UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO316: Applied Game Theory
Instructor: Christopher R. Dobronyi

Deferred Final Exam
September 2018

Duration: 3 Hours

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

No Aids Allowed

This examination paper consists of **20** pages and **8** questions. Please alert an invigilator to any discrepancy. The number in brackets at the start of each question is the number of points that the question is worth. Answer all questions.

To obtain credit, you must provide arguments that support your answers.

Question	Points
1	17
2	19
3	10
4	14

Question	Points
5	10
6	10
7	8
8	12

Total	100

1. (a) [5] Construct a strategic game with at least one pure strategy Nash equilibrium in which every pure strategy Nash equilibrium is Pareto efficient and no player is indifferent between any two action profiles. Describe your reasoning.

Player 2

		A	B
Player 1	X		
	Y		

(b) [5] Find all mixed strategy Nash equilibria of the following strategic game.

Player 2

		A	B	C
Player 1	X	$3, 2$	$0, 2$	$2, 0$
	Y	$1, 1$	$2, 1$	$2, 2$
	Z	$2, 0$	$3, 3$	$4, 0$

(c) [7] Find all values of α for which there exists at least one action that is strictly dominated by a mixed strategy, for some player. Describe your reasoning.

		Player 2		
		A	B	C
Player 1	X	$0, 0$	$0, \alpha$	$0, 0$
	Y	$0, 0$	$0, 0$	$0, \alpha$

2. There are two people. Each person must choose a real-valued number x such that $0 \leq x \leq 100$. Let x_i denote the choice of person i. The person whose choice is closest to the value $\frac{x_1+x_2}{4}$ wins. Each person prefers winning over tying and tying over losing.

   (a) [3] Formulate this situation as a strategic game.

   (b) [4] Argue that every action profile is Pareto efficient.

(c) [6] Show that every number larger than 50 is weakly dominated by the number 50. (Hint: Start by arguing that the value $\frac{x_1+x_2}{4}$ will never exceed the number 50.)

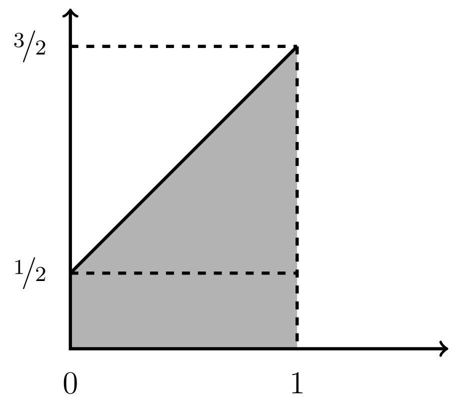
(d) [6] Find a pure strategy Nash equilibrium. (Hint: You can use iterated elimination of weakly dominated actions. If you cannot find a Nash equilibrium then you can describe how to find a Nash equilibrium using iterated elimination of weakly dominated actions for partial marks.)

3. There are eight citizens and three candidates. Citizens 1, 2 and 3 prefer candidate A to B to C. Citizens 4 and 5 prefer candidate B to C to A. Citizens 6, 7 and 8 prefer candidate C to A to B. A single candidate is elected by *single transferable vote*: Each citizen must submit a *ranking of all candidates*. If some candidate has more than 50% of the first-place votes then she wins. Else, remove the candidate with the fewest first-place votes and repeat.

   (a) [3] What is the outcome if each citizen submits her preference ordering as her ranking?

   (b) [7] Suppose that each citizen can submit any ranking. Show that the action profile in which each citizen submits her preference ordering as her ranking is not a Nash equilibrium.

4. Consider a first-price sealed-bid auction with two bidders: Each bidder has a value for an object that is being sold by an auctioneer. Each bidder knows her own value but does not know the value of the other bidder. Each bidder correctly believes that the other bidder's value is drawn independently from the following distribution:

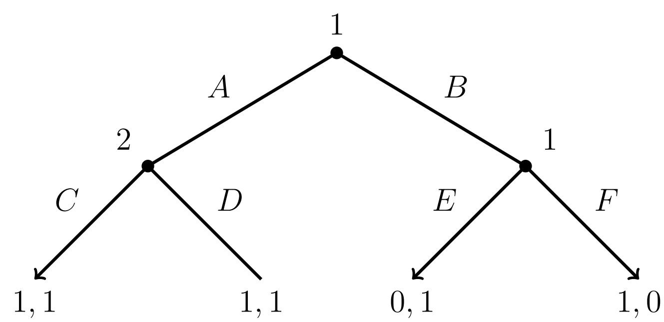


Each bidder chooses a non-negative bid. The bidder with the highest bid wins and receives the object. (You can ignore ties.) If a bidder wins then her payoff is her value minus her bid. If a bidder does not win then her payoff is zero.

(a) [8] Suppose that bidder 2's strategy is to bid βv_2, for each value $0 \leq v_2 \leq 1$ and some $\beta > 0$. Find the expected payoff of bidder 1 when she bids b_1, for each value $0 \leq v_1 \leq 1$.

(b) [6] Does the auction have a Nash equilibrium in which each bidder i's strategy is to bid her value v_i, for each possible value $0 \leq v_i \leq 1$? Explain why or why not.

5. Consider the following extensive game. (Careful: Player 1 moves after two histories.)



(a) [5] Find all Nash equilibria of this extensive game.

(b) [5] Find all subgame perfect equilibria of this extensive game.

6. Consider the following *ultimatum game*: There are two people. Person 1 chooses an offer x such that $0 \leq x \leq 100$. Person 2 observes this offer and chooses whether to *accept* or *reject*. If person 2 chooses to *accept* then player 1's payoff is $u(100 - x)$ and person 2's payoff is $u(x)$, for some strictly increasing utility function u.

   (a) [3] Formulate this situation as an extensive game.

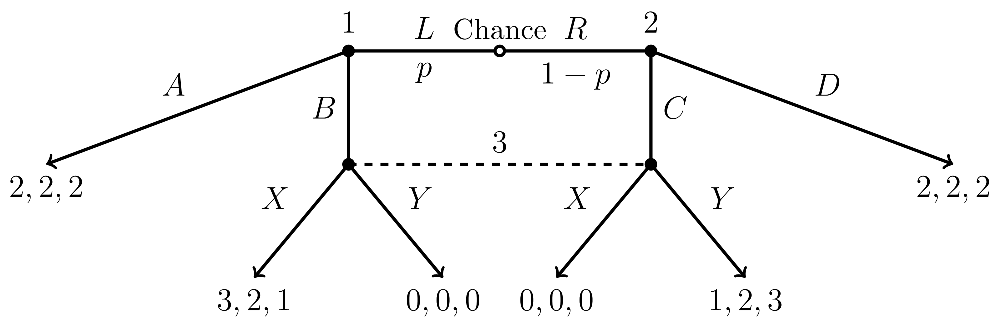
   (b) [3] Does there exist a Nash equilibrium in which person 1 offers a positive amount?

(c) [4] Find all subgame perfect equilibria of this extensive game.

7. [8] Find all values of the common discount factor $0 < \delta < 1$ (in terms of a, b and c) for which there exists a Nash equilibrium in the following infinitely repeated game in which both players use *All-C*. (The strategy *All-C* selects C in every period, for any history.)

		Player 2: C	Player 2: D
Player 1	C	a, a	b, c
	D	c, b	b, b

8. Consider the following extensive game with imperfect information.



(a) [6] Assume $p = \frac{1}{4}$. Is there a weak sequential equilibrium where player 1 chooses B after L?

(b) [6] Is there a value of the probability p for which there exists a (pure strategy) weak sequential equilibrium in which player 1 chooses B after L, and player 2 chooses C after R?

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