UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO316: Applied Game Theory
Instructor: Christopher R. Dobronyi

Midterm Exam
July 19, 2018

Duration: 1 Hour and 50 minutes

Last Name: ________________________________

First Name: ________________________________

Student Number: ________________________________

No Aids Allowed

This examination paper consists of **14** pages and **5** questions. Please alert an invigilator to any discrepancy. The number in brackets at the start of each question is the number of points that the question is worth. Answer all questions.

To obtain credit, you must provide arguments that support your answers.

Question	Points
1	6
2	30
3	23
4	13
5	18
Total	90

1. (a) [3] An individual has preferences over all non-negative integers. She prefers every positive number to zero. When comparing a pair of numbers different from zero, she prefers the number that is closer to zero. Construct a utility function that represents her preferences.

   (b) [3] An individual has preferences over all non-negative integers. She prefers every even number to every odd number and every larger number to every smaller number. Assume that there exists a utility function that represents her preferences. Find a contradiction.

2. (a) [6] Find all pure strategy Nash equilibria of the following strategic game. Check whether each equilibrium is Pareto efficient. Describe your reasoning.

		Player 2	
		A	B
Player 1	X	$1, 1$	$2, 4$
	Y	$1, 1$	$2, 1$

(b) [6] Is the indicated pair of mixed strategies a mixed strategy Nash equilibrium of the following strategic game? Describe your reasoning.

		Player 2	
		$A(1/2)$	$B(1/2)$
Player 1	$X(1/2)$	$1, 2$	$0, 0$
	$Y(1/2)$	$0, 0$	$1, 1$

(c) [12] Find all mixed strategy Nash equilibria of the following strategic game. (Recall: A player can assign a probability of 1 to a single action in a mixed strategy Nash equilibrium.)

		Player 2	
		A	B
Player 1	X	$3, 1$	$0, 1$
	Y	$1, 1$	$2, 2$

(d) [6] Construct a strategic game with at least one pure strategy Nash equilibrium in which exactly one player's action in equilibrium is weakly dominated by another action and exactly one player's action in equilibrium strictly dominates another action. Describe your reasoning.

Player 2

Player 1		A	B
	X		
	Y		

3. Consider a generalization of the traveler's dilemma: An airline has lost the suitcases of two travelers. The airline asks each traveler to specify a value for her suitcase in $\{2, \ldots, 100\}$. If the travelers specify the same value then each traveler is paid that value. If the travelers specify different values then the traveler with the smaller value is paid her value plus $x \geq 0$ and the traveler with the larger value is paid the smaller value minus x.

   (a) [3] Formulate this situation as a strategic game.

   (b) [4] Suppose $0 < x < 1$. Is specifying a value equal to 3 weakly dominated by specifying a value equal to 2? Describe your reasoning.

(c) [8] Suppose $x = 1$. Find all pure strategy Nash equilibria.

(d) [8] Does there exist a value of $x \geq 0$ that yields a Nash equilibrium in which the travelers specify different values? Explain.

4. [13] Consider a Cournot model with fixed cost: There are two firms. Each firm produces an identical good. Firm i can choose to produce any non-negative quantity q_i. Price is determined by inverse demand defined by $P(Q) = \alpha - Q$ for $Q \leq \alpha$ in which $Q = q_1 + q_2$ denotes total output. The cost of producing a quantity of q is

$$C_i(q) = \begin{cases} f + cq, & \text{if } q > 0, \\ 0, & \text{if } q = 0, \end{cases}$$

for firm i, for some fixed cost $f > 0$ and marginal cost c such that $0 < c < \alpha$. Find all values of $f > 0$ that yield a Nash equilibrium in which both firms choose a positive quantity. (Hint: Assume both firms choose a positive quantity. Find the Nash equilibrium. Input the equilibrium quantities into the profit function. Find the values of f that make this profit non-negative.)

5. Consider an election with *three* candidates. Each candidate can choose whether to enter the election or not. Every candidate that enters the election must choose a real-valued political position. There is a large number of citizens. Each citizen has a favourite political position. The distribution of citizens' favourite political positions is continuous with a unique median. Each citizen votes for the candidate whose political position is closest to her favourite political position. The candidate with the most votes wins. Each candidate prefers winning over tying, tying over not entering and not entering over losing.

   (a) [3] Is there a Nash equilibrium in which no candidate enters? Explain.

   (b) [3] Is there a Nash equilibrium in which exactly one candidate enters? Explain.

(c) [12] Is there a Nash equilibrium in which exactly two candidates enter? Explain.

This page is for rough work and will not be graded.

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