ECO316: Applied Game Theory

Lecture 9

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Review: Extensive Games

Today we will look at applications of extensive games:

An **extensive game** is a model consisting of
- a set of **players**
- a set of **terminal histories**
- a **player function**
- **preferences** over terminal histories, for each player

Review: Subgame Perfect Equilibrium

A strategy profile is a **subgame perfect equilibrium** (SPE) of an extensive game if every player i's strategy is optimal given the other players' strategies, in every subgame in which player i moves

Strategy profile: A list of strategies with one strategy per player

Review: How to Find a Subgame Perfect Equilibrium

- **Finite horizon**: Length of longest terminal history is finite
- Find an SPE using **backward induction** if finite horizon game

Backward induction:

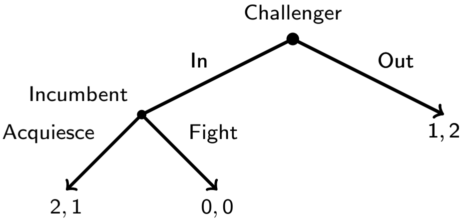
- Find optimal actions in every subgame of **length one**
- Fix one optimal action in each subgame of length one
- Find optimal actions in every subgame of **length two**
- Fix one optimal action in each subgame of length two
- Find optimal actions in every subgame of **length three**. . .

More Options Always Better?

- More alternatives are always better for isolated decision-maker
- The same is **not true** for a player in an **extensive game**: More possible actions at a non-terminal history in which a player moves is not necessarily better for that player in SPE

More Options Always Better? Example

Consider the **entry game** from lecture 8:



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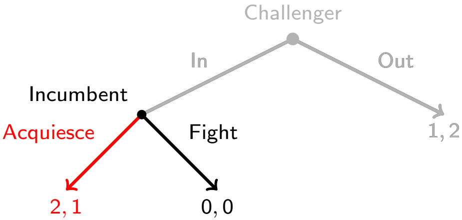

Backward induction:

- **Subgames of length one**:
  - Subgame after In: Acquiesce is optimal for Incumbent

More Options Always Better? Example

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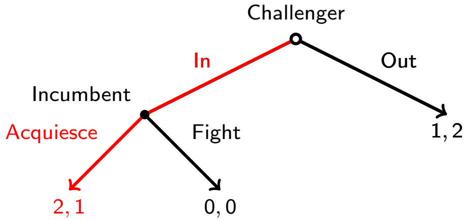
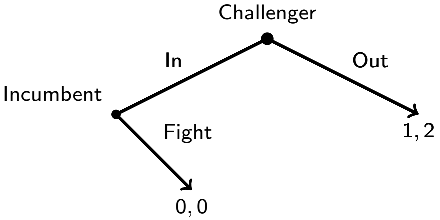


Backward induction:

- **Subgames of length one**:
  - Subgame after In: Acquiesce is optimal for Incumbent
- **Subgames of length two**:
  - Subgame after $\emptyset$: In is optimal for Challenger given Acquiesce
- **SPE**: (In,Acquiesce)

More Options Always Better? Example

Consider a **variant of the entry game** from lecture 8:



More Options Always Better? Example

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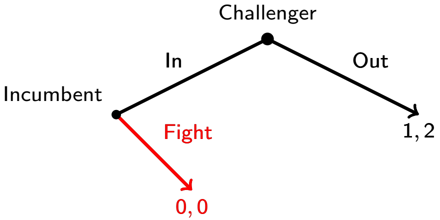

Backward induction:

- **Subgames of length one**:
  - Subgame after In: Fight is optimal for Incumbent (only option)

More Options Always Better? Example

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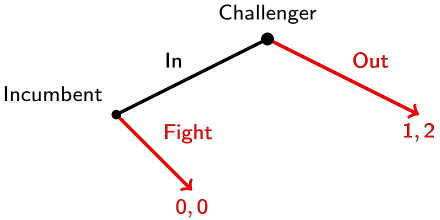


Backward induction:

- **Subgames of length one**:
  - Subgame after In: Fight is optimal for Incumbent (only option)
- **Subgames of length two**:
  - Subgame after $\emptyset$: Out is optimal for Challenger given Fight
- **SPE**: (Out,Fight)

Electoral Competition

Model of electoral competition with sequential entry:

- Candidate 1 chooses a political position
- Candidate 2 observes candidate 1's choice
- Candidate 2 chooses a political position

Electoral Competition: Extensive Game

- **Players**: Two candidates
- **Terminal histories**: all pairs of political positions (x_1, x_2)
- **Player function**: $P(\emptyset) = 1$ and $P(x_1) = 2$, for all x_1
- **Preferences**: win $\succ$ tie $\succ$ lose

- **Usual assumptions** about citizen's preferences and voting

Electoral Competition: Strategies

- **Candidate 1**: political position x_1^*
- **Candidate 2**: political position $x_2^*(x_1)$, for each position x_1

Electoral Competition: Backward Induction

Best position for candidate 2 given x_1:

- If $x_1 = m$ then enter at m and tie
- If $x_1 \neq m$ then enter closer to m and win

Best position for candidate 1 given candidate 2's strategy:

- $x_1 = m \Rightarrow$ candidate 2 will enter at $m \Rightarrow$ candidate 1 ties
- $x_1 \neq m \Rightarrow$ candidate 2 will enter near $m \Rightarrow$ candidate 1 loses
- Best position is m

Electoral Competition: Summary

Every SPE consists of strategies (s_1, s_2) such that

- $s_1(\emptyset) = m$
- $s_2(m) = m$ and $s_2(x_1) = x_2$ closer to m than x_1, for all $x_1 \neq m$

Many strategies satisfy this criterion. Examples:

- $s_1(\emptyset) = m$ and $s_2(x_1) = m$, for all x_1
- $s_1(\emptyset) = m$ and $s_2(x_1) = \frac{m+x_1}{2}$, for all x_1

Stackelberg Model

Variant of Cournot model in which firms act sequentially:

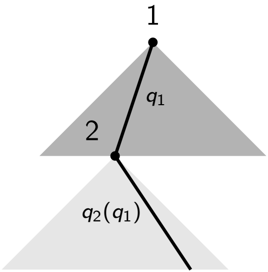
- Firm 1 chooses an output
- Firm 2 observes firm 1's choice of output
- Firm 2 chooses an output

Stackelberg Model: Extensive Game

- **Players**: two firms
- **Terminal histories**: all pairs of outputs (q_1, q_2)
- **Player function**: $P(\emptyset) = 1$ and $P(q_1) = 2$, for all q_1
- **Preferences**: profit for firm i defined by

$$\pi_i(q_1, q_2) = (\alpha - Q)q_i - cq_i, \text{ for } Q \leq \alpha \text{ and some } \alpha < c$$

Stackelberg Model: Strategies



- **Firm 1**: an output q_1^*
- **Firm 2**: an output $q_2^*(q_1)$, for each output q_1

Stackelberg Model: Backward Induction

Firm 2: optimal response $q_2^*(q_1)$ is the solution to

$$\max_{q_2} \left[(\alpha - q_1 - q_2)q_2 - cq_2\right], \text{ for each } q_1$$

As a result, firm 2's strategy is

$$q_2(q_1) = \frac{\alpha - c - q_1}{2}, \text{ for each } q_1$$

Stackelberg Model: Backward Induction

Firm 1: optimal quantity q_1 is the solution to

$$\max_{q_1} \left[\left(\alpha - q_1 - \left(\frac{\alpha - c - q_1}{2} \right) \right) q_1 - c q_1 \right],$$

As a result, firm 1's strategy is

$$q_1^* = \frac{\alpha - c}{2}$$

Stackelberg Model: Summary

Unique subgame perfect equilibrium:

$$q_1^* = \frac{\alpha - c}{2} \quad \text{and} \quad q_2^*(q_1) = \frac{\alpha - c - q_1}{2}$$

Quantities used in equilibrium:

$$q_1^* = \frac{\alpha - c}{2} \quad \text{and} \quad q_2^*(q_1^*) = \frac{\alpha - c}{4}$$

Profits in equilibrium:

$$\pi_1^* = \frac{(\alpha - c)^2}{8} > \frac{(\alpha - c)^2}{16} = \pi_2^*$$

Stackelberg Model: Commitment

Is firm 1's output a best response to firm 2's output? No!

Firm 1's best response $b_1(q_2)$ to quantity q_2 is the solution to

$$\max_{q_1} \left[(\alpha - q_1 - q_2)q_1 - cq_1\right]$$

As a result, firm 1's best response function is defined by

$$b_1(q_2) = \frac{\alpha - c - q_2}{2}$$

and her best response to firm 2's output in SPE is

$$b_1(q_2^*(q_1^*)) = \frac{\alpha - c - \left(\frac{\alpha - c}{4}\right)}{2} = \frac{3(\alpha - c)}{8} \neq \frac{\alpha - c}{2} = q_1^*$$

Stackelberg Model: Commitment

- Firm 1's output is **not a best response** to firm 2's output
- Firm 1 would like to **change its output** after firm 2 moves
- Firm 1 would **not** like the **opportunity** to change its output
    - Suppose firm 1 has opportunity to change output
    - First stage becomes irrelevant
    - Firm 2 effectively becomes the first-mover
    - Firm 1 is worse off in SPE
- Firm 1 prefers to be **committed** to not change its mind

First-Mover Advantage

Consider two games:

- **Strategic game**: Two-players with Nash equilibrium (a_1^*, a_2^*)
- **Extensive game**: Variation in which player 1 moves first

First-mover advantage:

- (a_1^*, a_2^*) Nash equilibrium $\Rightarrow a_2^*$ optimal given a_1^* in strategic game $\Rightarrow a_2^*$ optimal action after history a_1^* in extensive game
- If a_2^* is the only best response to a_1^* in the strategic game then a_1^* guarentees player 1 a payoff of $u(a_1^*, a_2^*)$ in extensive game
- In any extensive game that satisfies this criterion, player 1 gets a payoff $\geq u(a_1^*, a_2^*)$ in every subgame perfect equilibrium

First-Mover Advantage: Example

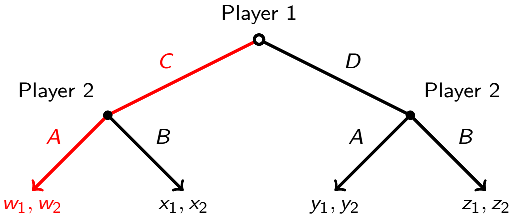
Player 2

Player 1		A	B
	C	w_1, w_2	x_1, x_2
	D	y_1, y_2	z_1, z_2

Suppose that $w_1 \geq y_1$ **and that** $w_2 > x_2$:

- (C, A) is a Nash equilibrium
- A is the only best response to C

First-Mover Advantage: Example



Suppose that $w_1 \geq y_1$ **and that** $w_2 > x_2$:

- (C, A) is a Nash equilibrium
- A is the only best response to C
- Player 2 will choose A if player 1 chooses C
- Player 1 is guarenteed payoff $\geq w_1$ because she can choose C
- Under some payoffs, player 1 can do better by choosing D

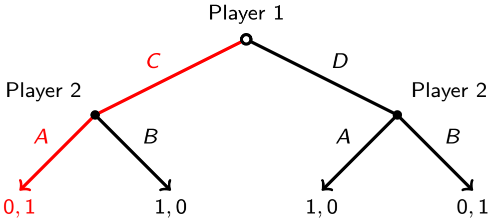
First-Mover Advantage: Another Example

Player 2

Player 1		A	B
	C	$0, 1$	$1, 0$
	D	$1, 0$	$0, 1$

No pure strategy Nash equilibrium

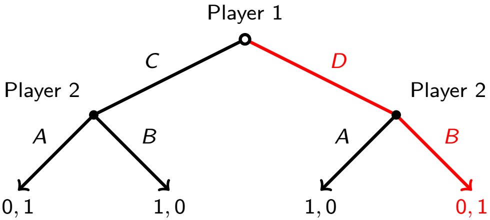
First-Mover Advantage: Another Example



No pure strategy Nash equilibrium

- If player 1 chooses C then player 2 will choose A

First-Mover Advantage: Another Example



No pure strategy Nash equilibrium

- If player 1 chooses C then player 2 will choose A
- If player 1 chooses D then player 2 will choose B
- In both cases, player 1 gets 0 and player 2 gets 1
- Player 2 has the **advantage** (criterion is important)

Ultimatum Game

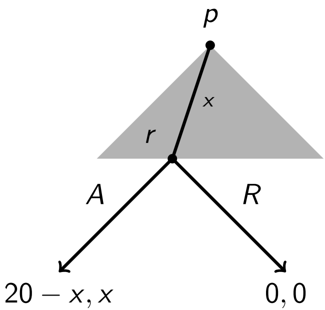
Experiment:

- **Two participants**: One **proposer** and one **responder**
- Proposer **offers** amount x between \$0 and \$20 to responder
- After proposer chooses offer, responder **accepts** or **rejects**
  - **Accept**: proposer earns $(20 - x)$, responder earns x
  - **Reject**: both participants earn 0

Ultimatum Game: Extensive Game

- **Players**: proposer (p) and responder (r)
- **Terminal histories**: (x, A) and (x, R) such that $0 \leq x \leq 20$
- **Player function**: $P(\emptyset) = p$ and $P(x) = r$, for all x
- **Preferences**:
  - $u_p(x, A) = 20 - x$ and $u_r(x, A) = x$
  - $u_p(x, R) = u_r(x, R) = 0$

Ultimatum Game: Strategies



- **Proposer**: an offer x^*
- **Responder**: a response A or R, for each offer x

Ultimatum Game: Backward Induction

Responder: two optimal strategies s_r and s_r' such that

$$s_r(x) = A, \quad \text{and} \quad s_r'(x) = \begin{cases} A, & \text{if } x > 0, \\ R, & \text{if } x = 0 \end{cases}$$

Proposer: responder uses $s_r \Rightarrow$ optimal for proposer to use

$$s_p(\emptyset) = 0,$$

and, responder uses $s_r' \Rightarrow$ no optimal strategy for proposer

Ultimatum Game: Summary

Subgame perfect equilibrium:

- Unique SPE
- $s_p(\emptyset) = 0$ and $s_r(x) = A$, for all x

Ultimatum Game: Experimental Evidence

Güth et al. (JEBO, 1982):

- Participants: graduate students in economics
- Many offers greater than 0
- About 20% of offers rejected
- Conclusion in paper: students not familiar with game theory
- But these are common findings
- Behavioural explanations? equity, fairness, spite, etc.

Ultimatum Game: Experimental Evidence

Henrich et al. (AER, 2001): (large maximum amount)

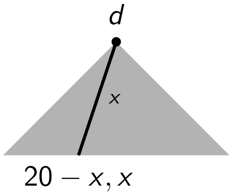
Group	Country	Avg. Offer	Rejection Rate
Machiguenga	Peru	26%	5%
Torguud	Mongolia	35%	5%
Tsimané	Bolivia	37%	0%
Sangu	Tanzania	41%	10%
Lamalera	Indonesia	58%	0%

Ultimatum Game: Experimental Evidence

Proctor et al. (2013): (participants are chimpanzees; two possible offers: one equitable division, one inequitable division)

Pair	% equitable choices
1	58%
2	71%
3	67%
4	92%

Ultimatum Game: Experimental Evidence



Forsythe et al. (1998):

- Dictator game: dictators offers, responder must accept
- Observe same results if ultimatum results are purely fairness
- Many still offer greater than 0
- But dictators offer less than proposers

Ultimatum Game: Experimental Evidence

Alternative explanation:

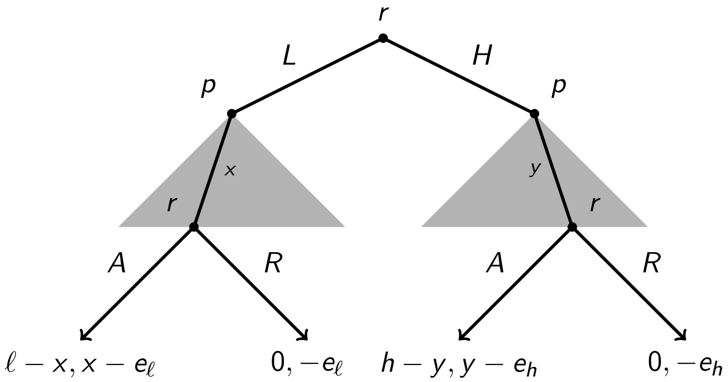
- Fear that responder will reject
- Consistent with fact that responders reject
- Why do responders reject?
- May not comprehend isolated nature of game
- Decisions often shaped by long-term relationships
- In long-term relationship, benefit to punishing low offers

Holdup Game

Variant of ultimatum game:

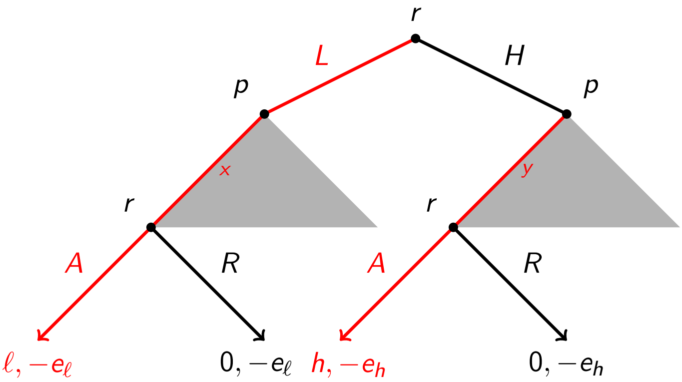
- Before playing ultimatum game, responder chooses effort level
- Responder can choose high effort or low effort
- Responder chooses high effort $\Rightarrow$ amount to split is larger
- Effort is costly

Holdup Game: Extensive Game



Assumtions: $\ell < h$ and $e_\ell < e_h$
Homework: Formulate the holdup game as an extensive game

Holdup Game: Backward Induction



- Responder accepts any offer, in each ultimatum game
- Proposer offers 0, in each ultimatum game
- Responder chooses L at start of game

Holdup Game: Summary

- Responder “held up” for all surplus her extra effort produces
- Responder will not choose to exert extra effort
- SPE is inefficient if $h - e_h > \ell - e_\ell$