UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO316: Applied Game Theory
Instructor: Christopher R. Dobronyi

Make-Up Midterm Exam
July 26, 2018

Duration: 1 Hour and 50 minutes

Last Name: ________________________________

First Name: ________________________________

Student Number: ________________________________

No Aids Allowed

This examination paper consists of **14** pages and **5** questions. Please alert an invigilator to any discrepancy. The number in brackets at the start of each question is the number of points that the question is worth. Answer all questions.

To obtain credit, you must provide arguments that support your answers.

Question	Points
1	6
2	30
3	23
4	13
5	18
Total	90

1. (a) [3] An individual has preferences over all non-negative integers. She prefers the number ten the most. She prefers every number larger than 10 to every number smaller than 10. She is indifferent between all numbers larger than 10 and indifferent between all numbers smaller than 10. Construct a utility function that represents her preferences.

   (b) [3] An individual has preferences over all non-negative integers. She prefers every smaller number to every larger number. Argue that her preferences are transitive.

2. (a) [6] Find all pure strategy Nash equilibria of the following strategic game. Is this strategic game Prisoner's Dilemma? Describe your reasoning.

		Player 2	
		X	Y
Player 1	X	$7, 4$	$1, 9$
	Y	$8, 0$	$2, 1$

(b) [6] Construct a strategic game with at least one pure strategy Nash equilibrium in which every pure strategy Nash equilibrium is Pareto efficient and no player is indifferent between any two action profiles. Describe your reasoning.

		Player 2	
		A	B
Player 1	X		
	Y		

(c) [12] Find all mixed strategy Nash equilibria of the following strategic game. (Recall: A player can assign a probability of 1 to a single action in a mixed strategy Nash equilibrium.)

Player 2

Player 1		A	B
	X	$2, 1$	$1, 1$
	Y	$0, 1$	$1, 2$

(d) [6] Find all action profiles in the following strategic game that survive iterated elimination of strictly dominated actions. Find all pure strategy Nash equilibria of the remaining game.

		Player 2		
		A	B	C
	X	$3, 2$	$0, 2$	$2, 0$
Player 1	Y	$1, 1$	$2, 1$	$2, 2$
	Z	$2, 0$	$3, 3$	$4, 0$

3. There are two people. Each person must choose a real-valued number x such that $0 \leq x \leq 100$. Let x_i denote the choice of person i. The person whose choice is closest to the value $\frac{x_1+x_2}{4}$ wins. Each person prefers winning over tying and tying over losing.

   (a) [3] Formulate this situation as a strategic game.

   (b) [5] Argue that every action profile is Pareto efficient.

(c) [8] Show that every number larger than 50 is weakly dominated by an action.

(d) [7] Find a pure strategy Nash equilibrium. (Hint: Use iterated elimination of weakly dominated actions. If you are unable to find a Nash equilibrium then you can describe how to find a Nash equilibrium using iterated elimination of weakly dominated actions for three marks.)

4. [13] Consider a Bertrand model with *three firms* and integer-valued prices. Each firm produces an identical good. The cost of producing a quantity of q is $C_i(q) = cq$, for each firm i, for some marginal cost $c > 0$. Firm i can choose any price p_i in cents (e.g. 1 cent, 2 cents, etc.). Demand for the good is defined by $D(p) = \alpha - p$, for all $p \leq \alpha$, for some $\alpha > c$. Consumers buy from the firm with the lowest price. (If $k > 1$ firms share the lowest price then each of these firms sells a quantity of $D(p)/k$.) Find all symmetric pure strategy Nash equilibria. (That is, find all pure strategy Nash equilibria in which $p_1 = p_2 = p_3$.)

5. Consider an election with *three* candidates. Each candidate can choose whether to enter the election or not. Every candidate that enters the election must choose a real-valued political position. There is a large number of citizens. Each citizen has a favourite political position. The distribution of citizens' favourite political positions is continuous with a unique median. Each citizen votes for the candidate whose political position is closest to her favourite political position. The candidate with the most votes wins. Each candidate prefers winning over tying, tying over not entering and not entering over losing.

   (a) [3] Show that all candidates must tie in any Nash equilibrium in which more than one candidate chooses to enter the election.

   (b) [3] Is there a Nash equilibrium in which three candidates enter at the same position? Explain.

(c) [12] Show that there does not exist a Nash equilibrium in which three candidates enter and at least two candidates have different positions. Describe your reasoning.

This page is for rough work and will not be graded.

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