* if and only if:

   $$x \leq y \text{ implies } f(x) \leq f(y).$$

   Note: in the case of data-flow analysis, the transfer indeed maps values from the domain of a lattice V back to V (it remains to be shown for each analysis that the transfer functions do actually satisfy this property).

   For a (semi)lattice, we can define a partial order on its domain V as follows:

   $$x \leq y \text{ if and only if } x \wedge y = x.$$

   Suppose we have a function f and a lattice with domain V that satisfies the following (for all $x, y \in V$).

   $$f(x \wedge y) \leq f(x) \wedge f(y).$$

   Let $a, b \in V$, such that $a \leq b$. Prove that $f(a) \leq f(b)$.

   For the following parts worth 1 point each:

   - 0.3 points if you leave it blank; otherwise:
   - 0 for incorrect, incomplete, or unclear reasoning.

   (a) (1 point) Rewrite $a \leq b$ in terms of the meet operator.

   (b) (1 point) Rewrite $f(x \wedge y) \leq f(x) \wedge f(y)$ in terms of the meet operator.

   (c) (1 point) Rewrite $f(a) \leq f(b)$ in terms of the meet operator.

   (d) (1 point) Complete the rest of the proof.

7. (3 points) A *partition* P of a set S is a set of subsets of S, such that every element $x \in S$ appears in one and only one subset of S in the partition P.

   For example, given the set $\{a, b, c, d, e\}$, the following are possible (but not all) partitions:

   - $\{\{a, b\}, \{c, d\}, \{e\}\}$.
   - $\{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$.
   - $\{\{a, b, c, d, e\}\}$.

   (Recall a partition is a set of (sub)sets.) Given two partitions P_1 and P_2 of a set S, we first define a partial order:

   $$P_1 \leq P_2 \text{ iff every element of } P_1 \text{ is a subset of an element of } P_2.$$

   Given two elements $x, y \in S$ and a partition P of S, we say $x \cong_P y$ (x and y are equivalent with respect to the partition P) iff x and y are in a single set in the partition P (an element of P is a set).

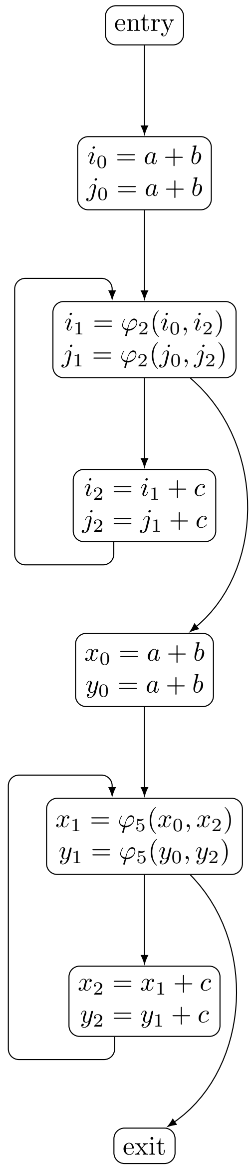
   (a) (1 point) Write down any lower bound for the following two partitions based on the partial order defined.

   - $P_0 = \{\{i_0, j_0, i_1, j_1\}\}$,
   - $P_1 = \{\{i_0, j_0\}, \{i_1, j_1\}\}$.
   - No penalty for error.

   (b) (2 points) Consider a new data-flow analysis for a program in SSA form. Recall that each variable/name in SSA form is unique; a name is defined by a single expression. Let S be the set of all names (and their corresponding expressions) in the program.

   - A value in the domain is a partition of S.
   - The analysis is forwards.
   - The meet of two values P_1, P_2 in the domain is the *greatest lower bound* of P_1 and P_2 (you can imagine computing this by brute force).
   - Assume every name/defining expression s takes the form of $s = \text{op}(u, v)$ where op is a deterministic, side-effect-free function in the program. The transfer function $f_s(P)$ is as follows.
     - s is a name/expression in the program. So it is in S. So it is in exactly one set in the partition P. Designate that set containing s as X.
     - The result $f_s(P)$ will be a partition containing all elements of P except X, and instead of X we will have X_1 and X_2, where we split X into the two sets X_1 and X_2.
     - We split X as follows: start by placing s in X_1. Then for all other names/expressions $s' = \text{op}'(u', v') \in X$:
     - We place s' in X_1 *only if* s and s' have the same operator, $u \cong_P u'$, and $v \cong_P v'$ (all three hold). We place s' in X_2 otherwise. (We loop over X once and that's it.)
     - For example, if $s = a - b$ and P is the set containing $\{s = a - b, y = c - d, z = e - f\}$, $\{a = ..., c = ..., f = ...\}$, and $\{b = ..., d = ..., e = ...\}$:
     - Then $f_s(P)$ contains $\{a = ..., c = ..., f = ...\}$ and $\{b = ..., d = ..., e = ...\}$ verbatim, and $\{s = a - b, y = c - d, z = e - f\}$ needs to be split.
     - We first put $s = a - b$ in X_1.
     - Consider $y = c - d$. It has the same operator. c (the first operand of y) was in the same set as a (the first operand of s) (referring back to P). d was in the same set as b. So we put $y = c - d$ in X_1.
     - Consider $z = e - f$. It has the same operator. But e (z's first operand) is not in the same set as a (s's first operand). So we put $z = e - f$ in X_2.
     - Our result $f_s(P)$ is the set containing $\{s = a - b, y = c - d\}$, $\{z = e - f\}$, $\{a = ..., c = ..., f = ...\}$, and $\{b = ..., d = ..., e = ...\}$.

Finally, consider the following program (suppose a, b, c are some previously defined values).



Consider $+$ to be the usual integer operator/function, and note that φ_2 and φ_5 are different from each other (but the two φ_2 in the same basic block are the same operator, and similarly for φ_5 in the other basic block).

Break the basic blocks into individual nodes per instruction.

Initialize OUT for all nodes to be the partition $\{\{i_0, j_0, i_2, j_2, x_0, y_0, x_2, y_2, i_1, j_1, x_1, y_1\}, \{a\}, \{b\}, \{c\}\}$.

For forwards analysis:

```
// initialization
while changed {
    for each node s in the CFG {
        IN[s] = meet OUT[s'] for all predecessors s';
        OUT[s] = f_s(IN[s]);
    }
}
```

Carry out the data-flow analysis for the first 4 nodes (after splitting the basic blocks) (you should have 4 INs and 4 OUTs).

- 0.5 if you leave it blank, otherwise
- -0.3 points per error.

(c) (1 bonus points) What does this data-flow analysis compute?

- No penalty for error.

(d) (1 bonus points) Why must the φ in different basic blocks be considered different operators/functions (φ_2 and φ_5)? *Hint*: think about how the algorithm could produce incorrect results.

- No penalty for error.