UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO316: Applied Game Theory
Instructor: Christopher R. Dobronyi

Final Exam
August 2018

Duration: 3 Hours

Last Name: ____________________

First Name: ____________________

Student Number: ____________________

No Aids Allowed

This examination paper consists of **20** pages and **8** questions. Please alert an invigilator to any discrepancy. The number in brackets at the start of each question is the number of points that the question is worth. Answer all questions.

To obtain credit, you must provide arguments that support your answers.

Question	Points
1	19
2	17
3	10
4	14

Question	Points
5	10
6	10
7	10
8	10

Total	100

1. A crime is observed by a group of n people. Each person must choose whether to report the crime or not. Each person obtains a benefit of b if at least one person reports the crime and a benefit of zero if no person reports the crime. Each person bears a cost of c if she chooses to report the crime and a cost of zero if she chooses not to report the crime. Assume that $0 < c < b$. Each person's payoff is her benefit minus her cost.

   (a) [3] Formulate this situation as a strategic game.

   (b) [3] Suppose that $n = 2$. Is this strategic game Prisoner's Dilemma? Explain.

(c) [5] Suppose that $n \geq 3$. Find all pure strategy Nash equilibria of this strategic game.

(d) [8] Suppose that $n \geq 3$. Find a (symmetric) mixed strategy Nash equilibrium in which each person chooses to report with the same probability. (Hint: The probability that at least one other person reports is equal to one minus the probability that no other person reports.) You can answer the question assuming that there are only two people for partial marks.

2. (a) [5] Find all pure strategy Nash equilibria of the following strategic game. Check whether each equilibrium is Pareto efficient. Describe your reasoning.

Player 2

		A	B	C
Player 1	X	$3, 2$	$2, 3$	$2, 1$
	Y	$1, 1$	$1, 0$	$2, 3$

(b) [7] Does the following strategic game have a mixed strategy Nash equilibrium in which each player assigns a probability of zero to action Z and positive probability to actions X and Y?

Player 2

		X	Y	Z
	X	$2, 1$	$0, 0$	$3, 0$
Player 1	Y	$0, 0$	$1, 2$	$2, 0$
	Z	$0, 5$	$0, 3$	$4, 4$

(c) [5] In the following strategic game, does there exist an action of either player that is strictly dominated by a mixed strategy? Describe your reasoning.

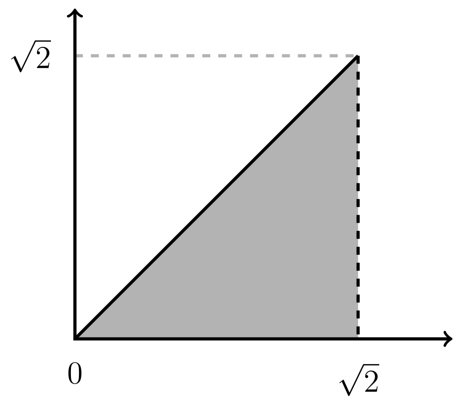
		Player 2		
		A	B	C
Player 1	X	$3, 1$	$0, 3$	$2, 0$
	Y	$1, 1$	$2, 0$	$2, 3$

3. There are three citizens and two candidates. Citizens 1 and 2 support candidate A. Citizen 3 supports candidate B. A single candidate is elected by *approval voting*: Each citizen must choose to vote for a *set of candidates* (citizens cannot abstain) and the candidate with the most votes wins. Each citizen prefers for the candidate that she supports to win over tie over lose.

   (a) [3] Does there exist a Nash equilibrium in which candidate B wins?

   (b) [7] Find all actions that are weakly dominated by another action for citizen 1.

4. Consider a first-price sealed-bid auction with two bidders: Each bidder has a value for an object that is being sold by an auctioneer. Each bidder knows her own value but does not know the value of the other bidder. Each bidder correctly believes that the other bidder's value is drawn independently from the following distribution:

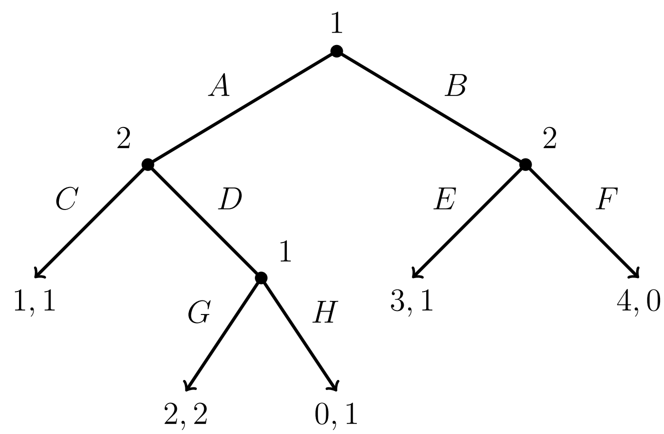


Each bidder chooses a non-negative bid. The bidder with the highest bid wins and receives the object. (You can ignore ties.) If a bidder wins then her payoff is her value minus her bid. If a bidder does not win then her payoff is zero.

(a) [7] Suppose that bidder 2's strategy is to bid βv_2, for each value $0 \leq v_2 \leq \sqrt{2}$ and some $\beta > 0$. Find the expected payoff of bidder 1 when she bids b_1, for each value $0 \leq v_1 \leq \sqrt{2}$.

(b) [7] Does the auction have a Nash equilibrium in which each bidder i's strategy is to bid βv_i, for each value $0 \leq v_i \leq \sqrt{2}$ and some $\beta > 0$? If yes, find the value of β. If no, explain why.

5. Consider the following extensive game.



(a) [5] Find all Nash equilibria of this extensive game.

(b) [5] Find all subgame perfect equilibria of this extensive game.

6. Consider a Bertrand model with sequential choices and only two possible prices: There are two firms. Each firm produces an identical good. The cost of producing a quantity of q is cq for each firm, for some $c > 0$. First, firm 1 chooses a price p_1 in $\{1, 2\}$. Second, firm 2 observes p_1 and chooses a price p_2 in $\{1, 2\}$. Demand for the good is defined by $D(p) = 3 - p$, for all $p \leq 3$. Consumers buy from the firm with the lowest price. (The firms split demand evenly if $p_1 = p_2$.)

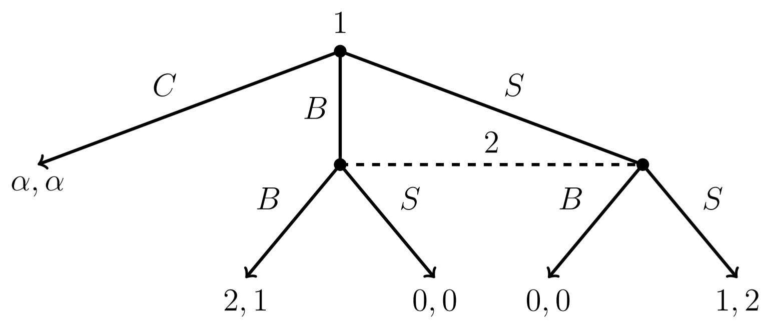
   (a) [3] Formulate this situation as an extensive game.

(b) [7] Find all values of $c > 0$ that yield a subgame perfect equilibrium in which both players end up choosing a price equal to two.

7. [10] Find all values of α for which there exists a Nash equilibrium in the following infinitely repeated game in which both players use *Unrelenting Punishment* and the common discount factor $\delta > 0$ is less than or equal to $1/2$. (*Unrelenting Punishment* selects C initially and, in every future period, selects C if and only if the other player chose C in every previous period. Your final answer should not be in terms of the discount factor δ; you must find the values of α that ensure that a Nash equilibrium exists, for any discount factor δ such that $0 < \delta \leq 1/2$.)

		Player 2	
		C	D
Player 1	C	$2, 2$	$1, \alpha$
	D	$\alpha, 1$	$1, 1$

8. Consider the following variant of *Bach or Stravinsky* with imperfect information in which person 1 moves first and has the option to make an observable cancellation of the event:



(a) [4] Find the values α that yield a weak sequential equilibrium in which both people choose B.

(b) [6] Suppose that $\alpha = 1$. Is there a weak sequential equilibrium in which person 1 chooses C?

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